

Chapters 23 and 24: Electric Potential and Capacitance

1. The concepts covered in this chapter are:

1. Electric potential
2. Electric potential energy
3. Capacitance
4. Capacitors arranged in series and parallel configuration
5. Air- and dielectric-filled capacitors
6. Potential energy stored in capacitors

2. Potential and Potential Energy.

In the previous chapters we learned that charges exert a force on each other according to Coulomb's law. This means that such forces that occur among the charges can change a given configuration of charges: in the field view of electrostatic interaction the electric fields of various charges interact to make the charges move, i.e., the electric field does work on the charge configuration). The configuration can also be changed by an external agent, i.e., work can be performed by an external agent. This change in charge configuration will show up as a change in the potential energy of the system (the charge configuration).

The **change in the potential energy** *between* (or *of*) two charges is defined as

$$\Delta W_{\text{field}} = -\Delta W_{\text{ext}} = -\Delta U \quad \dots[1]$$

$$-\Delta U_{ab} = U_a - U_b = \int_a^b \vec{F} \cdot d\vec{r} = q \int_a^b \vec{E} \cdot d\vec{r} \quad \dots[2]$$

The **potential difference** $\Delta V = V_b - V_a$ is defined as:

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{r} \quad \dots[3]$$

Potential can be defined as the potential energy per unit charge (recall a similar definition of the electric field $\mathbf{E}$ as the force per unit charge, $q\mathbf{E} = \mathbf{F}$)

A point charge Q , creates an electric potential $V(r)$ at a distance r from it given by:

$$V(r) = kQ/r \quad (\text{referenced to infinity, i.e., } V(\infty) = 0) \quad \dots[4]$$

The **net potential** at a given point due to a collection of charges is simply the algebraic sum of individual potentials

$$V = k \sum_{i=1}^n \frac{Q_i}{r_i} \quad \dots[5]$$

Potential energy of two point charges is given by

$$U_{12} = kQ_1Q_2/r \quad \dots[6]$$

Potential energy of a collection or configuration of point charges is the algebraic sum of the potential energy of all the pairs of charges you can form in the configuration. For example, in Solved Example 1 below, U_i is the potential energy of the 3-charge configuration.

Potential energy of a single charge in a collection or configuration of point charges is the algebraic sum of the potential energy of all the pairs the given charge forms in the configuration. For example, Solved Example 1 the potential energy of Q_1 in part [a] would be the sum $U_{12} + U_{13}$.

Points to remember:

1. Make sure you understand the difference between potential and potential energy, and between potential energy *of* a charge configuration and the potential energy of a charge *in* a charge configuration.
2. A single charge will create a potential at a point in space, but you need at least two charges for the potential energy to have any meaning.
3. Potential and potential energy are scalars.
4. Eq. [3] can also be expressed as :

$E_x = -dV/dx$ (and similar relations in the y- and z-directions) .This allows one to calculate the components of $\mathbf{E}$ (a vector) by knowing the potential $V(x,y,z)$ which is a scalar. Please see sec. 23-5 of your Text for additional details.

3. Capacitance

The capacitance C is defined as $C = Q/V$. C is always positive. When we talk about the charge on a capacitor we mean the magnitude of the charge on one of its plates.

The capacitors can be arranged in a series or a parallel arrangement (see sec 24-2 of your Text for diagrams of capacitors in parallel and series combinations)

In a series arrangement:

[i] the net capacitance C is given by;

$$1/C = 1/C_1 + 1/C_2 + 1/C_3 + \dots \quad \dots[7]$$

[ii] the magnitude of the charge on every capacitor is the same.

[iii] the potential difference across each capacitor is different and is given by $V_i = Q/C_i$.

In a parallel arrangement:

[i] the net capacitance C is given by;

$$C = C_1 + C_2 + C_3 + \dots \quad \dots[8]$$

[ii] the potential difference V across each capacitor is the same.

[iii] the charge on every capacitor is different and is given by $Q_i = V C_i$.

The capacitance of a parallel plate capacitor of plate area A and separation d is given by:

$$C = A\epsilon_0/d \quad \dots[9]$$

The energy stored in a capacitor is given by

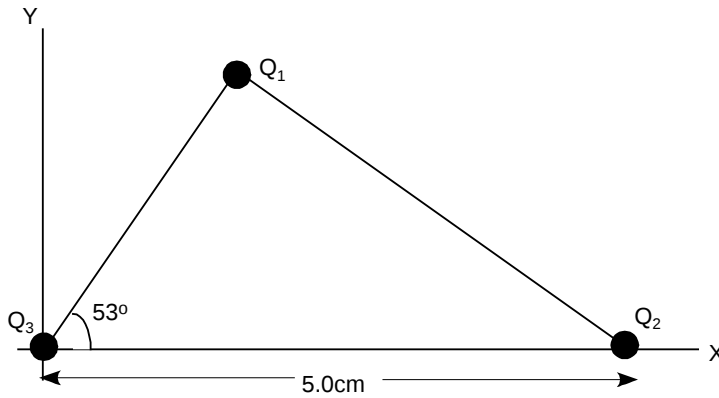
$$U = \frac{1}{2} CV^2 \quad \dots[10]$$

By using $C = Q/V$, eq [10] can be cast in terms of other parameters of interest:

$$U = \frac{1}{2} QV \text{ or } U = \frac{1}{2} Q^2/C$$

Insertion of a dielectric medium in a capacitor will change its capacitance by a factor of κ , where κ is the dielectric constant of the medium, $C_\kappa = \kappa C_0$.

4. Solved Examples.



1. Three charges are arranged at the vertices of a right-angle triangle as shown, with $Q_1 = 4\mu\text{C} = 2Q_2$, and $Q_3 = -3\mu\text{C}$.

[a] Determine the total potential energy of the three-charge system.

[b] Determine the work done by an external force to move Q_2 from its present position to infinity.

[a] The initial potential energy,

$$U_i = 9 \times 10^9 \left(\frac{-12 \times 10^{-12}}{3 \times 10^{-2}} - \frac{6 \times 10^{-12}}{5 \times 10^{-2}} + \frac{8 \times 10^{-12}}{4 \times 10^{-2}} \right) = -2.88 \text{ J}$$

[b] after Q_2 has been removed to infinity, the potential energy of the remaining charges is

$$U_f = 9 \times 10^9 \left(\frac{-12 \times 10^{-12}}{3 \times 10^{-2}} \right) = -3.60 \text{ J}$$

the work done by the external force is given by $\Delta W = \Delta U = U_f - U_i = -3.60 - (-2.88) = -0.72 \text{ J}$

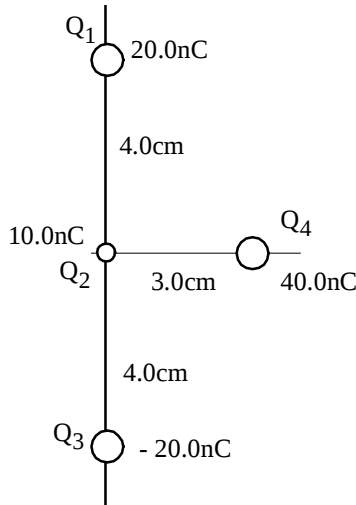
Note: in the problem above we have used $\Delta W = \Delta U = U_f - U_i$, where ΔU is the change in the potential energy of the charge configuration. We could have also done the problem by looking at it from the point of view of Q_2 (in a manner of speaking, charges don't really have a point of view). In this case ΔU would be the change in the potential energy of Q_2 .

$$\Delta U = U_f(Q_2) - U_i(Q_2) = 0 - (U_{12} + U_{23}) = -9 \times 10^9 (-1.2 + 2.0) 10^{-10} = -0.72 \text{ J}$$

2. Consider three charges Q_1 , Q_2 , and Q_3 placed as shown in the diagram below(disregard Q_4 for the moment)

[a] Calculate the electric potential energy of the three-charge configuration.

[b] A fourth particle of mass $m = 2.0 \times 10^{-13} \text{ kg}$ and charge, $Q_4 = 40.0 \text{ nC}$ is placed on the x- axis 3.0 cm from Q_2 and released from rest. Find the speed of Q_4 when it has moved very far away from the three-charge (Q_1 , Q_2 , and Q_3) configuration.



This is a rather longish method but has several explanatory details. The problem can also be solved in a much more compact way. The shorter method follows the long one.

- (a) In an empty universe, the 20-nC charge can be placed at its location with no energy investment. At a distance of 4 cm, it creates a potential

$$V_1 = \frac{k_e q_1}{r} = \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(20 \times 10^{-9} \text{ C})}{0.04 \text{ m}} = 4.50 \text{ kV}$$

To place the 10-nC charge there we must put in energy

$$U_{12} = q_2 V_1 = (10 \times 10^{-9} \text{ C})(4.5 \times 10^3 \text{ V}) = 4.50 \times 10^{-5} \text{ J}.$$

Next, to bring up the -20-nC charge requires energy

$$\begin{aligned} U_{23} + U_{13} &= q_3 V_2 + q_3 V_1 = q_3 (V_2 + V_1) \\ &= -20 \times 10^{-9} \text{ C} (9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \left(\frac{10 \times 10^{-9} \text{ C}}{0.04 \text{ m}} + \frac{20 \times 10^{-9} \text{ C}}{0.08 \text{ m}} \right) \\ &= -4.50 \times 10^{-5} \text{ J} - 4.50 \times 10^{-5} \text{ J} \end{aligned}$$

The total energy of the three charges is

$$U_{12} + U_{23} + U_{13} = \boxed{-4.50 \times 10^{-5} \text{ J}}.$$

- (b) The three fixed charges create this potential at the location where the fourth is released:

$$V = V_1 + V_2 + V_3 = (9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \left(\frac{20 \times 10^{-9}}{\sqrt{0.04^2 + 0.03^2}} + \frac{10 \times 10^{-9}}{0.03} - \frac{20 \times 10^{-9}}{0.05} \right) \text{ C/m}$$

$$V = 3.00 \times 10^3 \text{ V}$$

Energy of the system of four charged objects is conserved as the fourth charge flies away:

$$\left(\frac{1}{2} mv^2 + qV \right)_i = \left(\frac{1}{2} mv^2 + qV \right)_f$$

$$0 + (40 \times 10^{-9} \text{ C})(3.00 \times 10^3 \text{ V}) = \frac{1}{2} (2.00 \times 10^{-13} \text{ kg}) v^2 + 0$$

$$v = \sqrt{\frac{2(1.20 \times 10^{-4} \text{ J})}{2 \times 10^{-13} \text{ kg}}} = \boxed{3.46 \times 10^4 \text{ m/s}}$$

The shorter method.

$$[a] U = U_{12} + U_{23} + U_{13} = U_{13} = -9 \times 10^9 \times 400 \times 10^{-18} / 8 \times 10^{-2} = -4.5 \times 10^{-5} \text{ J}$$

(why is $U_{12} + U_{23} = 0$?)

[b] $(U + KE)_i = (U + KE)_f$ (conservation of energy from last term)

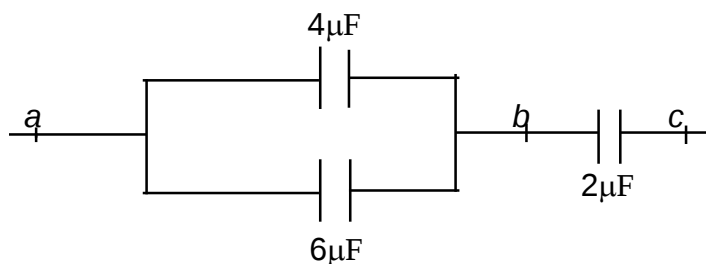
$$U_i = KE_f$$

$$U_i = U_{14} + U_{34} + U_{24} = U_{24} = 9 \times 10^9 \times 400 \times 10^{-18} / 3 \times 10^{-2} = 12.0 \times 10^{-5} \text{ J} = \frac{1}{2} \times 2 \times 10^{-13} v^2$$

(why is $U_{14} + U_{34} = 0$?)

$$v = [12]^{1/2} 10^4 \text{ m/s} = 3.46 \times 10^4 \text{ m/s}$$

3. Consider the three capacitors connected as shown. A potential difference of $V_{ac} = 120 \text{ V}$ is maintained between points a and c .



[a] What is the potential difference

V_{bc} across the $2 \mu\text{F}$ capacitor?

[b] Determine the charge on the

$4 \mu\text{F}$ capacitor.

[c] How much energy is stored in the $6 \mu\text{F}$ capacitor ?

[a] The $4\mu\text{F}$ and the $6\mu\text{F}$ capacitors can be combined into a single equivalent capacitor of capacitance, $10\mu\text{F}$.

$$V_{ab} = Q/10\mu\text{F} \quad V_{bc} = Q/2\mu\text{F} \quad \text{or } V_{ab}/V_{bc} = 2/10 = 1/5;$$

$$5V_{ab} = V_{bc}$$

Also $V_{ab} + V_{bc} = 120\text{V}$, solving the last two equations gives $V_{ab} = 20\text{V}$, and $V_{bc} = 100\text{V}$

$$\text{[b]} \quad Q_4 = V_{ab} 4 \times 10^{-6} = 8 \times 10^{-5} \text{ C}$$

$$\text{[c]} \quad U = \frac{1}{2}(6.0 \times 10^{-6})(V_{ab})^2 = 1.2 \times 10^{-3} \text{ J}$$

4. Four capacitors, each with a capacitance $C = 4.00\mu\text{F}$, are connected as shown. The potential difference $V_{ab} = 25.0\text{V}$.

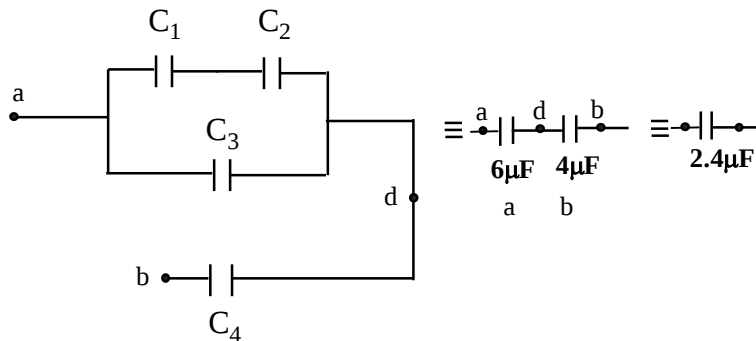
[a] What is the equivalent capacitance between a and d ?

[b] What is the potential difference between a and d ?

[c] What is the charge on C_4 ?

[d] What is the charge on C_1 ?

[e] What is the total electric energy stored in the four-capacitor system?



$$\text{[a]} \quad C_{ad} = (C_1 C_2 / C_1 + C_2) + C_3 = 6.0\mu\text{F}$$

(the resultant of C_1 and C_2 connected in series is connected in parallel with C_3)

$$\text{[b]} \quad V_{ad}/V_{db} = C_{db}/C_{ad} = 4/6 \quad \text{or} \quad V_{db} = 1.5V_{ad} \quad \dots[1]$$

$$\text{Also note that } V_{ad} + V_{db} = 25.0\text{V} \quad \dots[2]$$

From eqs. [1] and [2]

$$V_{ad} + 1.5V_{ad} = 25.0V \quad \text{or} \quad V_{ad} = 10.0V \quad \text{and} \quad V_{db} = 15.0V$$

$$[c] \quad Q_4 = C_4 * V_{db} = 4.0\mu F * 15.0V = 60.0\mu C$$

$$[d] \quad Q_1 = C_1 * (V_{ad}/2) = 4.0\mu F * 5.0V = 20.0\mu C$$

(Note: $V_{ad}/2$ is the potential across C_1)

$$[e] \quad U = 0.5C_{net} V_{ab}^2 = 0.5 * 2.4\mu F * (25.0)^2 = 7.5 * 10^{-4} J$$

Concept Questions

1. A spherical balloon contains a positively charged object at its center. **(i)** As the balloon is inflated to a greater volume while the charged object remains at the center, does the electric potential at the surface of the balloon **(a)** increase, **(b)** decrease, or **(c)** remain the same? **(ii)** Does the electric flux through the surface of the balloon **(a)** increase, **(b)** decrease, or **(c)** remain the same?

Answer: (i), (b). The electric potential is inversely proportional to the radius (see Eq. 20.11). (ii), (c). Because the same number of field lines passes through a closed surface of any shape or size, the electric flux through the surface remains constant.

$$V = k_e \frac{q}{r}$$

2. **(i)** In a certain region of space, the electric potential is zero everywhere along the x axis. From this information, we can conclude that the x component of the electric field in this region is **(a)** zero, **(b)** in the $+x$ direction, or **(c)** in the $-x$ direction. **(ii)** In a certain region of space, the electric field is zero. From this information, we can conclude that the electric potential in this region is **(a)** zero, **(b)** constant, **(c)** positive, or **(d)** negative.

Answer: (i), (a). If the potential is constant (zero in this case), its derivative along this direction is zero. (ii), (b). If the electric field is zero, there is no change in the electric potential and it must be constant. This constant value *could be* zero, but it does not *have to be* zero.

3. A capacitor stores charge Q at a potential difference V . If the voltage applied by a battery to the capacitor is doubled to $2V$, **(a)** the capacitance falls to half its initial value and the charge remains the same, **(b)** the capacitance and the charge both fall to half their initial values, **(c)** the capacitance and the charge both double, or **(d)** the capacitance remains the same and the charge doubles.

Answer: (d). The capacitance is a property of the physical system and does not vary with applied voltage. According $C=Q/V$ if the voltage is doubled, the charge is doubled.

4. Two capacitors are identical. They can be connected in series or in parallel. **(i)** If you want the *smallest* equivalent capacitance for the combination, **(a)** do you connect them in series, **(b)** do you connect them in parallel, or **(c)** do the combinations have the same capacitance? **(ii)** Each capacitor is charged to a voltage of $10V$. If you want the largest combined potential difference across the combination, **(a)** do you connect them in series, **(b)** do you connect them in parallel, or **(c)** do the combinations have the same potential difference?

Answer: (i), (a). When connecting capacitors in series, the inverse of the capacitances add, resulting in a smaller overall equivalent capacitance. (ii), (a). When capacitors are connected in series, the voltages add, for a total of $20V$ in this case. If they are combined in parallel, the voltage across the combination is still $10V$.

5. You have three capacitors and a battery. In which of the following combinations of the three capacitors will the maximum possible energy be stored when the combination is attached to the battery? **(a)** When in series the maximum amount is stored. **(b)** When parallel the maximum amount is stored. **(c)** Both combinations will store the same amount of energy.

Answer: (b). For a given voltage, the energy stored in a capacitor is proportional to C : $U = C(V)^2/2$. Therefore, you want to maximize the equivalent capacitance. You do so by connecting the three capacitors in parallel so that the capacitances add.

6. When the potential difference between the plates of a capacitor is doubled, the magnitude of the electric energy stored in the capacitor :

[a] is doubled

[b] is halved

[c] remains the same

[d] quadrupled

7. Three charges are placed on the three corners of an equilateral triangle as shown.. The total potential energy of these charges is:

- [a] $-kQ^2/r$ [b] $-1.5kQ^2/r$ [c] zero [d] $+1.5kQ^2/r$

