1. In this set we establish some simple properties of the Fourier transform. For a function $f(t)$, the Fourier transform $\mathscr{F}(f(t)) = F(\omega)$ is given by the formula
$$\mathscr{F}(f(t)) = F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}\, dt.$$
Assume that the Fourier transform of $g(t)$ is $G(\omega)$, and let the inverse Fourier transform be denoted by $\mathscr{F}^{-1}(F(\omega)) = f(t)$.
   (a) Show $\mathscr{F}(af(t) + bg(t)) = aF(\omega) + G(\omega)$.
   (b) Show $\mathscr{F}(f(t-c)) = e^{i\omega c}F(\omega)$.
   (c) Show $\mathscr{F}(e^{icx}f(x)) = F(\omega - c)$.
   (d) Show $\mathscr{F}(f(at)) = \frac{1}{a}F(\omega/a)$, if $a > 0$. What is the relevant formula if $a < 0$?
   (e) Show $\mathscr{F}(f'(t)) = i\omega F(\omega)$. Hence figure out what $\mathscr{F}(f^{(2014)}(t))$ is.

2. Let the Fourier transform of a probability distribution $g(t)$ be $G(\omega)$. Solve the functional equation
$$f(t-1) + f(t+1) = g(t)$$
for the unknown function $f(t)$. Note: your answer will be in terms of the inverse Fourier transform $\mathscr{F}^{-1}$.

3. Using the standard trick of squaring and changing to polar coordinates, evaluate the integral
$$I = \int_{-\infty}^{\infty} e^{-t^2}\, dt.$$
Let $g(t) = I^{-1}e^{-t^2}$, so that $g$ represents a probability distribution. Compute the Fourier transform of $g$. Hint: there is a quadratic present, so the first thing you should think of trying is completing the square.