1. Find the Fourier series for the function $f(x) = \cos^2 x$. Note: There is an easy way to do this. Now, by using Euler's formula, find the Fourier series for $\cos^3 x$.

2. Find the Fourier series expansion of the function

$$f(x) = \begin{cases} 0 & -\pi \leq x \leq 0 \\ x^2 & 0 \leq x \leq \pi \end{cases}$$

Use this series to show that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

3. (a) Let $\zeta$ be nonintegral. Obtain the expansion

$$\cos \zeta x = \frac{\sin \pi\zeta}{\pi\zeta} + \sum_{n=1}^{\infty} (-1)^n \frac{2\zeta \sin \pi\zeta}{\pi(\zeta^2 - n^2)} \cos nx, \qquad -\pi \leq x \leq \pi.$$

(b) Deduce from this result that

$$\cot \pi\zeta = \frac{1}{\pi} \left( \frac{1}{\zeta} - \sum_{n=1}^{\infty} \frac{2\zeta}{n^2 - \zeta^2} \right)$$

when $\zeta$ is nonintegral.

4. Let $F$ be the matrix for the DFT

$$F = [F]_{kt} = e^{-i2\pi kt/N}$$

Using what you know about the orthogonality of the basis vectors, find the matrix for the inverse operator $F^{-1}$.

5. Once again, let $F$ be the matrix for the DFT and let $R = F^2$. It is known that $R^2 = I$, the identity matrix (well, up to a constant multiple, which we will ignore). Use this fact to find an expression for the inverse DFT $F^{-1}$ in terms of $F$ and $R$.

6. This computation illustrates why you would want to employ the FFT as opposed to just doing one big matrix multiplication.

   Suppose your signal had $32,768$ $(2^{15})$ samples. Count the number of multiplications and additions in the two cases

   (a) the straight matrix multiplication $X = Fx$.

   (b) the decimation in time algorithm that does repeated merge sorting by doubling the length (and halving the number) of the merged files.