Lecture Notes

$\varphi(n)$ when n has powers in its prime factorization.

Theorem: $\varphi(p^2) = p(p-1)$. if p is prime.

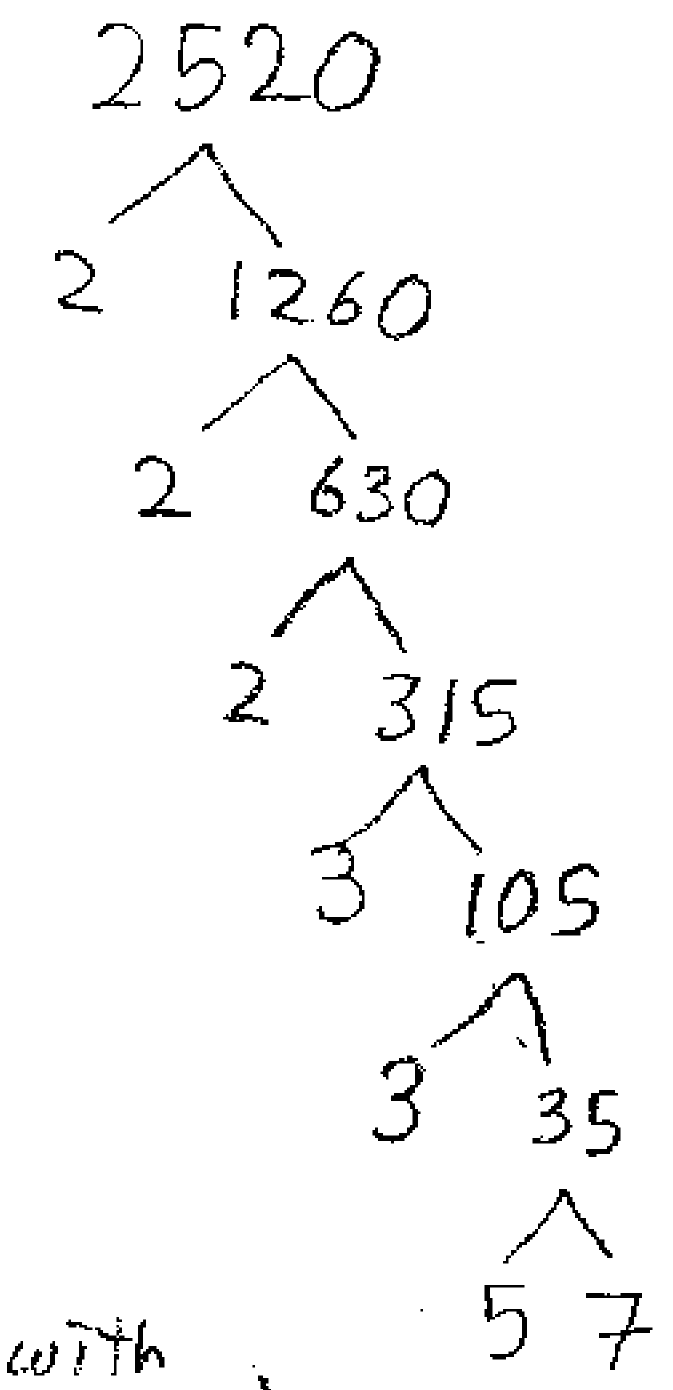
Example: $\varphi(9) = \varphi(3^2) = 3\times(3-1) = 3\times 2 = 6$.

Theorem: $\varphi(p^k) = p^{(k-1)}(p-1)$. if p is prime.

Example: $\varphi(16) = \varphi(2^4) = 2^{(4-1)}\times(2-1) = 2^3\times 1 = 8$.

Now we can find $\varphi(n)$ for any number!
Just find the prime factorization and apply our Theorems.

Example: Find $\varphi(2520)$.



$2520 = 2^3 \times 3^2 \times 5 \times 7$.

$\varphi(2520) = \varphi(2^3\times 3^2 \times 5 \times 7)$

$= \varphi(2^3) \times \varphi(3^2) \times \varphi(5) \times \varphi(7)$

$= 2^2(2-1) \times 3(3-1) \times (5-1) \times (7-1)$

$= 4 \times 6 \times 4 \times 6 = 576$

Corollary: $\mathbb{Z}_{2520}$ has 576 units (elements with inverses).

Corollary: $\mathbb{Z}_{2520}$ is not a field. (In a field everything has an inverse).