

Lesson #3: Sep. 20 2014

Greatest Common Divisor:

The greatest common divisor of two positive integers a and b is the largest positive integer that is a divisor of BOTH a and b .

Proper mathematical definition: $\gcd(a,b)$ is a number, let's call it x ; such that if m divides a and m also divides b , then m divides x also.

Example: The greatest common divisor of 30 and 20 is 10.

Indeed, since 5 divides both 30 and 20, observe that 5 divides $10 = \gcd(30,20)$ as predicted. The same holds for 2.

We can find \gcd as follows:

$$30 = 2 \times 3 \times 5$$

$$20 = 2 \times 2 \times 5$$

Take minimum size of sets of each factor and multiply them together to get:

$$\gcd(30,20) = 2 \times 5 = 10$$

Least Common Multiple:

The least common multiple of two positive integers a and b is the smallest positive integer that is a multiple of BOTH a and b .

Proper mathematical definition: $\text{lcm}(a,b)$ is a number, let's call it x ; such that if a divides m and b also divides m , then x divides m also.

Example: The least common multiple of 5 and 6 is 30.

Indeed, since 5 and 6 both divide 120, observe that $30 = \gcd(30,20)$ also divides 120 as predicted. The same holds for 180.

We can find $\text{lcm}(5,6)$ as follows:

$$5 = 5$$

$$6 = 2 \times 3$$

Take maximum size of sets of each factor and multiply them together to get:

$$\text{lcm}(30,20) = 2 \times 3 \times 5 = 30$$