

Financial Mathematics

Principles of Capitalism

- 1) Value of Money
- 2) Time value of money.
- 3) Goods/services have value

Rate of Return:

$$= \frac{\text{money at end of investment}}{\text{money invested}}$$

$$= \frac{\text{money gained}}{\text{money invested}} + 1$$

Interest Rate:

Rate of return on money invested in a bank.

Borrowing rate vs. Lending rate.

Must be REGULATED

B₁, B₂ - businesses.

B ₂ ↓ \ B ₁ →	Only B ₁ benefit	B ₁ benefit + economy	Economy #1	
Only B ₂ benefit	B ₁ • B ₂ •	B ₁ ↑ B ₂ ↑	B ₁ • B ₂ ↑	• - no growth ↑ - slow growth ↑↑ - fast growth
B ₂ benefit + economy	B ₁ ↑ B ₂ ↑	B ₁ ↑ B ₂ ↑	B ₁ ↑ B ₂ ↑	<p>For B₂ this is optimal growth in all cases averaged.</p>
Economy #1	B ₁ ↑ B ₂ •	B ₁ ↑ B ₂ ↑	B ₁ ↑ B ₂ ↑	

Conclusion:

Do what is best for your own business first, plus what benefits the economy.

For B₁ this is optimal growth in all cases averaged

Investment: $A \xrightarrow[1 \text{ year}]{\text{Principle}} A \times (1+r) = A + A \times r$ ^{interest accrued}
 $r = \text{interest rate}$.

Example: $\$100 \xrightarrow[1 \text{ year @ 5\%}]{} \$100 \times (1+0.05) = \$105$
 $= \$100 + \5 ^{interest accrued}.

Simple interest: $A \xrightarrow[n \text{ years}]{} A \times (1+rn)$

Compound interest: $A \xrightarrow[n \text{ years}]{} A \times (1+r)^n$

Interest compounded m-thly: $\left(1 + \frac{r_{(m)}}{m}\right)^m = (1+r)$

$$r_m = \left[(1+r)^{\frac{1}{m}} - 1\right] \times m$$

Discount factor = $v = \frac{1}{1+r}$ = present value of 1 unit of investment.

Annuity (unit) for n payments = (due)

$$1 + v + v^2 + v^3 + \dots + v^{n-1} = \frac{1-v^n}{1-v}$$

Annuity (unit) for n payments = (immediate)

$$v + v^2 + v^3 + v^4 + \dots + v^n = v \left(\frac{1-v^n}{1-v} \right) = \frac{1-v^n}{r}$$

$$\left(\text{Since } v \cdot \left(\frac{1}{1-v} \right) = \frac{\left(\frac{1}{1+r} \right)}{1 - \left(\frac{1}{1+r} \right)} = \frac{\left(\frac{1}{1+r} \right)}{\left(\frac{r}{1+r} \right)} = \frac{1}{r} \right)$$