Trigonometry

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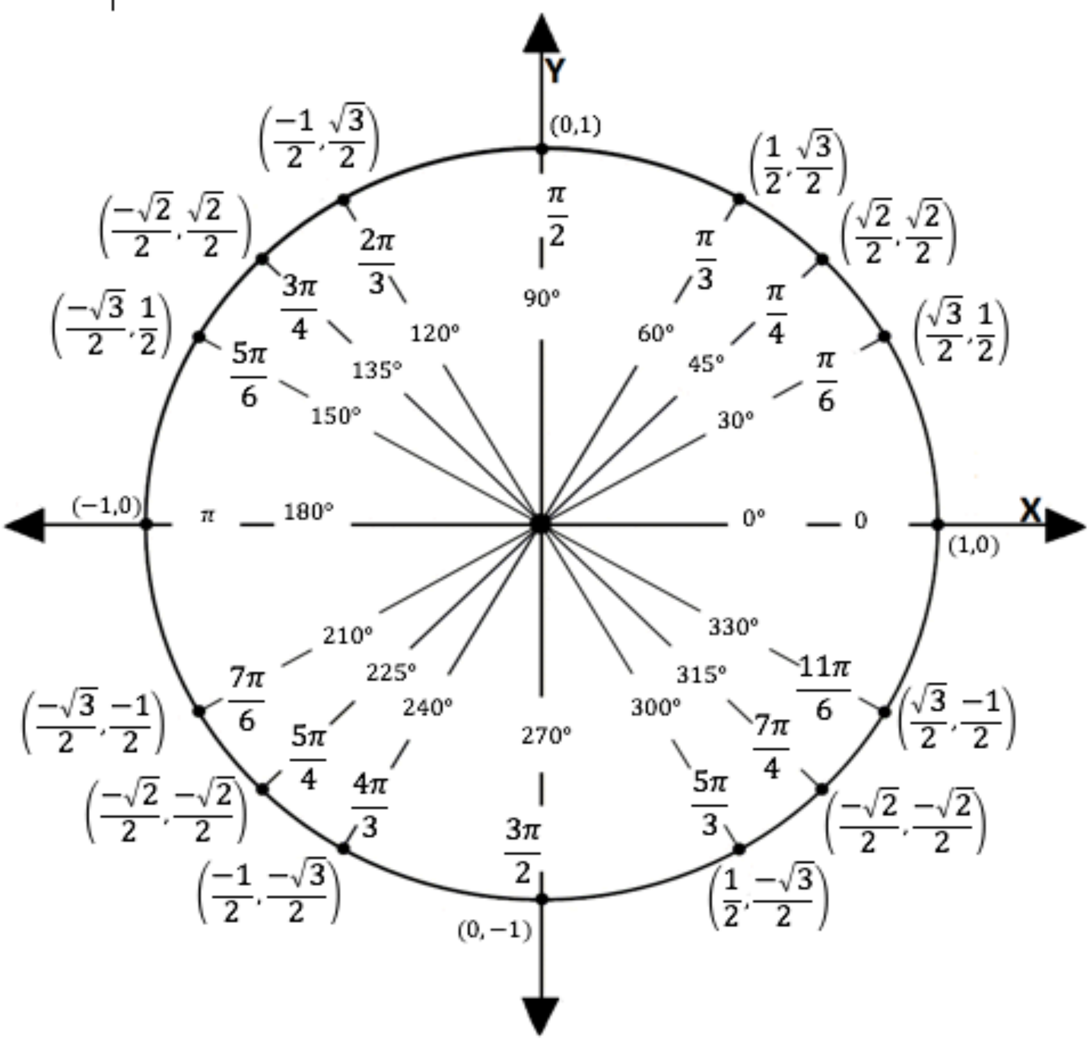
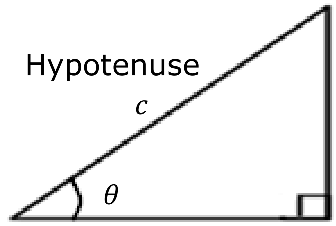


<u>Trigonometric Functions</u>:

Of an <u>acute angle</u>:

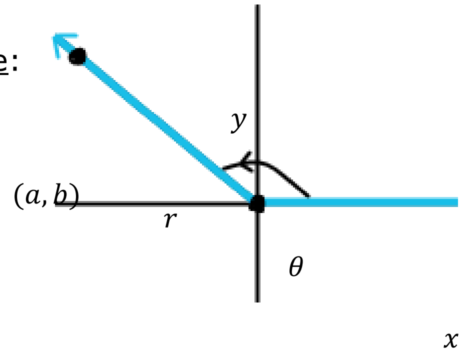


$$\sin\theta = \frac{b}{c} = \frac{Opposite}{Hypotenuse} \qquad \cos\theta = \frac{a}{c} = \frac{Adjacent}{Hypotenuse}$$

$$\tan\theta = \frac{b}{a} = \frac{Opposite}{Adjacent} \qquad \csc\theta = \frac{c}{b} = \frac{Hypotenuse}{Opposite}$$

$$\sec\theta = \frac{c}{a} = \frac{Hypotenuse}{Adjacent} \qquad \cot\theta = \frac{a}{b} = \frac{Adjacent}{Opposite}$$

Of a general angle:



$\sin\theta = \frac{b}{r}$ $\cos\theta = \frac{a}{r}$ $\tan\theta = \frac{b}{a}, a \neq 0$

$\csc\theta = \frac{r}{b}, a \neq 0$ $\sec\theta = \frac{r}{a}, a \neq 0$ $\cot\theta = \frac{a}{b}, a \neq 0$

Trigonometric Identities:

Fundamental Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

Half-Angle Formulas:

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$

Double-Angle Formulas:

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = 2\cos^2\theta - 1$

$\cos(2\theta) = 1 - 2\sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

Even-Odd Identities:

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$

$\sec(-\theta) = \sec\theta$

$\cot(-\theta) = -\cot\theta$

Product-to-Sum Formulas:

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

$\sin\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

Sum and Difference Formulas:

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

Sum-to-Product Formulas:

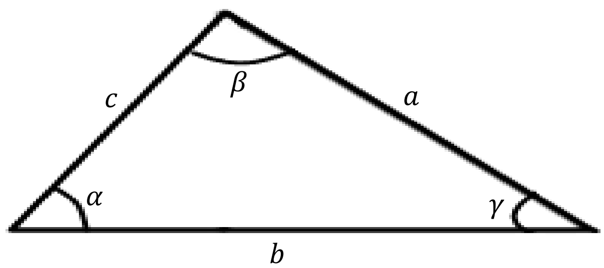
$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

Solving Triangles:



Law of Sines: $\frac{\sin\ \alpha}{a} = \frac{\sin\ \beta}{b} = \frac{\sin\ \gamma}{c}$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cdot \cos \gamma$