Euclidean Algorithm (Extended).

Suppose we want to find gcd (640, 84)

Can find it with <u>Method 1</u>: $\left.\begin{array}{l} 640 = 2^7 \times 5 \\ 84 = 2^2 \times 3 \times 7 \end{array}\right\} \Rightarrow gcd = 2^2 = 4.$

Or with <u>Method 2</u>:

$\boxed{640}^{r_1} = \boxed{84}^{r_2} \times 7^{q_3} + \boxed{52}^{r_3}$

$\boxed{84} = \boxed{52} \times 1^{q_4} + \boxed{32}^{r_4}$

$\boxed{52} = \boxed{32} \times 1^{q_5} + \boxed{20}^{r_5}$

$\boxed{32} = \boxed{20} \times 1^{q_6} + \boxed{12}^{r_6}$

$\boxed{20} = \boxed{12} \times 1^{q_7} + \boxed{8}^{r_7}$

$\boxed{12} = \boxed{8} \times 1^{q_8} + \boxed{4}^{r_8}$

$\boxed{8} = \boxed{4} \times 2^{q_9} + \boxed{0}^{r_9}$

The last non-zero r_i is the gcd

So gcd (640, 84) = 4.

"Extended" Euclidean Algorithm.

q	640	84	r	
none	1	0	640	r_1
none	0	1	84	r_2
7	1	−7	52	r_3
1	−1	8	32	r_4
1	2	−15	20	r_5
1	−3	23	12	r_6
1	5	−38	8	r_7
1	−8	61	4	r_8
2	21	−160	0	r_9

Notice: In this table, after you fill in the q column and the r column, and the $\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$ under the $\begin{array}{c|c} 640 & 84 \\ \hline & \end{array}$, each number left to calculate is found from taking the number 2 steps above, and subtracting the product of the number 1 step above times the q in the same row.

The above table gives us the numbers so that for example:

$r_3 = 52 = 640 \times (1) + 84 \times (-7)$

or

$r_6 = 12 = 640 \times (-3) + 84 \times (23)$

or most importantly, gcd(640, 84) = 4

and

$r_8 = 4 = 640 \times (-8) + 84 \times (61)$.