Lecture Notes

$\varphi(n)$ counts the number of positive integers i such that $\gcd(i,n)=1$ and $i<n$.

Example: $\varphi(8)$.

$\gcd(8,7)=1$
$\gcd(8,6)=2$
$\gcd(8,5)=1$
$\gcd(8,4)=4$
$\gcd(8,3)=1$
$\gcd(8,2)=2$
$\gcd(8,1)=1$

4 have $\gcd(8,i)=1$.

Therefore $\varphi(8)=4$.

Theorem: $\varphi(p)=p-1$ if p is a prime number.
(because every number i from 1 to $p-1$ has $\gcd(i,p)=1$).

Example: $\varphi(5)=4$. $\varphi(29)=29-1=28$. (29 and 5 are prime).

Theorem: $\varphi(p\times q)=\varphi(p)\times\varphi(q)$ if $\gcd(p,q)=1$.

Example: $\varphi(15)=\varphi(5\times 3)=\varphi(5)\times\varphi(3)=4\times 2=8$.

We can find $\varphi(n)$ for any n once we know its prime factors (this is assuming no prime factor shows up more than once.)

Example: $\varphi(105)=\varphi(5\times 3\times 7)=\varphi(5)\times\varphi(3)\times\varphi(7)$
$=4\times 2\times 6=48$