THE QUANT EDGE:
Testing trading systems on data randomly generated by stochastic models

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1 Introduction

1.1 Background

The investment industry has, in recent years, become one of the most technologically advanced industries in the North American economy. In Canada and the United States alone, investment-related activities amounts for up to 20% of national GDP. Due to the emergence of hedge funds and arbitrage houses, fundamental analysis, that is, for example, evaluating purchasing an asset based on corporate valuation and understanding the business, is largely becoming secondary to quantitative methods that model the behavior of financial assets. Quantitative stochastic analysis seeks to use mathematical and statistical models to not only fit a parameter-based model to historical asset time series data, but also to simulate additional data for testing trading algorithms, since time series data is relatively scare in the marketplace, as many new stocks appear on a weekly basis on the NASDAQ as new companies go public. Indeed, stochastic models can even be used to predict future movement of stock prices, although great care must be taken when risking investors' money in doing so, as choice of model can drastically affect probabilistic measures of future asset prices.

Additionally, in an increasingly energy-dependent economy, the trading of futures has become a huge industry. Hedging is tremendously important not only to businesses attempting to lock in prices, but also to governments seeking concrete strategies to countering economic exposure to energy prices. However, just as insurance policies incur a cost in the payment of premiums, so does hedging. Many trading firms are also arbitrage houses, following classic trend following algorithms. These firms identify a rather unique characteristic of commodities markets in general: mean reversion. Mean-reverting assets tend to return to some value over long periods of time. Clearly, a commodity which has some intrinsic value on the marketplace and global economy may expect to have futures prices revert to this value, up to inflation effects and macroeconomic shifts. Supply and demand generated fluctuations, sometimes periodic, sometimes not, are generally the principal drivers of the fluctuations in prices of commodities futures. There are a number of stochastic models which exhibit mean reversion which are used in modeling these unique assets.

Yet another motivation for stochastic modeling is the growth of the usage of volatility-based strategies by arbitrage houses. Volatility-based strategies pay little attention to whether an asset is increasing or even maintaining value over the horizon of say, several years, but rather seek profit by short term holds, buying and selling the stock and capitalizing on price fluctuations within a few weeks, days, or even within a day (referred to as day trading). Clearly, more price fluctuations, more volatility in stochastic math parlance, implies more potential for profits. Modeling periods of high volatility identifies not only potential for profit, but also the risk of decreased volatility, which may trap an arbitrageur in an unfavorable position from which he must unload his assets. [1, 3, 4, 7]

1.2 Objectives

The purpose of this investigation is to collect evidence on the subject of whether or not randomly simulated data is reflective of actual asset paths. This is a complex question whose answer lies in recognizing the fundamental assumptions of investment and trading firms which employ large-scale stochastic analysis and proving of trading systems.

The underlying hypothesis of advocates of random asset data generation is as follows: Randomly simulated data based on parameters obtained by fitting a stochastic model to historical stock data can serve as a “good” proving ground for trading strategies. We shall test the validity of this hypothesis; of course, concerning what exactly the word “good” means in various areas of the trading industry. We will test whether the returns obtained using basic trading strategies on simulations are comparable to running those strategies on actual market data. We shall present conclusions to our results.

We are focusing on answering three main questions in this project:

1. Will trading strategies proven to perform well (or poorly) on randomly simulated data perform comparably well (or poorly) on actual future market data?
2. Does past asset behavior serve as a good indicator for future asset behavior in terms of trading strategy performance?
3. Which stochastic models among our list of models generate random data that maintains realistic behavior?

1.3 Implications

The outcomes we shall present are very relevant to the investment industry. A positive answer to Questions 1 and 2 makes the case for the value of stochastic modeling in the investment industry. Also, positive support for specific models addressed in Question 3 should encourage investors with goals similar to those outlined above to consider stochastic analysis as a core element of their investment strategy development and risk management. [1]

2 Methodology

We select a number of models described in Section 4 and calibrate them to some sets of data. We select historical prices of commodities and equities dating from before the first trading day of Jan 1, 2012. We will calibrate models like the Geometric Brownian Motion (GBM) or Ornstein Uhlenbeck (OU) process to the data. Next, we simulate a number of possible future asset paths using our calibrated stochastic models. Then we run a series of trading algorithms related by one parameter on the simulated data, and average out the return performance of the algorithm for each value of that parameter. Finally, we run the trading algorithm for each parameter on the actual data realized in the stock market since Jan 1, 2012. We shall compare and analyze qualitative and quantitatve correlation between the performance of the algorithms on the simulated data with the performance of the same algorithms on the actual realized market data.

3 Data sources

Below are listed the assets used, together with which models we fit to the pre-Jan 1, 2012 data:

3.1 Equity

We have selected Apple Computers (AAPL - NASDAQ, Jan 1, 2012 to April 12, 2016) and TransCanada Corporation (TRP - TSX, Jan 1, 2012 to April 12, 2016), all easily found on Yahoo! Finance, to fit GBM and GMRP models to.

3.2 Commodities

We have selected daily Brent crude spot closing prices (Chicago Mercantile Exchange - CME Jan 1, 2012 to June 19, 2014) and daily IPE Natural Gas Index closing prices (CME, Jan 1, 2012 to June 19, 2014). This data was easily found on the RBS (Royal Bank of Scotland) website free of charge. We will fit generalized Ornstein Uhlenbeck mean reverting processes to this data.

4 Models used and calibration methods

4.1 Non-mean reverting models

One of the most commonly used equity models is the Geometric Brownian Motion (GBM). A GBM is the solution S_t to the following stochastic differential equation (SDE):

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{1}$$

where μ and σ are the mean and variance parameters, respectively, t is time, and W_t is a standard Brownian motion.
A standard Brownian motion is a continuous function W_t such that for any acceptable values of t,

$$W_t - W_s = W(t) - W(s) \sim \mathscr{N}(0, 1)$$

i.e., is a normally distributed random variable.
The solution to the SDE (1) is derived using Ito's formula (this is a standard derivation in stochastic calculus), resulting in the following solution:

$$S_t = S_0 \exp\left((\mu - \frac{\sigma^2}{2})t + \sigma W_t\right) \tag{2}$$

where W_t is a standard Brownian motion.

Calibration

Under a GBM model as in (1) we have that the log returns

$$\ln\left(\frac{S_{t+1}}{S_t}\right)$$

are normally distributed with mean $\mu$and variance σ. Thus, given an data series equally spaced in time S_i, $i = 0, 1, 2, ..., n$ we may examine the data series

$$\ln\left(\frac{S_{i+1}}{S_i}\right),\ i = 0, 1, 2, ..., (n-1)$$

and calculated the mean and variance to estimate the GBM parameters μ and σ. [11, 3]
We provide both a calibration and simulation code for GBM in Sections 10.1 and 10.2 respectively.

4.2 Mean reverting models

In modeling assets such as commodities, it is desirable to reflect the phenomenon of mean reversion; that is, the tendency to revert to a long term average price. To this end, we use the generalized Ornstein-Uhlenbeck (OU) process, which is the solution to the following SDE:

$$dS_t = \lambda(\mu - S_t)dt + \sigma dW_t \tag{3}$$

where λ is the mean reversion rate, $\mu$is the mean, and $\sigma$is the volatility.
We can rewrite (3) as

$$e^{\lambda t}(dS_t + \lambda S_t dt) = e^{\lambda t}(\lambda\mu dt + \sigma dW_t)$$

whence

$$d(e^{\lambda t}S_t) = e^{\lambda t}(\lambda\mu dt + \sigma dW_t)$$

and integrating from 0 to t on both sides yields

$$e^{\lambda t}S_t - S_0 = \int_0^t e^{\lambda t}\lambda\mu dt + \int_0^t e^{\lambda t}\sigma dW_t$$

so we have

$$S_t = S_0 e^{-\lambda t} + e^{-\lambda t}(e^{\lambda t} - 1)\mu + e^{-\lambda t}\int_o^t e^{\lambda t}\sigma dW_t$$

or

$$S_t = S_0 e^{-\lambda t} + (1 - e^{-\lambda t})\mu + e^{-\lambda t}\int_o^t e^{\lambda t}\sigma dW_t$$

from where we calculate the mean of S_t given S_0 as the following:

$$S_0 + \mu(1 - e^{-\lambda t})$$

and with variance (use Ito's isometry, see [9]) of S_t given S_0 as the following:

$$\sigma\sqrt{\left(\frac{1 - e^{-2\lambda t}}{2\lambda}\right)}$$

Hence, (3) yields the following representation:

$$S_t = S_0 e^{-\lambda t} + \mu(1 - e^{-\lambda t}) + \sigma\sqrt{\left(\frac{1 - e^{-2\lambda t}}{2\lambda}\right)}\mathcal{N}(0,1) \tag{4}$$

where $\mathcal{N}(0,1)$ is the normally distributed random variable with mean 0 and variance 1.
We will use this representation in our calibration methods that follow. [12, 3]

Calibration

Our representation of S_t, the solution to (3), in (4) leads to the following relationship between S_{i+1} and S_i, where S_i are the daily close prices:

$$S_{i+1} = aS_i + b + \epsilon$$

Note that using daily close prices will yield a constant time step of 1 period (1 day). Fitting a linear regression yields

$$a = e^{-\lambda}$$
$$b = \mu(1 - e^{-\lambda})$$
$$sd(\epsilon) = \sigma\sqrt{\left(\frac{1-e^{-2\lambda}}{2\lambda}\right)}$$

which yields explicit formulas for λ, μ, and σ for our OU model as:

$$\lambda = -\ln a$$
$$\mu = \frac{b}{1-a}$$
$$\sigma = sd(\epsilon)\sqrt{\frac{-2\ln a}{(1-a^2)}}$$

We provide both a calibration and simulation code for OU in Sections 10.3 and 10.4 respectively.

4.3 Markov-modulated models

A discretized Geometric Markov Renewal Process (GMRP) is a Markov-modulated asset model. It can be approximated under a variety of assumptions to behave essentially as a GBM, as shown in [8]. But why use such a model, where volatility in the purest form of the model does not even have a convenient form to work with? GMRP's primary redeeming factor is that it incorporates positive market feedback. Since a GBM is derived in one manner as the limiting case as the number of interval subdivisions goes to infinity of the classical, celebrated Cox-Ross-Rubenstein binomial asset model, it is clear that phenomena such as running up the ask price or running down the bid price are not incorporated into a GBM or binomial model's fundamental assumptions. Also, buying sprees or frenzied selling sprees are common features as well. Phenomena such as these are often found in financial markets such as highly liquid equities and penny stocks.

We focus on fixed time increment discretized GMRP in this study (henceforth referred to as simply GMRP). Suppose we are modeling an asset on the time period $[0, T]$ with N subdivisions of size δ. Then the GMRP model S_t is given by

$$S_t = S_0 \prod_{k=1}^{C(t)} (1 + \rho(x_k)) \tag{5}$$

where S_0 is the starting asset price at time $t = 0$, $C(t)$ is the number of subdivisions of size δ contained in the interval $[0, t]$, rounded down, $x_k,\ k = 1, 2, ..., C(t)$ is a Markov chain with n states with transition matrix M, and $\rho_k := \rho(k),\ k = 1, 2, ..., n$ is a function defined on the n states of the Markov chain with transition matrix M, and $\rho_k > -1,\ k = 1, 2, ..., n$.

Calibration

The GMRP model can be calibrated in one way as follows: We take all the observed returns $S_i/S_{i-1},\ i = 2, 3, ..., k$ for k observations. We find the 0, 25, 50, 75, and 100th percentiles of the data and bin them as 1,2,3,4,5 - the states of our Markov chain. We define ρ on each Markov state by the average of those returns in that Markov state bin.
To calibrate the Markov chain, we have the standard MLE (maximum likelihood estimator) method we have the following estimates for the $(i, j)^{th}$ entry of the transition probability matrix M:

$$M_{i,j} \approx \hat{p}_{i,j} = n_{ij} / \sum_{j=1}^{N} n_{ij}$$

where n_{ij} is the number of times the observed Markov chain enters state j from state i, and there are N Markov states in total.
This is one way to calibrate a GMRP, and this is the method we shall use in this paper. [2, 8]
We provide both a calibration and simulation code for GMRP in Sections 10.5 and 10.6 respectively.

4.4 Stochastic volatility modeling

One of the most commonly used mean mean-reverting methodologies to model stochastic volatility is GARCH (Generalized Autoregressive Conditional Heteroskedasticity). The idea behind GARCH is that the volatility on a given day is a function of the volatilities on previous days and the returns on previous days.
Clearly, we have that for a data set of returns (or log returns, depending on the choice of model) $x_t,\ t = 0, 1, 2, ...$ we have that, after removing the mean, the returns x_t/x_{t-1} look like $\sigma_t \epsilon_t$. A GARCH (p, q) model models the volatility at time t as follows:

$$\sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i x_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma_{t-j}^2, \quad a_i, b_j \geq 0 \tag{6}$$

We will focus on the particular case where we only consider the most recent time step. This is where $p = 1$ and $q = 1$. This results in Equation (6) appearing in what we call the GARCH (1,1) formulation as

$$\sigma_t^2 = a_0 + a_1 x_t^2 + b_1 \sigma_t^2, \quad a_1, b_1 \geq 0 \tag{7}$$

Under GARCH (1,1), the conditional expectation given time-t information is:

$$E_t[\sigma^2_{t+n}] = a_0\Big(\frac{1-(a_1b_1)^n}{1-a_1-b_1}\Big) + (a_1+b_1)^n\sigma^2_t$$

Stability occurs when

$$a_1 + b_1 < 1$$

and convergence of forecasts of volatility occurs when

$$\lim_{n\to\infty} E_t[\sigma^2_{t+n}] = \frac{a_0}{1-a_1-b_1}$$

with average conditional variance given by

$$E[x^2_{t+1}] = a_0 + a_1E[x^2_t] + b_1E[\sigma^2_t]$$

hence

$$E[x^2_t] = \frac{a_0}{1-a_1-b_1}$$

GARCH (1,1) volatility is one of a number of volatility models used, often in conjunction with a specific model for the asset path. For practical applications, we will not focus on creating a comprehensive stochastic mathematical model, but rather, we include GARCH methodology as part of a complete discussion of stochastic models, and to indicate how GARCH volatility modeling can add a layer of realism to an easy-to-apply model. [6, 10] Below we present a method of calibrating a time series with removed mean to the GARCH (1,1) model.

Calibration

We proceed with GARCH (1,1) calibration with a standard MLE (Maximum Likelihood Estimator) method.
We write the log-likelihood function as

$$\mathscr{L}(\theta) = \sum_{t=1}^{T} \ln\Big(p(x_t|\sigma_t:\theta)\Big)$$

where $p(x_t|\sigma_t:\theta)$ is the normal density

$$p(x_t|\sigma_t:\theta) = \frac{1}{\sqrt{2\pi\sigma^2_t}}\exp\Big(-\frac{x^2_t}{2\sigma^2_t}\Big)$$

i.e.

$$\mathscr{L}(\theta) = \sum_{t=1}^{T} -\ln\sqrt{2\pi} - \frac{x^2_t}{2\sigma^2_t} - \frac{1}{2}\ln(\sigma^2_t)$$

where

$$\sigma_t^2 = a_0 + a_1 x_{t-1}^2 + b_1 \sigma_{t-1}^2$$

and for σ_1, to start off, we can use the fitted parameter for σ from the standard GBM calibration (see Section 4.1), if that is our model, for example.
So, we can phrase our problem as an optimization problem with constraints (due to stability concerns) as follows:

$$\hat{\theta} = \arg \max_{(a_0, b_1, b_1)} \mathscr{L}(\theta), \text{ where } a_1 \geq 0, b_1 \geq 0, a_1 + b_1 < 1$$

The above GARCH (1,1), as aforementioned, can be used individually to model volatility or in conjunction with another model, such as GBM [6, 10]. We have included volatility modeling for completion and for limited discussion purposes. However, as volatility-based strategies are, as earlier explained, intended for shorter-term trading strategies, they fall outside the scope of the types of trading strategies we intend to focus on, which are longer term trend following and moving average crossovers strategies, as outlined in Section (5).

5 Trading algorithms

The investment industry is huge and diverse. Many firms approach investment with a fundamental philosophy, whereby key indicators of a business' financial health and growth potential are examined, primarily from financial statements. Many other traders focus on quantitative trading, using trading strategies or trading algorithms, which is our primary goal to examine in this paper. Indeed, most investment firms use a combination of both approaches. Just as the industry is diverse, so also trading systems and trading algorithms used in the investment industry are very, very many and extremely varied. It would be nigh impossible to describe all kinds, so we stick to the simple ones. One thing that most trading algorithms have in common is the notion of generating buy and sell signals. These signals are generally indicated by some quantitative time-series based statistic achieving or exceeding a certain level, or perhaps the crossing over of two such time series. Most investors separate funds into two categories: those that include short selling (that is, selling negative amounts of a stock, a somewhat more risky venture than "going long", as it often requires a margin account) and those which do not. We stick to long-only strategies. The most commonly used long position strategies are moving average crossovers. These involve the price S_t breaking through a n-day moving average as a buy signals, while the reverse would be a sell signal. Most strategies include a stop-loss figure, such as an immediate sell if funds fall below a certain percentage of the initial investment. [5]
Below are two very general trading algorithms we will use to study the quality of randomly simulated data in this paper.

5.1 Price crossing MA1

Let MA1 be some n-day moving average. One commonly used buy signal is generated when the price S_t crosses above MA1. We adopt the algorithm so that the price should stay above MA1 for 2 trading days to generate a buy signal, and stay below 2 days to generate a sell signal. This helps avoid "false signals". We also maintain the convention of generating a sell signal when the asset price falls below some stop-loss percentage, say 90%, of what it was purchased for. It should be noted that setting n too low yields too many false signals and hasty buy decisions, often resulting in losses. Similarly, setting n too high may result in an over-conservative or overly risk-averse approach to investing, resulting in losing out on chances for profit.

5.2 MA1 crossing MA2

Another commonly used moving average crossover strategy is one where two moving averages are used, MA1 and MA2. A buy signal is generated when MA1 crosses above MA2. We adopt this algorithm where the number of days taken into consideration for the moving average MA2 is twice that of MA1. Similarly to the first algorithm presented, we attempt to reduce false signal generation by requiring the crossover to be maintained for 2 trading days before the signal is generated. We again maintain that a sell signal is generated when the asset price falls below some stop-loss percentage of what it was purchased for. Similar behavior for selection for n is observed in this algorithm as well.

We have Octave codes to implement both of the above algorithms presented in Section 10.

6 Trading system testing on random data

We will examine the performance of moving average strategies as outlined in Section 5 by running OCTAVE codes to define our data sets of vectors of price paths for our selected assets. For a given stochastic model, we execute a MA1 crossover (see section 5.1) algorithm for number of different n-values on exactly 6 sets of $randomly\, simulated$ data calibrated to data ranging from the first trading day of 2008 through to the last trading day of 2011, together with the $actual\, data\, realized$ over the first trading day of 2012 to the last trading day of 2013. We do the same for a MA2/MA1 crossover (see section 5.2) trading algorithm. We present the results in tables.

The idea is that if an investment company only had access to the 2008-2011 data, and had to select the n-parameter in their MA1 or MA1/MA2 strategy, they would use the simulated data to select such a value for the parameter n. Supposing they choose their n-value in such a manner, we examine what would happen in the "future", i.e.

2012-2013, upon execution of this selected strategy, compared to all other strategy n-values. If the selected n-value strategy arrived at by using the randomly simulated data outperforms the other data, this is strong evidence to support the great value of using randomly simulated data as a proving ground for trading strategies. To better see the results, we will plot the actual market returns against the random simulation average returns and plot a line of best fit to help us understand if we are actually seeing correlation.

7 Results and analysis

7.1 Geometric Brownian Motion modeling

We fit GBM μ and σ parameters to 2008 through 2011 NASDAQ:AAPL historical prices. We execute a MA1 crossover algorithm on 6 simulated 2-year GBM runs and evaluate the percentage returns, as seen in the table below. We take the average of the MA n value strategy's performance over the 6 sims and compare it to the actual realized portfolio gain when run over the realized data from 2012 through 2013. See results in Table 1. The simulated plots and historical data used for calibration can be seen in Figure 2. The fitted line's R^2 value is low, but the presence of a positive trend is encouraging. Applying the same technique, only this time using the MA2/MA1 algorithm, we obtain the results in Table 2 and Figure 3. The negative trend line and low R^2 statistic is indicative that the simulation testing was not effective in MA2/MA1 strategy n-value picking in this case. Following an analogous procedure for the MA1 for algorithm for TSX:TRP shares with the same calibration and actual realization periods, we obtain the results in Table 3 and Figure 5. The resulting line trends slightly positively, with a very poor R^2 value of 0.076. However, the MA2/MA1 algorithm n-selection proves to be much better, providing a strong positive trend with an R^2 value at relatively strong 0.36.

7.2 Ornstein-Uhlenbeck Process modeling

We now proceed with OU parameter fitting, modeling, and algorithm calibration for commodities as we did for equities above. After parameters fitting and simulations for Brent crude spot price, we had rather ambiguous results for 1-20 moving average selection correlation with actual performance on the MA1 crossover strategy. The R^2 statistic of the line fit to the plot of the actual return of strategies compared to averaged simulation return was 0.0183. However, the fitted line still had a positive (if miniscule) slope (see Table 5 and Figure 8). However, we had much more favorable results with the MA2/MA1 crossover with a strong positive trend with R^2=0.1132 (where an n-value indicates crossover indicators using n and 2n length moving averages), indicating that Brent crude's longer term trends are more readily captured and modeled in an OU framework (see Table 6 and Figure 9). Natural gas OU modeling proved to be an interesting case, as the unique shape of the price graph (see Figure 10) was not capable of being completely captured by the OU model. However, since our focus is on the similarity of trading system behavior on the price path and not exactly the price path itself, we applied an OU fit and proceeded as usual. Both data sets (Tables 7 and 8) presented weak yet consistently positive indicators that enough behavior is being captured in simulation to use as a proving ground for MA1 and MA2/MA1 crossover algorithms. This suggests that the long-term mean reverting behavior captured in an OU model is strong enough to capture certain aspects of the time series, even without capturing shorter-term nuances such as seen in Figure 10 (clearly not completely captured in simulations).

7.3 GMRP modeling

Lastly, we turn to the recent, interesting Markov-based model: the GMRP. For brevity, we apply this only to NASDAQ:TRP stock, using exactly the same procedure as described above for GBM and outlined in Figures 3 and 4. The obvious unrealistic behavior is that all of the returns are one of four values. However, despite the model itself exhibiting this unrealistic behavior, it can be seen that, although the R^2 values remain small, there is a consistent positive linear relationship between the yields of strategies tested on the GMRP-simulated data and the same strategies on the actual realized data. This may very well be due to the positive feedback exhibited in the GMRP (due to the Markov chain behavior) capturing some essential aspects of the market drivers which produce trends. It should be noted that the strongest similarity was found in the MA2/MA1 crossover case, which indicates the GMRP model is capturing longer (i.e. 20-40 day) trends more accurately than shorter (2-20 day) trends for this particular asset.

8 Conclusions and further work

8.1 Model evaluations

Below we present evaluations of the three stochastic models presented.

8.1.1 GBM

The GBM's constant drift parameter μ means unrealistic growth long-term. It is clear from the GBM modeling of AAPL shares that the growth exhibited in 2008-2011 was not sustained in 2012-2013. However, several iterations of the GBM simulation exhibited continuous growth in AAPL share value. Although variance was not extensively analyzed in the paper, it should be noted that the constant variance σ parameter in GBM modeling is not realistic. It would be proper to calibrate GBM to several different consecutive data sets first, and model the larger time step time series of μ to eliminate the problem of this variable being a constant. However, GBM is good for modeling out a few months of an asset's behavior. We saw some weak correlation between algorithm performance on simulations and on real data for small-n moving averages, so there is some justification for using GBM modeling to test trading systems.

TRP shares proved more reliable in terms of being able to use simulations as proving grounds for trading algorithms. We attribute this to the lower value of μ for this stock, and to more stable price levels over the period in question. We also recommend that in future investigations, many more than six simulations should be used, at least 25-30, as this is to what we attribute the low R^2 values. This would also give a much more solid answer as to whether there is consistency between returns on simulations and returns on the real market.

8.1.2 OU

OU modeling provides good long-term stability- the principle drawback of GBM. OU modeling does, to a large degree, capture commodities behavior, but in the form we presented, it fails to model nuances like the unique behavior of natural gas futures shown in Figure 10. The addition of a random term for μ, the long term value to which the process mean-reverts, may be used to incorporate supply and demand modeling. The issue of skewed volatility profiles resulting from our modeling of the logarithm of the price rather than the actual price stream turned out to not be a problem, as consistency was noted between algorithm performance on simulations and real data. However, this is clearly a problem which should be addressed for any trader considering high-frequency commodities trading. We recommend that many more simulations be done as we recommended for GBM modeling to ensure the weak correlation we saw between simulation and real data returns was not merely a coincidence.

8.1.3 GMRP

GMRP performance exhibited a good capacity to model run ups/downs in price. Despite the limited realism due to only 4 distinct daily return values, the GMRP simulations modeled TransCanada assets well enough to exhibit some degree of correlation between simulation yields and real market yields on algorithms. To address the problem of the return values, it is recommended to speed up the simulations to include multiple (smaller) movements per day to generate diversity in close-to-close returns. It is also recommended to run many more GMRP simulations in order to make a more definitive judgment on whether or not GMRP simulations are a good enough proving ground for trading algorithms.

8.2 Answers to our original questions

Below we summarize the answers to out questions presented in Section (1.2).

1. Will trading strategies proven to perform well (or poorly) on randomly simulated data perform comparably well (or poorly) on actual future market data? The answer is YES- but far from a strong YES. Many more simulations should be run in order to make this conclusion, as R^2 statistics presented in this paper are in many cases so low that it is debatable as to whether the fitted line even has meaning. However, consistency was noted in the slope of the line being positive, indicating a positive answer to out original question.

2. Does past asset behavior serve as a good indicator for future asset behavior in terms of trading strategy performance? The answer is YES, when we are clear what behaviors we are talking about. In some cases like with AAPL stock the high growth rate observed in past data (and reflected in a high GBM μ parameter calibrated to that data) was not sustained. However, in commodities, mean reversion was indeed maintained. There is also weak evidence that the positive feedback GMRP attempts to capture was indeed reflected in GMRP simulations of TRP stock, as some comparison between trading system performance was observed there.

3. Which stochastic models among our list of models generate random data that maintains realistic behavior? The answer is YES, all of them - GBM, OU, and GMRP, but this is dependent on what is to be modeled. We have presented evidence (albeit very weak) that all are useful for trading strategy selection based on simulations from calibrated models.

We conclude that although the results presented in this paper are indications to justify the use of modeling and simulations based on those models to test trading algorithms, much more work must be done to investigate this complex and important field of applied mathematics.

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9 Tables and graphs

9.1 GBM testing

Below are tables and graphs from GBM model testing on equities.

MA n	sim1	sim2	sim3	sim4	sim5	sim6	Av	Actual
2	53.54	72.1	145.28	-0.74	-16.99	149.47	67.11	33.5
3	30.76	51.5	71.45	-27.25	3.24	129.51	43.20	29.08
4	29.2	66.93	21.92	16.14	9.46	126.85	45.08	10.43
5	71.76	59.32	67.67	3.57	-5.69	128.45	54.18	14.1
6	61	54.34	88.55	18.01	-13.08	121.88	55.12	38.14
7	60.03	27.05	93.46	14.81	-14.3	103.26	47.39	23.71
8	49.59	31.06	79.84	19.82	-22.06	110.53	44.80	56.54
9	43.67	39.59	115.73	6.34	-22.7	110.59	48.87	36.68
10	53.45	49.06	90.62	27.61	-24.18	130.99	54.59	30.42
11	45.3	44.36	92.93	32.56	-19.61	110.58	51.02	27.35
12	36.26	53.73	101.99	38.01	-15.42	91.59	51.03	31.53
13	42.66	48.72	92.71	22.58	-22.98	111.95	49.27	27.12
14	25.82	38.32	82.6	18.14	-19.72	101.35	41.09	29.11
15	41.44	35.88	75.76	29.5	-27.54	92.06	41.18	28.19
16	39.25	30.79	62.45	48.41	-23.3	92.13	41.62	-5.17
17	43.47	22.78	54.2	44.95	-26.26	90.25	38.23	-10.86
18	33.42	33.05	67.58	43.73	-21.87	76.27	38.70	-10.54
19	40.69	28.5	75.12	52.9	-21.01	86.55	43.79	-6.73
20	40.12	29.42	81.55	55.63	-18.18	78.44	44.50	-2.95

Table 1: MA1 crossover strategy implemented on 6 GBM sims for AAPL stock. The GBM is calibrated to 2008-2011 data; actual is based on 2012-2013 performance. The column labeled “MA n” displays the n-value of the algorithm; “sim1” gives the return for that strategy in percentage (similarly for “sim2” through “sim6”), “Av” provides the average strategy return across the 6 simulations, and “Actual” provides the actual realized return by the corresponding strategy. A line fitted to y=“Actual” vs x=“Av” yields y = 1.3158x - 42.397, with $R^2 = 0.2361$.

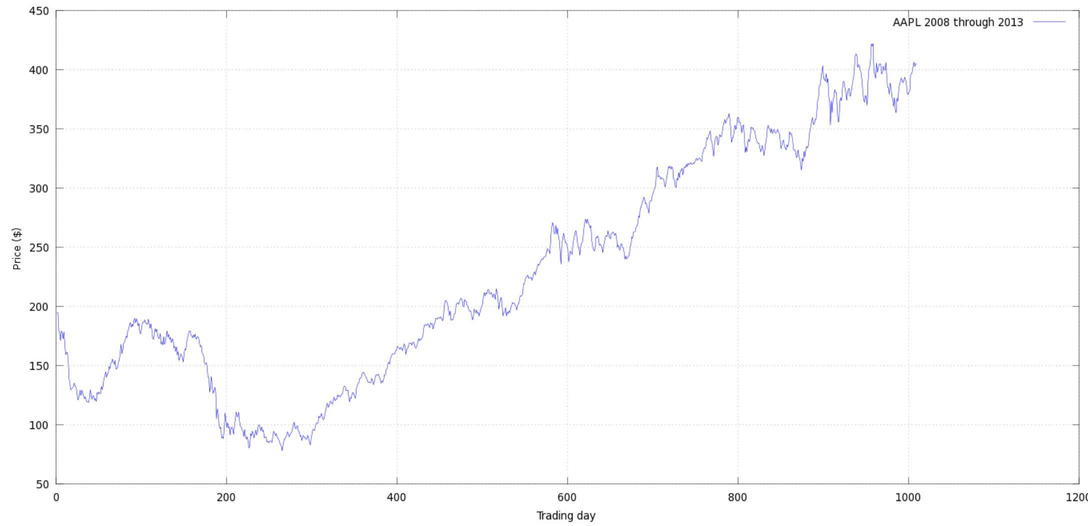

Figure 1: 2008 through 2011 historical AAPL price path used for calibrations

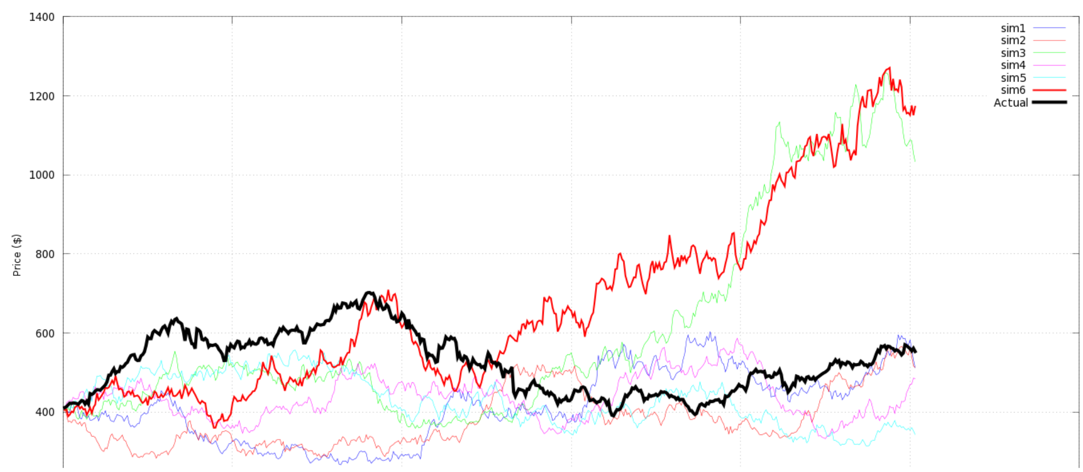

Figure 2: Plot of GBM calibrated simulated paths for AAPL used in Table 1, together with actual realized 2012 through 2013 AAPL price path.

MA n	sim1	sim2	sim3	sim4	sim5	sim6	Av	Actual
2	-20.47	-1.29	13.3	46.06	17.71	-12.75	7.09	27.22
3	19	26.16	-18.1	45.11	7.25	14.26	15.61	25.85
4	-9.51	11.9	-8.32	34.49	-26.57	21.01	3.83	17.95
5	1.18	-8.58	-11.83	66.35	-3.63	17.64	10.19	33.85
6	-0.63	22.82	26.94	13.53	35.74	28.14	21.09	-3.41
7	-2.83	33.95	10.71	26.68	40.78	29.34	23.11	-9.34
8	8.73	18.13	-7.73	11.2	36.16	39.32	17.64	-0.81
9	-21.69	16.15	10.94	4.75	33.7	65.66	18.25	19.95
10	-39.01	31.15	-1.46	-8.01	34.52	51.69	11.48	8.38
11	-37.94	-5	2.31	-4.14	28.97	64.42	8.10	-5.5
12	-16.57	4.15	0.27	6.53	31.71	50.02	12.69	0.98
13	-15.54	24.15	64.78	13.24	52.4	57.17	32.70	1.79
14	-22.31	24.53	44.85	8.61	26.83	55.96	23.08	15.67
15	-39.97	31.71	35.44	-0.72	2.16	59.4	14.67	22.57
16	-14.12	22.65	27.77	1.94	12.9	34.44	14.26	28.12
17	-8.42	14.75	33.06	-3.04	7.35	46.85	15.09	26.56
18	5.35	35.9	35.03	13.79	25.74	38.76	25.76	32.66
19	6.73	14.89	47.5	22.12	24.33	37.5	25.51	29.25
20	10.23	16.52	30.97	4.4	42.86	24.29	21.55	26.34

Table 2: MA2/MA1 crossover strategy implemented on 6 GBM sims for AAPL stock. The GBM is calibrated to 2008-2011 data; actual is based on 2012-2013 performance. The column labeled “MA n” displays the n-value of the algorithm; “sim1” gives the return for that strategy in percentage (similarly for “sim2” through “sim6”), “Av” provides the average strategy return across the 6 simulations, and “Actual” provides the actual realized return by the corresponding strategy. A line fitted to y=“Actual” vs x=“Av” yields y = -0.2076x + 19.203, with $R^2 = 0.0114$.

Figure 3: Plot of GBM calibrated simulated paths for AAPL used in Table 2, together with actual realized 2012 through 2013 AAPL price path.

MA n	sim1	sim2	sim3	sim4	sim5	sim6	Av	Actual
2	7.61	48.19	-0.07	-6.95	-13.5	-11.04	4.04	-3.12
3	5.14	73.55	-7.84	9.14	-14.59	-21.09	7.39	-1.75
4	-0.27	51.64	-1.6	-0.86	-19.32	-10.27	3.22	-7.2
5	2.99	49.52	2.3	-8.47	-14.62	-4.69	4.51	-3.82
6	24.11	56.48	5.61	8.31	-16.8	-3.47	12.37	2.76
7	32.7	51.37	11.22	-10.88	-20.39	8.66	12.11	-4.07
8	22.62	43.23	10.33	-0.84	-18.88	-0.79	9.28	-3.89
9	31.65	25.18	18.33	-5.87	-24.38	8.77	8.95	-3.32
10	19.27	29.86	15.77	-12.3	-25.57	14.61	6.94	-0.17
11	18.39	12.46	16.58	-14.33	-20.11	14.11	4.52	-6.67
12	17.93	7.35	12.36	-11.35	-12.71	11.6	4.20	0.98
13	8.67	6.86	11.89	-9.42	-9.93	14.99	3.84	1.66
14	12.85	8.23	8.75	6.34	-18.12	21.53	6.60	3.37
15	14.2	-0.57	7.87	5.82	-20.41	21.99	4.82	1.63
16	17.1	-2.29	9.49	5.24	-14.94	29.71	7.39	-4.28
17	7.69	-10.35	5.08	2.25	-12.95	24.21	2.66	-3.78
18	6.49	-15.96	7.43	3.07	-15.2	26.43	2.04	-2.96
19	1.63	-13.24	1.37	1.5	-16.71	41.28	2.64	-3.09
20	3.57	-10.75	-0.11	-2.34	-21.72	27.4	-0.66	-5.37

Table 3: MA1 crossover strategy implemented on 6 GBM sims for TRP stock. The GBM is calibrated to 2008-2011 data; actual is based on 2012-2013 performance. The column labeled “MA n” displays the n-value of the algorithm; “sim1” gives the return for that strategy in percentage (similarly for “sim2” through “sim6”), “Av” provides the average strategy return across the 6 simulations, and “Actual” provides the actual realized return by the corresponding strategy. A line fitted to y=“Actual” vs x=“Av” yields y = 0.2523x - 3.6868, with $R^2 = 0.076$.

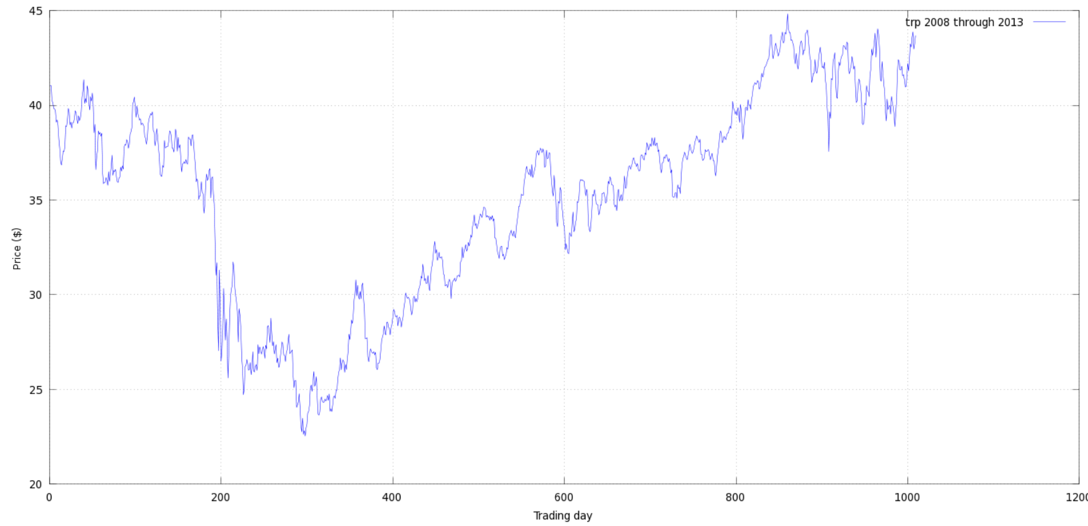

Figure 4: 2008 through 2011 historical TRP price path used for calibrations

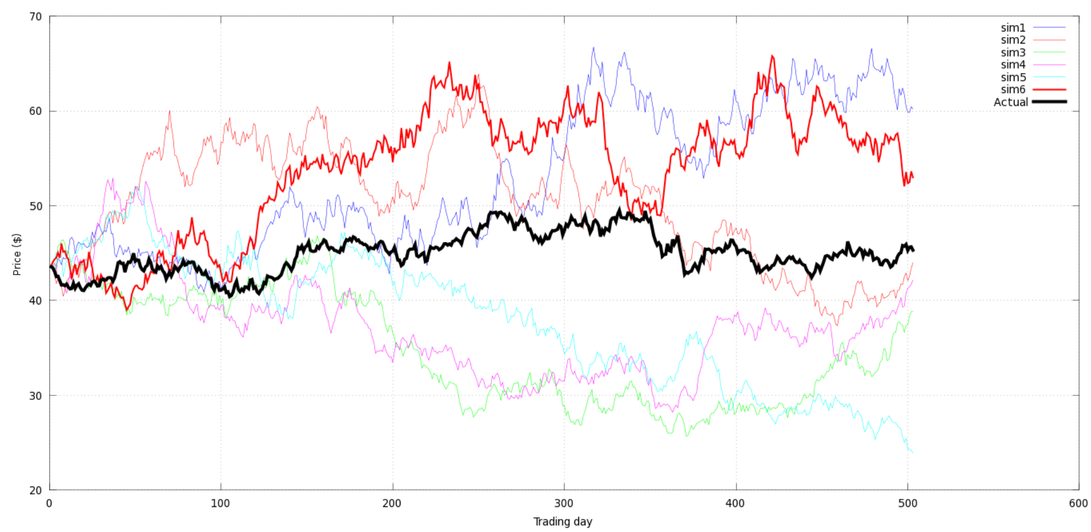

Figure 5: Plot of GBM calibrated simulated paths for TRP used in Table 3, together with actual realized 2012 through 2013 TRP price path.

MA n	sim1	sim2	sim3	sim4	sim5	sim6	Av	Actual
2	9.31	35.09	-18.3	3.86	22.38	22.22	12.43	-22.05
3	-1.35	44.16	-18.12	48.35	5.35	51.03	21.57	-4.25
4	-4.41	18.66	-5.89	26.64	-2.82	43.19	12.56	-6.65
5	-22.67	17.67	-4.05	30.17	-3.7	81.71	16.52	6.85
6	-29.01	25.88	31.95	10.08	-4.85	86.74	20.13	10.69
7	-6.85	29.79	29.91	11.2	-0.79	90.81	25.68	6.07
8	4.88	64.18	49.01	18.61	-5.69	84.58	35.93	11.74
9	1.47	59.03	15.79	-5.46	11.85	67.23	24.99	13.25
10	-3.01	49.31	30.94	1.87	3.43	61.46	24.00	-5.26
11	-31.23	34.26	17.88	9.51	-7.62	46.62	11.57	-4
12	-19.12	31.47	-8.59	54.05	-9.65	35.19	13.89	-4.75
13	-19.21	5.74	5.69	51.34	-1.91	26.2	11.31	-17.64
14	-20.74	35.21	17.36	48.19	-15.05	33.35	16.39	-13.08
15	-22.87	31.03	8.15	62.39	10.75	35.09	20.76	-17.83
16	-20.78	12.74	13.9	74.01	1.26	38.91	20.01	-11.38
17	-21.02	9.33	-4.68	63.71	2.34	44.42	15.68	-2.33
18	-29.89	17.99	9.84	61.2	2.93	49.25	18.55	-4.27
19	-26.15	16.54	6.8	59.04	-1.09	58.95	19.02	-4.71
20	-26.12	17.76	0.11	49.01	0.98	62.48	17.37	-5.4

Table 4: MA2/MA1 crossover strategy implemented on 6 GBM sims for TRP stock. The GBM is calibrated to 2008-2011 data; actual is based on 2012-2013 performance. The column labeled “MA n” displays the n-value of the algorithm; “sim1” gives the return for that strategy in percentage (similarly for “sim2” through “sim6”), “Av” provides the average strategy return across the 6 simulations, and “Actual” provides the actual realized return by the corresponding strategy. A line fitted to y=“Actual” vs x=“Av” yields y = 0.9964x - 22.74, with R^2= 0.3547 .

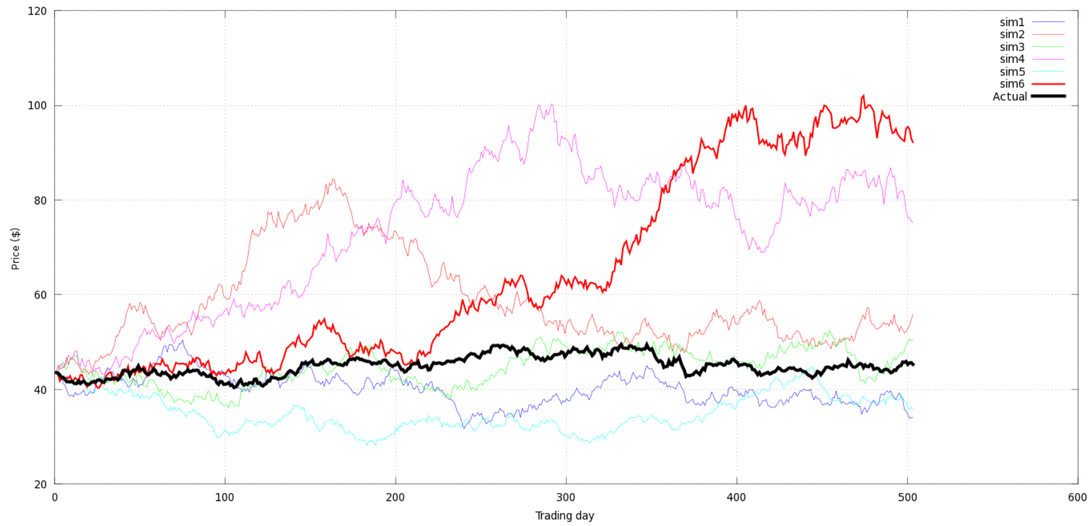

Figure 6: Plot of GBM calibrated simulated paths for TRP used in Table 4, together with actual realized 2012 through 2013 TRP price path.

9.2 OU testing

Below are tables and graphs from OU model testing on commodities.

MA n	sim1	sim2	sim3	sim4	sim5	sim6	Av	Actual
2	-30.21	-32.83	23	36.46	-14.17	-2.64	-3.40	-18.18
3	-18	-45.16	-21.14	70.32	-19.54	26.01	-1.25	-21.94
4	-31.47	-24.94	-12.45	64.68	-53.54	33.31	-4.07	-6.75
5	-15.21	-3.59	-16.29	68.38	-49.09	59.65	7.31	-1.52
6	-18.86	-25.33	-1.38	92.35	-45.47	59.36	10.11	-4.85
7	-24.38	-27.79	-14.72	113.33	-58	69.87	9.72	-2.31
8	-25.36	-24.39	-3.84	102.41	-45.92	72.07	12.50	-13.85
9	-18.81	-28.85	-10.91	76.98	-41.21	71.85	8.18	-9.01
10	-20.52	-27.56	-4.27	80.61	-30.31	80.69	13.11	-7.9
11	-26.35	-27.42	-4.01	55.31	-36.47	77.21	6.38	3.88
12	-22.05	-20.88	-8.12	49.03	-33.99	74.69	6.45	8.33
13	-20.98	-9.41	-13.34	42.89	-28.88	37.52	1.30	0.56
14	-25.91	-7.66	-12.05	17.37	-34.52	31.93	-5.14	-3.25
15	-31.99	-9.5	-11.44	23.51	-22.99	39.44	-2.16	-3.43
16	-29.3	-20.27	-13.48	26.38	-21.28	39.44	-3.09	-2.87
17	-29.27	-28.98	-16.74	23.39	-14.89	33.51	-5.50	-3.19
18	-25.48	-21.19	-11.61	26.33	-16.03	35.77	-2.04	-8.36
19	-22.31	-10.95	-12.51	34.86	-11.17	39.97	2.98	-7.44
20	-25.9	-12.05	-5.84	47.35	-10.05	48.56	7.01	-3.53

Table 5: MA1 crossover strategy implemented on 6 OU sims for Brent crude spot. The OU is calibrated to 2008-2011 data; actual is based on 2012-2013 performance. The column labeled “MA n” displays the n-value of the algorithm; “sim1” gives the return for that strategy in percentage (similarly for “sim2” through “sim6”), “Av” provides the average strategy return across the 6 simulations, and “Actual” provides the actual realized return by the corresponding strategy. A line fitted to y=“Actual” vs x=“Av” yields y = 0.152x - 6.0255, with R^2 = 0.0183.

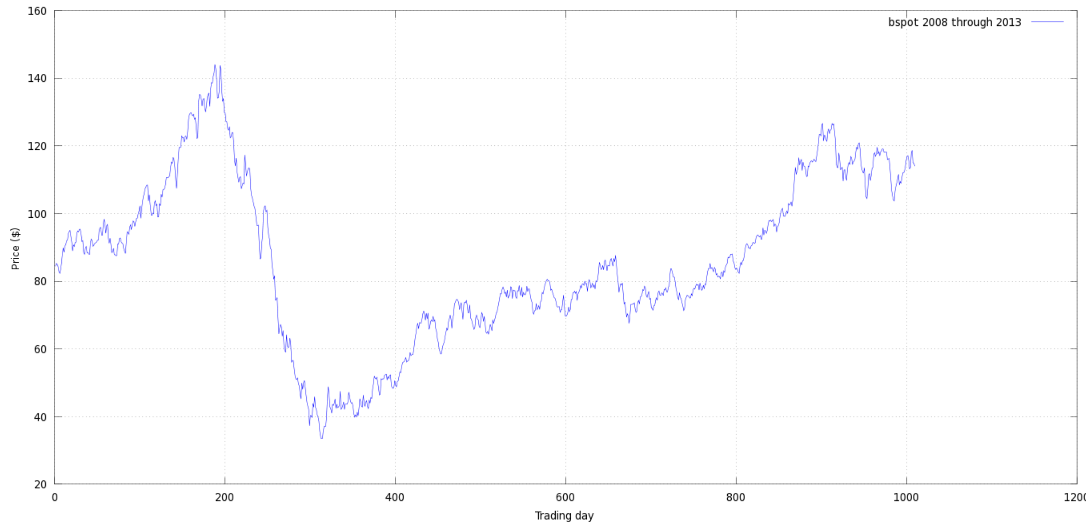

Figure 7: 2008 through 2011 historical Brent crude spot price path used for calibrations

Figure 8: Plot of OU calibrated simulated paths for TRP used in Table 5, together with actual realized 2012 through 2013 TRP price path.

MA n	sim1	sim2	sim3	sim4	sim5	sim6	Av	Actual
2	-17.33	-36.92	-12.14	9.25	-2.92	-19.43	-13.25	6.35
3	-22	-11.6	-8.49	43.57	107.56	-32.59	12.74	-1.86
4	-27.67	-27.54	13.92	16.42	41.94	14.59	5.28	-5.67
5	7.22	-11.46	1.9	-17.93	30.91	7.55	3.03	7.37
6	-22.07	-28.51	17.94	30.16	99.7	-23.97	12.21	10.7
7	-29.25	-39.03	-19.77	31.42	77.4	-19.64	0.19	-8.16
8	-13.33	-30.74	-11.02	18.21	69.07	-21.25	1.82	-7.02
9	-30.73	-14.96	-17.8	2.2	67.85	-34.21	-4.61	-0.97
10	-30.06	-18.1	-4.75	13.54	152.52	-43.88	11.55	-2.39
11	-22.29	-17.88	-1.39	-3.91	134.98	-43.83	7.61	1.25
12	-18.55	0.52	0.12	-15.65	107.98	-41.54	5.48	16.24
13	-28.85	-0.78	-7.19	11.49	113.5	-41.42	7.79	14.82
14	-8.45	41.49	-7.24	18.01	103.08	-37.5	18.23	22.57
15	-8.19	20.64	5.13	1.74	30.34	-40	1.61	17.64
16	1.41	-0.8	24.45	6.7	64.79	-34.92	10.27	22.26
17	-2.14	4.1	38	-4.51	55.69	-39.89	8.54	24.25
18	21.83	3.7	36.79	-22.65	54.9	-41.1	8.91	5.19
19	10.88	6.45	35.64	-7.49	42.67	-40.48	7.95	4.97
20	15.33	13.53	37.01	8.78	31.85	-48.05	9.74	30.93

Table 6: MA2/MA1 crossover strategy implemented on 6 OU sims for Brent crude spot.
The OU is calibrated to 2008-2011 data; actual is based on 2012-2013 performance. The column labeled "MA n" displays the n-value of the algorithm; "sim1" gives the return for that strategy in percentage (similarly for "sim2" through "sim6"), "Av" provides the average strategy return across the 6 simulations, and "Actual" provides the actual realized return by the corresponding strategy. A line fitted to y="Actual" vs x="Av" yields y = 0.5593x + 4.9526, with R^2 = 0.1132.

Figure 9: Plot of OU calibrated simulated paths for TRP used in Table 6, together with actual realized 2012 through 2013 TRP price path.

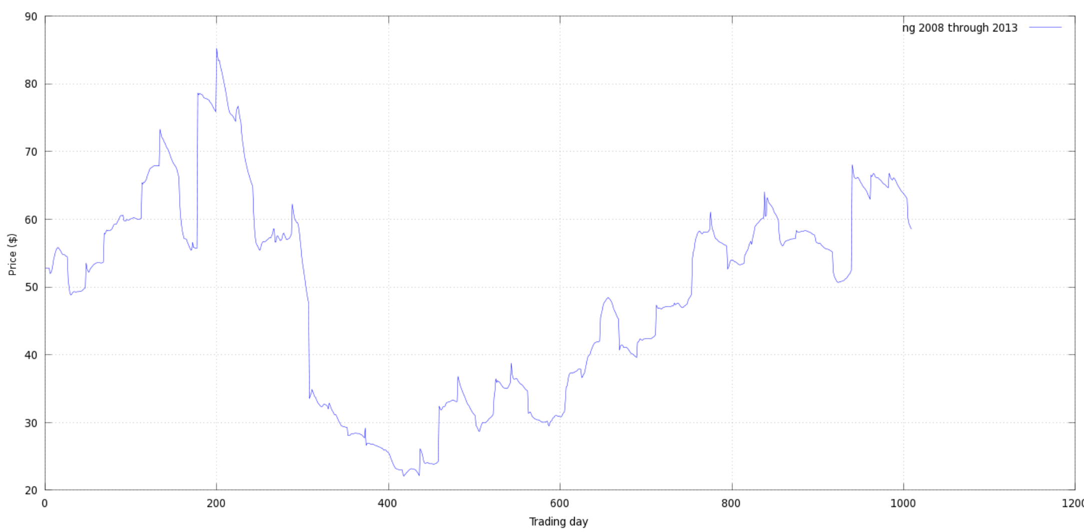

Figure 10: 2008 through 2011 historical IPE natural gas settlement path used for calibrations

MA n	sim1	sim2	sim3	sim4	sim5	sim6	Av	Actual
2	-30.16	5.88	-9.66	-54.68	-48.88	-12.56	-25.01	52.75
3	-19.14	1.12	-19.99	-48.78	-34.72	11.8	-18.29	47.76
4	-4.68	-1.76	-23.95	-38.78	-17.16	-9.79	-16.02	45.9
5	-20.26	-24.55	-18.83	-46.47	-36.15	-5.59	-25.31	44.75
6	-19.48	-12.7	-30.55	-45.05	-34.56	-15.66	-26.33	42.86
7	-27.18	-13.03	-32.73	-41	-36.87	-9.96	-26.80	45.13
8	-27.94	-20.21	-35.44	-38.77	-33.43	16.48	-23.22	44.07
9	-30.36	-16.31	-29.75	-34.26	-28.44	18.37	-20.13	42.24
10	-31.5	-21.13	-26.39	-40.49	-28.81	25.77	-20.43	44.31
11	-27.37	-28.32	-29.86	-42.22	-26.76	-0.37	-25.82	43.73
12	-32.12	-26.51	-28.25	-41.72	-36.15	-4.87	-28.27	46.85
13	-30.38	-26.57	-25.74	-52.08	-47.58	-8.38	-31.79	46.83
14	-30.57	-23.77	-23.16	-54.23	-48.24	-5.44	-30.90	43.41
15	-24.44	-19.46	-27.34	-54.01	-47.66	-4.39	-29.55	44.03
16	-19.67	-21.73	-25.12	-55.93	-52.2	-10.72	-30.90	40.05
17	-26.62	-25.74	-6.23	-55.25	-51.06	-2.6	-27.92	39.38
18	-21.58	-23.36	-2.1	-56.47	-42.34	0.93	-24.15	43.5
19	-21.93	-18.28	2.19	-51.03	-43.52	14.19	-19.73	45.07
20	-25.31	-25.04	3.52	-50.46	-40.48	27.11	-18.44	43.71

Table 7: MA1 crossover strategy implemented on 6 OU sims for IPE natural gas. The OU is calibrated to 2008-2011 data; actual is based on 2012-2013 performance. The column labeled “MA n” displays the n-value of the algorithm; “sim1” gives the return for that strategy in percentage (similarly for “sim2” through “sim6”), “Av” provides the average strategy return across the 6 simulations, and “Actual” provides the actual realized return by the corresponding strategy. A line fitted to y=“Actual” vs x=“Av” yields y = 0.1144x + 47.368, with R^2 = 0.0353.

MA n	sim1	sim2	sim3	sim4	sim5	sim6	Av	Actual
2	-25.34	24.09	-33.26	-33.72	-25.24	34.74	-9.79	35.02
3	-18.54	16.92	-17.45	-0.08	-14.84	14.68	-3.22	32.76
4	-19.17	4.93	-14.12	5.95	-24.89	49.43	0.36	34.46
5	-4.58	-28.29	23.54	28.87	13.85	54.16	14.59	41.65
6	8.89	-14.82	20.74	-18.89	10.22	-21.06	-2.49	33.5
7	-13.36	31.26	28.22	-3.42	19.62	37.72	16.67	27
8	-28.53	-15.87	41.26	-16.29	4.39	16.91	0.31	27.26
9	-37.13	-3.15	23.81	-15.79	11.6	56.49	5.97	38.74
10	-29.94	-22.79	5.61	-8.83	-14.7	75.98	0.89	37.49
11	-39.5	-39.05	-15.47	16.41	-29.22	104.97	-0.31	26.86
12	-46.44	-23.43	-15.85	1.11	-23.19	-8.9	-19.45	23.18
13	-34.21	-28.55	-22.25	-3.11	-11.67	81.11	-3.11	20.7
14	-25.59	-37.87	-16.66	-3.61	-8.8	108.08	2.59	20.65
15	-32.28	-40.02	-14.29	-17.07	-23.4	47.28	-13.30	20.03
16	-22.37	-35.07	-12.62	21.3	18.86	50.38	3.41	19.18
17	-31.08	-27.08	-11.92	7.55	-12.33	49.8	-4.18	12.05
18	-26.46	-27.88	-24.27	1.71	-6.85	29.32	-9.07	13.1
19	-29.06	-15.32	-23.48	1	-16.82	49.36	-5.72	13.03
20	-43.44	-14.68	-33.62	17.65	-8.06	52.38	-4.96	8.13

Table 8: MA1 crossover strategy implemented on 6 OU sims for IPE natural gas. The OU is calibrated to 2008-2011 data; actual is based on 2012-2013 performance. The column labeled "MA n" displays the n-value of the algorithm; "sim1" gives the return for that strategy in percentage (similarly for "sim2" through "sim6"), "Av" provides the average strategy return across the 6 simulations, and "Actual" provides the actual realized return by the corresponding strategy. A line fitted to y="Actual" vs x="Av" yields y = 0.483x + 26.298, with R^2 = 0.1735.

9.3 GMRP testing

Below are tables and graphs from GMRP model testing (on equities).

MA n	sim1	sim2	sim3	sim4	sim5	sim6	Av	Actual
2	44.46	42.68	21.39	-10.95	15.32	35.72	24.77	-3.12
3	31.43	41.29	-1.08	-0.82	3.46	2.38	12.78	-1.75
4	8.21	44.75	-5.02	6.36	1.08	-0.66	9.12	-7.20
5	11.87	42.06	-7.16	-2.92	-2.74	-6.17	5.82	-3.82
6	25.13	38.56	-3.18	-14.27	14.75	-0.1	10.15	2.76
7	34.82	29.07	-2.95	-19.48	3.21	7.15	8.64	-4.07
8	30.56	38.96	8.12	-10.94	12.63	19.17	16.42	-3.89
9	26.49	41.96	-0.15	-8.55	6.15	10.76	12.78	-3.32
10	22.02	52.46	6.44	-9.57	6.48	4.62	13.74	-0.17
11	22.13	53.04	5.66	-10.49	7.8	15.96	15.68	-6.67
12	19.48	41.09	3.16	-3.29	5.1	13.68	13.20	0.98
13	11.75	37.86	3.86	-6.44	8.88	8.46	10.73	1.66
14	13.89	42.14	2.71	-1.04	13.16	20.18	15.17	3.37
15	10.75	29.91	-7.91	-0.02	9.91	16.02	9.78	1.63
16	6.91	23	-9.86	0.29	1.09	12.32	5.63	-4.28
17	10.8	17.52	-11.59	-5.9	1.41	13.89	4.36	-3.78
18	14.45	18.47	-8.74	-3.62	-2.87	18.4	6.02	-2.96
19	22.21	18.05	-15.16	1.99	-5.98	28.64	8.29	-3.09
20	25.17	21.58	-15.04	-5.18	-9.78	22.85	6.60	-5.37

Table 9: MA1 crossover strategy implemented on 6 GMRP sims for Transcanada shares.
The GMRP is calibrated to 2008-2011 data; actual is based on 2012-2013 performance. The column labeled "MA n" displays the n-value of the algorithm; "sim1" gives the return for that strategy in percentage (similarly for "sim2" through "sim6"), "Av" provides the average strategy return across the 6 simulations, and "Actual" provides the actual realized return by the corresponding strategy. A line fitted to y="Actual" vs x="Av" yields y = =0.1046x - 3.4219, with R^2 = 0.0277.

MA n	sim1	sim2	sim3	sim4	sim5	sim6	Av	Actual
2	-7.35	5.68	52.05	-31.46	27.83	24.04	11.80	-22.05
3	-6.02	8.78	28.97	-15.32	-5.14	-6.59	0.78	-4.25
4	5.59	-3.68	57.23	-17.14	-3.09	9.7	8.10	-6.65
5	14.09	-11.83	37.66	3.94	9	19.79	12.11	6.85
6	18.48	-11.07	39.38	-14.36	-6.13	5.77	5.35	10.69
7	7.44	-3.72	34.36	-23.29	1.05	0.43	2.71	6.07
8	12.28	-10.94	41.85	-23.77	27.05	2.55	8.17	11.74
9	25.02	2.28	28.38	-27.02	10.75	2.26	6.95	13.25
10	13.82	-14.66	29.11	-20.74	2.48	0.21	1.70	-5.26
11	13.44	-22.71	42.5	-6.24	6.44	13.55	7.83	-4
12	24.54	-20.22	35.9	0.59	5.14	-24.19	3.63	-4.75
13	18.06	-24.13	15.39	-1.14	3.72	-29.01	-2.85	-17.64
14	-5.73	-29.32	20.99	-5.11	-4.55	-16.37	-6.68	-13.08
15	24.49	-26.92	29.13	-3.32	1.88	-35.41	-1.69	-17.83
16	28.47	-35.58	29.23	-5.47	-1.56	-19	-0.65	-11.38
17	35.08	-23.06	27.71	-7.15	1.76	-18.55	2.63	-2.33
18	31.89	-13.48	25.26	-7.96	0.3	-13.71	3.72	-4.27
19	29.59	-2.16	18.26	-2.11	-1.63	-20.25	3.62	-4.71
20	29.21	-19.01	8.98	-8.58	7.75	-25.46	-1.19	-5.4

Table 10: MA2/MA1 crossover strategy implemented on 6 GMRP sims for Transcanada shares
The GMRP is calibrated to 2008-2011 data; actual is based on 2012-2013 performance. The column labeled “MA n” displays the n-value of the algorithm; “sim1” gives the return for that strategy in percentage (similarly for “sim2” through “sim6”), “Av” provides the average strategy return across the 6 simulations, and “Actual” provides the actual realized return by the corresponding strategy. A line fitted to y=“Actual” vs x=“Av” yields y = 0.8433x - 6.8777, with R^2 = 0.1714.

Figure 11: Plot of GMRP calibrated simulated paths for TRP used in Table 9, together with actual realized 2012 through 2013 TRP price path.

Figure 12: Plot of GMRP calibrated simulated paths for TRP used in Table 12, together with actual realized 2012 through 2013 TRP price path.

10 Codes

10.1 GBM calibration

Please contact Zach Moyer for access to codes.

10.2 GBM simulation

Please contact Zach Moyer for access to codes.

10.3 OU Calibration

Please contact Zach Moyer for access to codes.

10.4 OU simulation

Please contact Zach Moyer for access to codes.

10.5 GMRP calibration

Please contact Zach Moyer for access to codes.

10.6 GMRP simulation

Please contact Zach Moyer for access to codes.

10.7 Trading systems

10.7.1 Price crosses MA1 trading algorithm simulator

Please contact Zach Moyer for access to codes.

10.7.2 MA1 crosses MA2 trading algorithm simulator

Please contact Zach Moyer for access to codes.

10.8 Sample code for generating tables (code for Table 1)

Please contact Zach Moyer for access to codes.