1. (a) Show that $u(x, y) = x^2 - y^2$ is a harmonic function.
   (b) Find a function $v(x, y)$ with the properties that
      i. $v(x, y)$ is a harmonic function.
      ii. $f(z) := u(x, y) + iv(x, y)$ is an analytic function.

   Such a function v is called a *harmonic conjugate* of u.
   (c) Compute the angle between the gradients of u and v.

2. Let C be the boundary of the square with vertices $0, 1, 1+i, i$ traversed counterclockwise. Compute the contour integral

$$\int_C \pi e^{\pi \bar{z}}\, dz.$$

3. Find the real and imaginary parts of the function

$$f(z) = i \tanh iz$$

in terms of the variables x and y.

4. Find the Laurent expansion of the function

$$f(z) = \frac{1}{(4z^2 + 1)(4 + z^2)}$$

that converges in the annulus centred on the origin and contains the unit circle.

5. Let $C = \{z \mid |z| = 2\}$ traversed in the positive sense. Show that

$$\int_C \tan z\, dz = -4\pi i.$$