

Multiplication Table.

\mathbb{Z}_5	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

MULTIPLICATIVE INVERSES
IN \mathbb{Z}_n .

Inverse of 2 is 3

because $2 \times 3 = 1 \pmod{5}$.

In \mathbb{Z}_5 , every element has an inverse.

\mathbb{Z}_9	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	1	3	5	7
3	3	6	0	3	6	0	3	6
4	4	8	3	7	2	6	1	5
5	5	1	6	2	7	3	8	4
6	6	3	0	6	3	0	6	3
7	7	5	3	1	8	6	4	2
8	8	7	6	5	4	3	2	1

Not all elements have inverses.

2 has an inverse, since $2 \times 6 = 1 \pmod{9}$

But 3 has no inverse!

Observe that any number that has $\gcd = 1$ with 9 has an inverse. This is if and only if.

For example: $\gcd(2, 9) = 1$ therefore 2 has an inverse in \mathbb{Z}_9 .
 $\gcd(6, 9) = 3 \neq 1$ therefore 3 has NO inverse in \mathbb{Z}_9 .

An element $m \in \mathbb{Z}_n$ has a (multiplicative) inverse if and only if $\gcd(m, n) = 1$.