The Electric Potential

- The potential energy per unit charge, U/q_o, is the **electric potential**
  - The potential is independent of the value of q_o
  - The potential has a value at every point in an electric field
- The electric potential is $$V = \frac{U}{q_o}$$

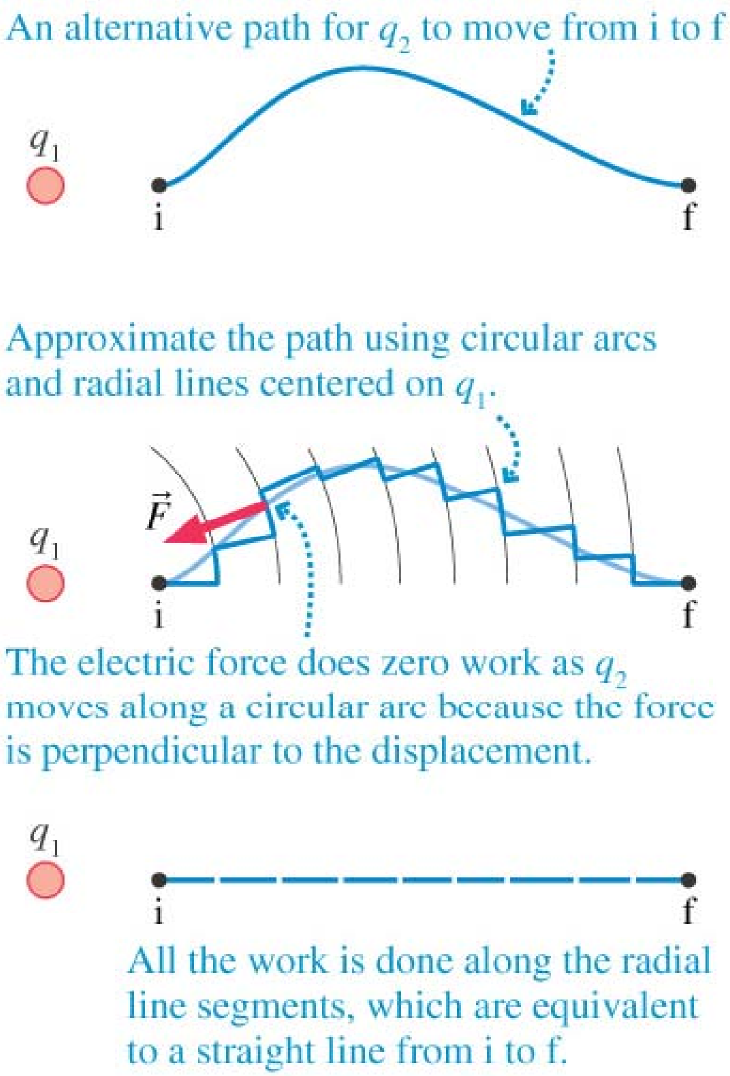
The Electric Potential

- The potential is a scalar quantity
  - Since energy is a scalar
- As a charged particle moves in an electric field, it will experience a change in potential

$$\Delta V = \frac{\Delta U}{q_o} = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

- The *difference* in potential is the meaningful quantity
- We often take the value of the potential to be zero at some convenient point in the field

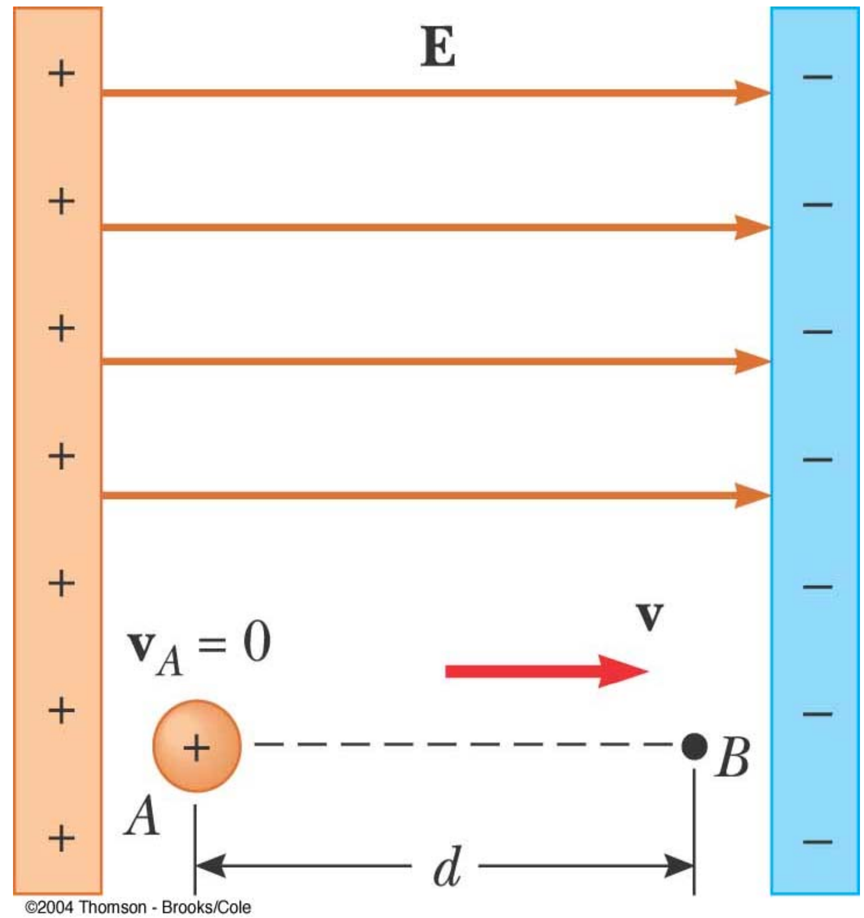
The Electric Force is a Conservative Force



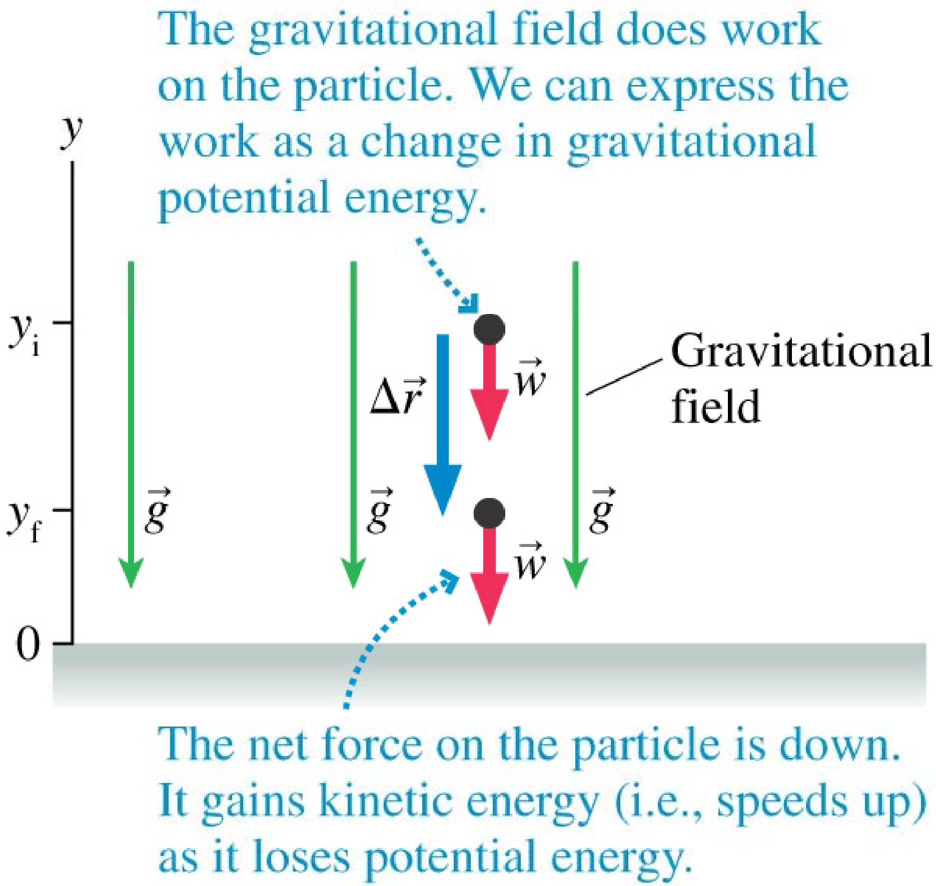
Potential energy can be defined only if the force is conservative, meaning that the work done on the particle as it moves from position *i* to position *f* is independent of the path followed between *i* and *f*.

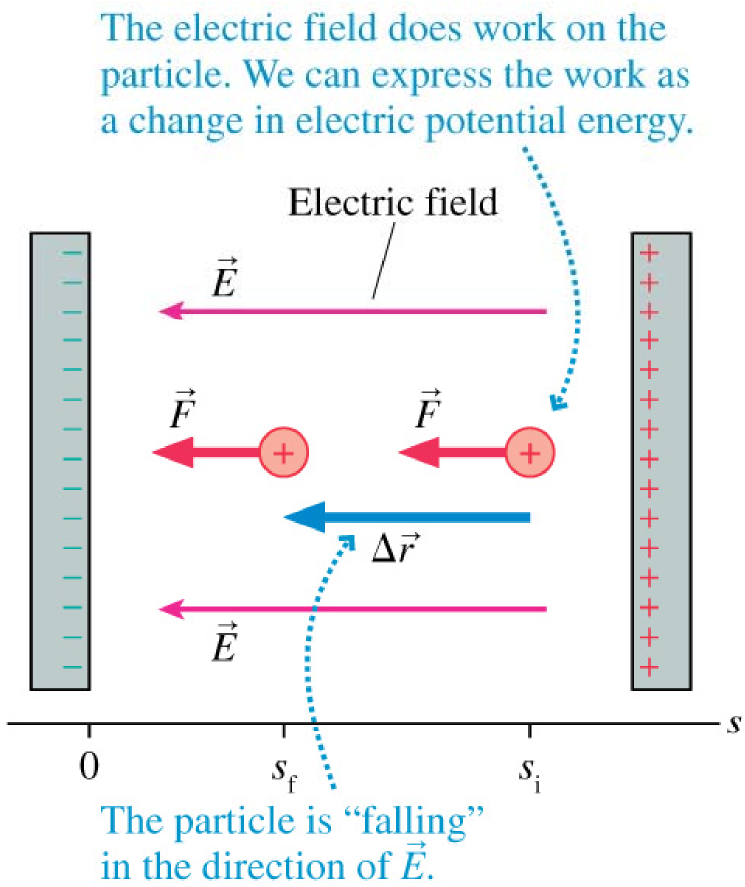
Charged Particle in a Uniform Field

- A positive charge is released from rest and moves in the direction of the electric field
- The change in potential is negative
- The change in potential energy is negative
- The force and acceleration are in the direction of the field

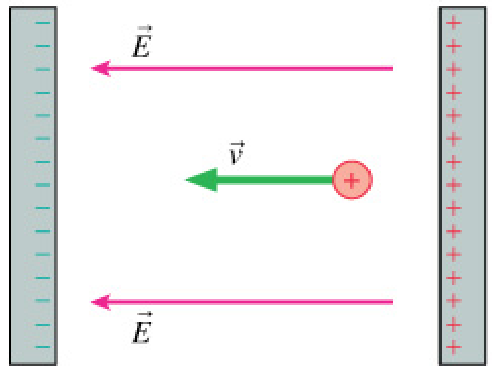


A Uniform Field

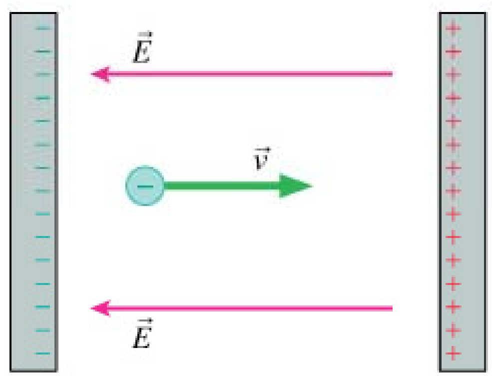




A Uniform Field

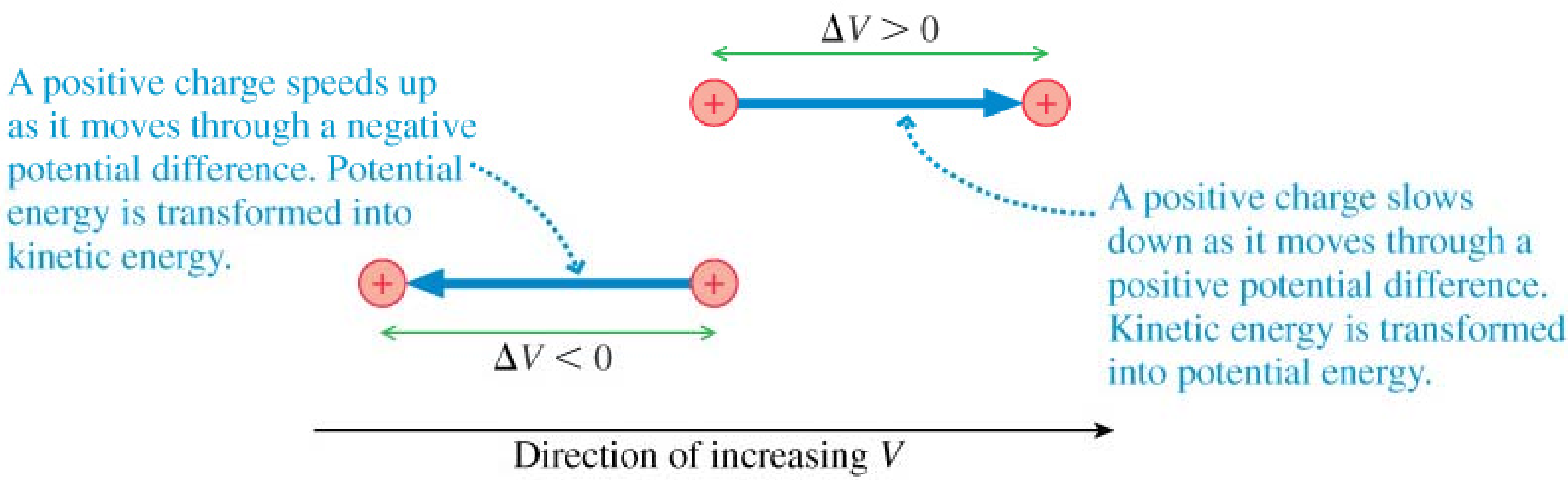


The potential energy of a positive charge decreases in the direction of $\vec{E}$. The charge gains kinetic energy as it moves toward the negative plate.



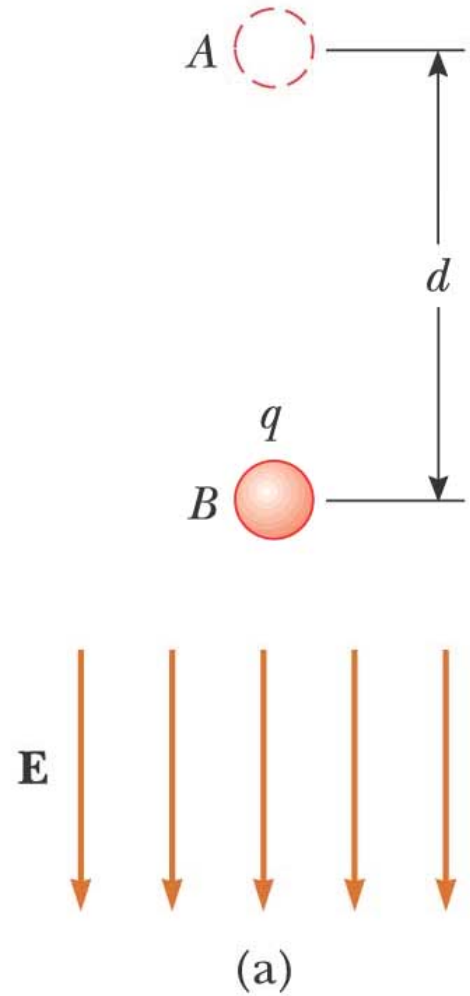
The potential energy of a negative charge decreases in the direction opposite to $\vec{E}$. The charge gains kinetic energy as it moves away from the negative plate.

The Electric Potential



Energy and the Direction of Electric Fi

- When the electric field is directed downward, point *B* is at a lower potential than point *A*
- When a positive test charge moves from *A* to *B*, the charge-field system loses potential energy



More About Directions

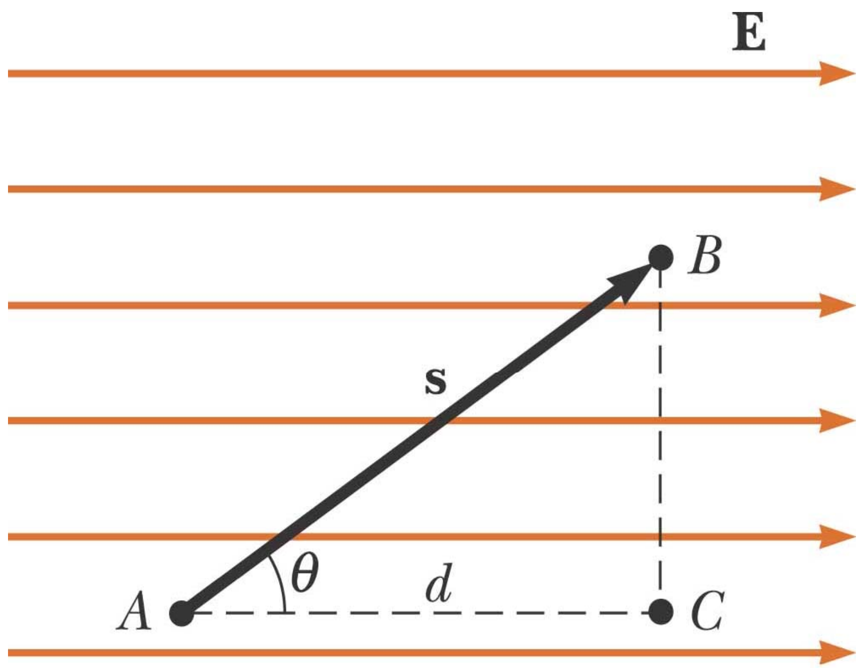
- A system consisting of a positive charge and an electric field **loses** electric potential energy when the charge moves in the direction of the field
  - An electric field does work on a positive charge when the charge moves in the direction of the electric field
- The charged particle gains kinetic energy equal to the potential energy lost by the charge-field system
  - Another example of Conservation of Energy

Directions

- If q_o is negative, then ΔU is positive
- A system consisting of a negative charge and an electric field *gains* potential energy when the charge moves in the direction of the field
  - In order for a negative charge to move in the direction of the field, an external agent must do positive work on the charge

Equipotentials

- Point *B* is at a lower potential than point *A*
- Points *B* and *C* are at the same potential
- The name **equipotential surface** is given to any surface consisting of a continuous distribution of points having the same electric potential



Work and Electric Potential

- Assume a charge moves in an electric field without any change in its kinetic energy
- The work performed on the charge is

  $W = \Delta V = q\, \Delta V$

  Units:

- 1 V = 1 J/C
  – V is a volt
  – It takes one joule of work to move a 1-coulomb charge through a potential difference of 1 volt

Electron-Volts

- One ***electron-volt*** is defined as the energy a charge-field system gains or loses when a charge of magnitude *e* (an electron or a proton) is moved through a potential difference of 1 volt
  - 1 eV = 1.60 x 10^{-19} J

Potential and Point Charges

- The electric potential is independent of the path between points *A* and *B*
- It is customary to choose a reference potential of $V = 0$ at $r_A = \infty$
- Then the potential at some point *r* is

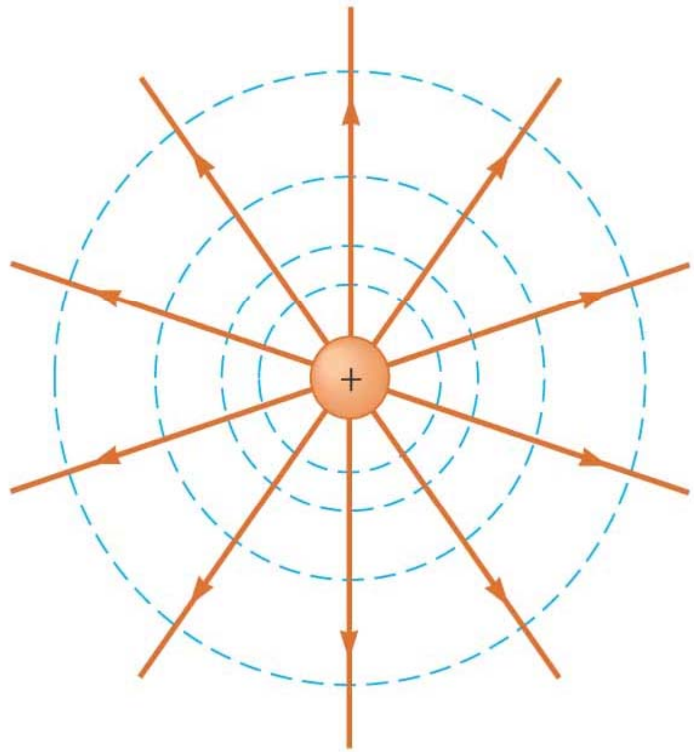
$$V = k_e \frac{q}{r}$$

  – The superposition principle:

$$V = k_e \sum_i \frac{q_i}{r_i}$$

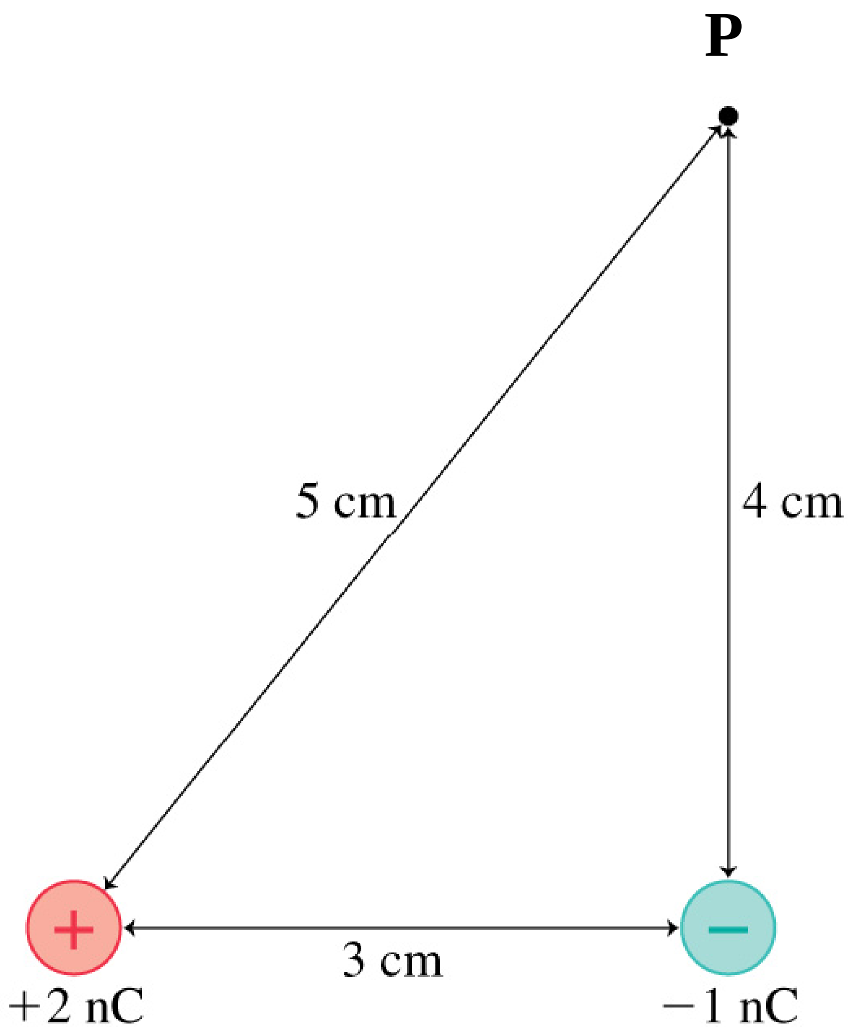
E and *V* for a Point Charge

- The equipotential lines are the dashed blue lines
- The electric field lines are the brown lines
- The equipotential lines are everywhere perpendicular to the field lines



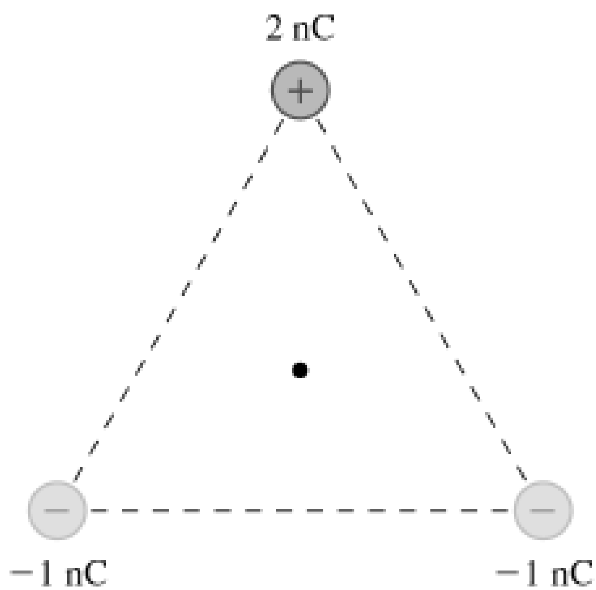
(b)

Quiz 1: What is the potential at P?



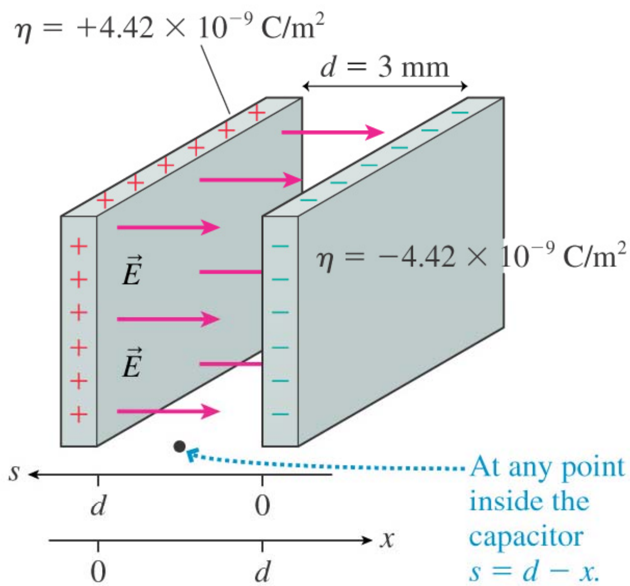
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

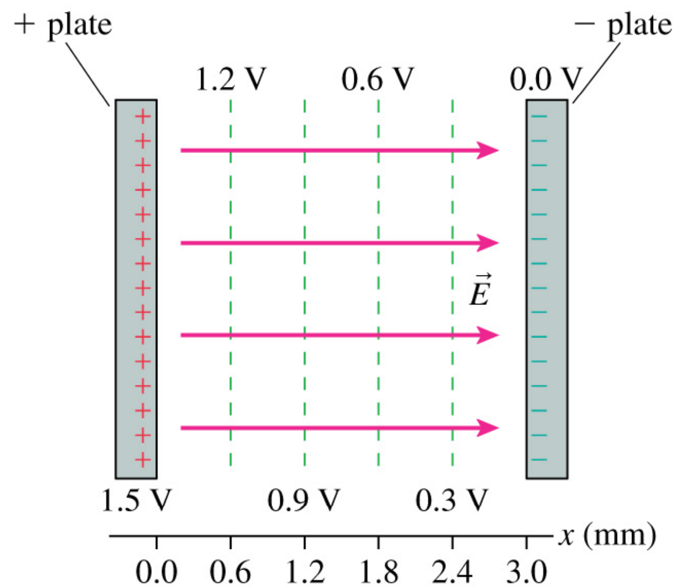
1) Three charges form an equilateral triangle with 5.7 cm long sides. What is the electric potential at the point indicated with the dot?



A) zero B) 1600 V C) 1100 V D) 550 V

The Electric Potential Inside a Parallel-Plate Capacitor





$$U_{elec} = qEs$$

$$V = Es$$

$$\Delta V_C = V_+ - V_- = Ed$$

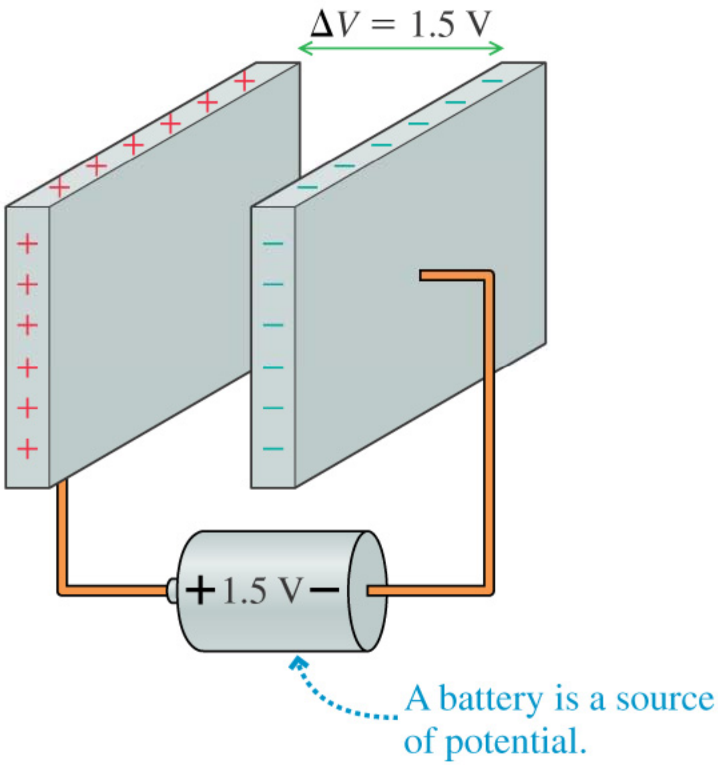
$$E = \frac{\Delta V_C}{d}$$

Potential Difference in a Uniform Field

- The equations for electric potential can be simplified if the electric field is uniform:

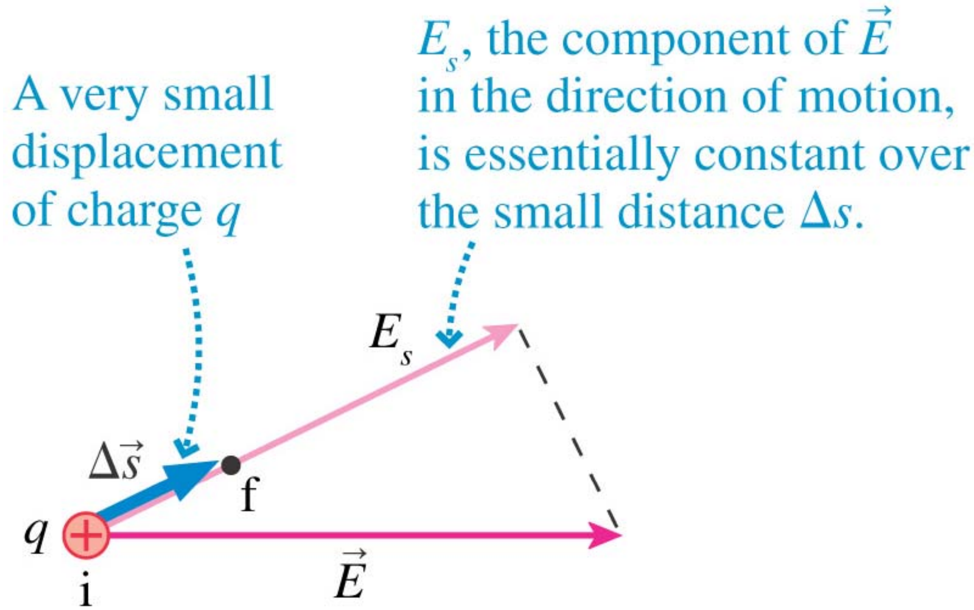
$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -E\int_A^B d\mathbf{s} = -Ed$$

- The negative sign indicates that the electric potential at point *B* is lower than at point *A*



Finding the Electric Field from the Potential

$$E = -\frac{\Delta V_C}{\Delta s}$$



Electric Field from Potential

- In general, the electric potential is a function of all three dimensions
- Equipotential surfaces must always be perpendicular to the electric field lines passing through them
- Given $V(x, y, z)$ you can find E_x, E_y and E_z as partial derivatives

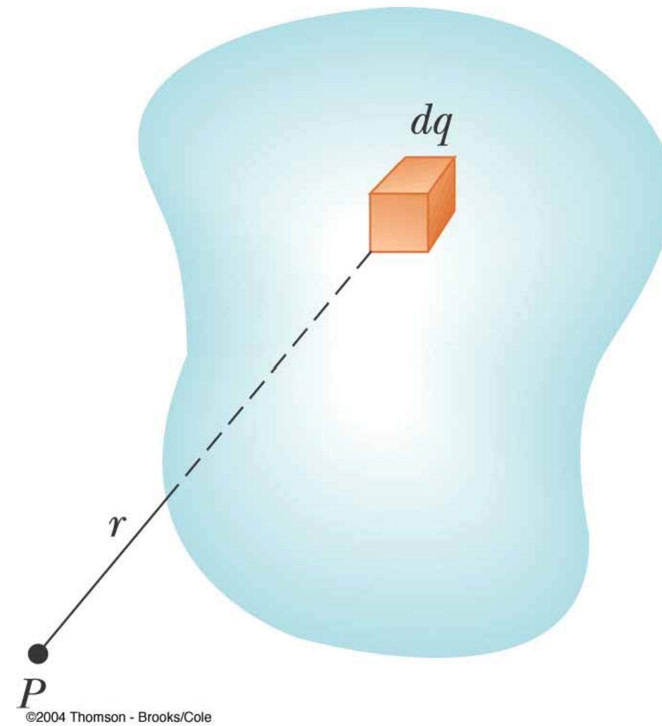
$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

Electric Potential for a Continuous Charge Distribution

- Consider a small charge element *dq*
  - Treat it as a point charge
- The potential at some point due to this charge element is

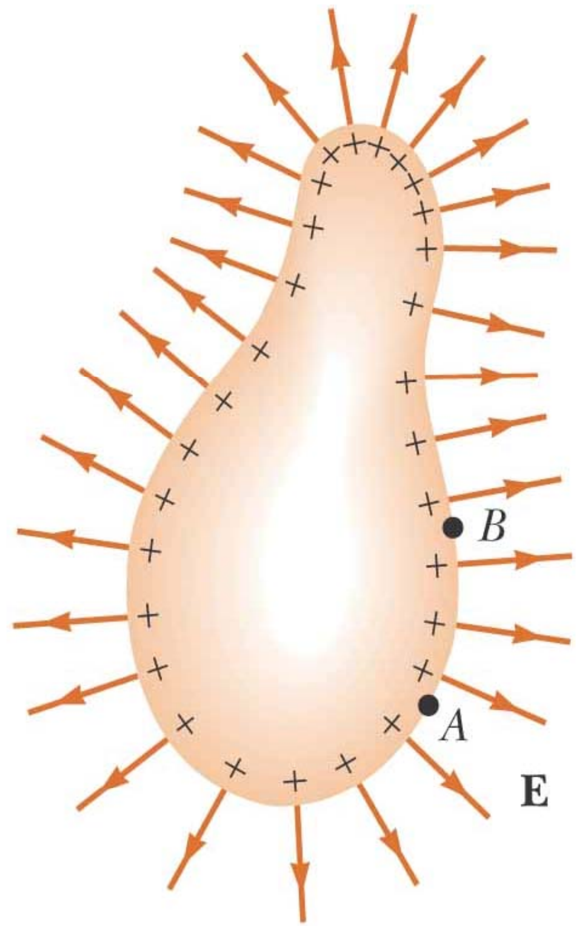
$$dV = k_e \frac{dq}{r}$$

$$V = k_e \int \frac{dq}{r}$$



V Due to a Charged Conductor

- Consider two points on the surface of the charged conductor as shown
- **E** is always perpendicular to the displacement $d\mathbf{s}$
- Therefore, $\mathbf{E} \cdot d\mathbf{s} = 0$
- Therefore, the potential difference between A and B is also zero

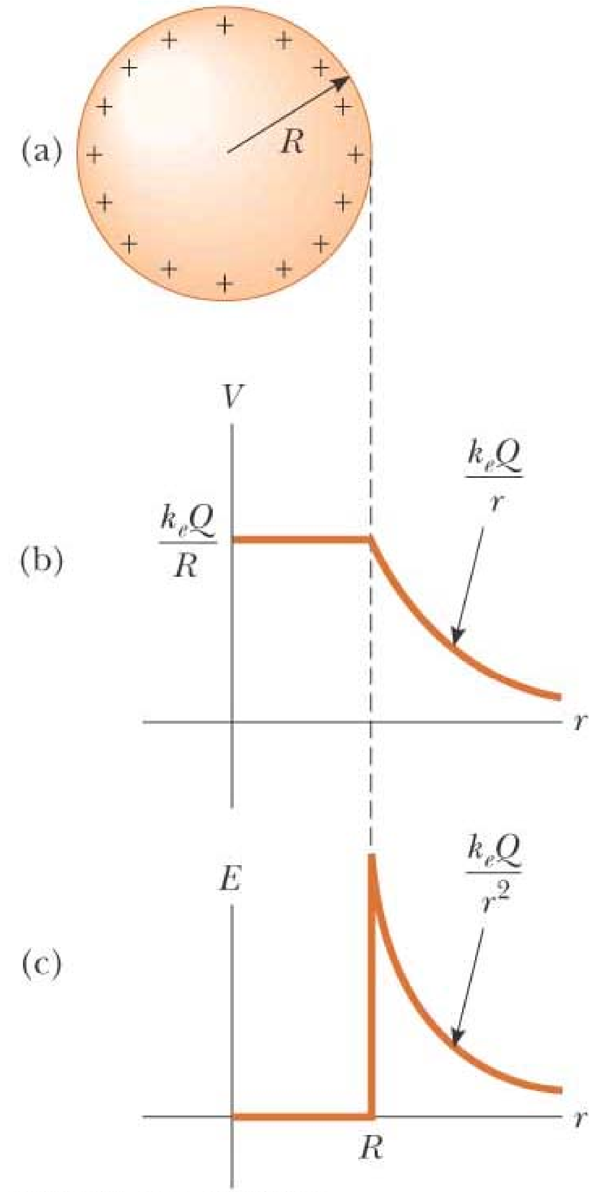


V Due to a Charged Conductor

- V is constant everywhere on the surface of a charged conductor in equilibrium
  - $\Delta V = 0$ between any two points on the surface
- The surface of any charged conductor in electrostatic equilibrium is an equipotential surface
- Because the electric field is zero inside the conductor, we conclude that the electric potential is constant everywhere inside the conductor and equal to the value at the surface

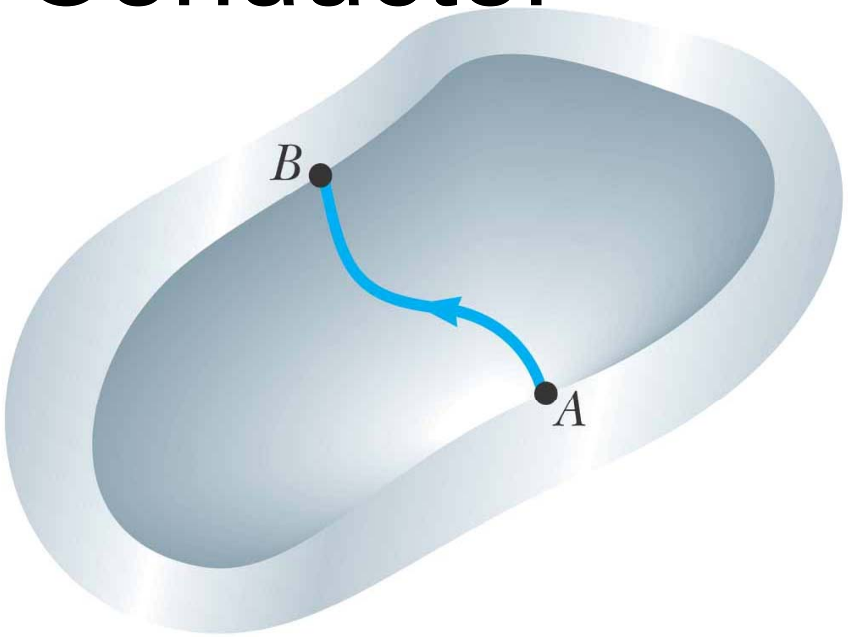
E Compared to *V*

- The electric potential is a function of *r*
- The electric field is a function of r^2
- The effect of a charge on the space surrounding it:
  - The charge sets up a vector electric field which is related to the force
  - The charge sets up a scalar potential which is related to the energy



Cavity in a Conductor

- Assume an irregularly shaped cavity is inside a conductor
- Assume no charges are inside the cavity
- The electric field inside the conductor must be zero

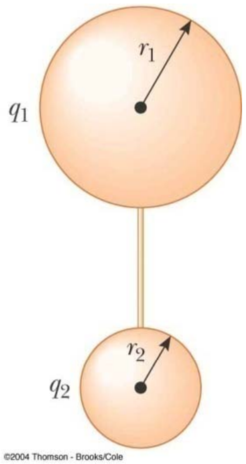


Cavity in a Conductor

- The electric field inside does not depend on the charge distribution on the outside surface of the conductor
- For all paths between *A* and *B*,

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$

- A cavity surrounded by conducting walls is a field-free region as long as no charges are inside the cavity

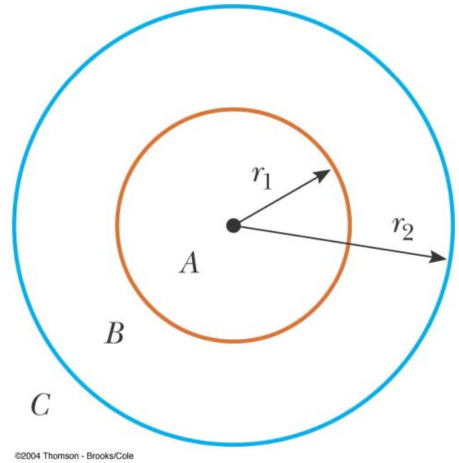


$$V = k_e \frac{q_1}{r_1} = k_e \frac{q_2}{r_2}$$

$$\frac{q_1}{q_2} = \frac{r_1}{r_2}$$

$$E_1 = k_e \frac{q_1}{r_1^{\,2}} \qquad E_2 = k_e \frac{q_2}{r_2^{\,2}}$$

$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$



GENERAL PRINCIPLES

Sources of *V*

The electric potential, like the electric field, is created by charges.

Two major tools for calculating V are

- The potential of a point charge $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$
- The principle of superposition

Multiple point charges

Use superposition: $V = V_1 + V_2 + V_3 + \ldots$

Continuous distribution of charge

- Divide the charge into point-like ΔQ.
- Find the potential of each ΔQ.
- Find V by summing the potentials of all ΔQ.

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a charge density and an integration coordinate. Calculating V is usually easier than calculating $\vec{E}$ because the potential is a scalar.

Consequences of *V*

A charged particle has potential energy

$$U = qV$$

at a point where source charges have created an electric potential V.

The electric force is a conservative force, so the mechanical energy is conserved for a charged particle in an electric potential:

$$K_f + U_f = K_i + U_i$$

The potential energy of **two point charges** separated by distance r is

$$U_{q_1+q_2} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

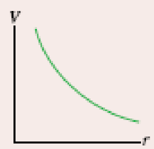
The **zero point** of potential and potential energy is chosen to be convenient. For point charges, we let $U = 0$ when $r \to \infty$.

The potential energy in an electric field of an **electric dipole** with dipole moment $\vec{p}$ is

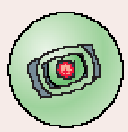
$$U_{\text{dipole}} = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

APPLICATIONS

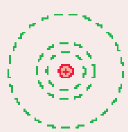
Graphical representations of the potential:



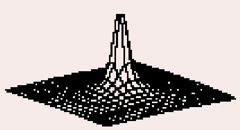
Potential graph



Equipotential surfaces



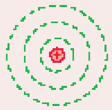
Contour map



Elevation graph

Sphere of charge Q

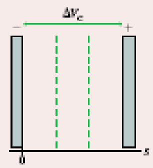
Same as a point charge if $r \geq R$.



Parallel-plate capacitor

$V = Es$, where s is measured from the negative plate. The electric field inside is

$$E = \Delta V_C / d$$



Units

Electric potential: 1 V = 1 J/C

Electric field: 1 V/m = 1 N/C