

# Chapter 14

## Kirchhoff's laws

**Worksheet**

**Worked examples**

**Practical: Determining the e.m.f. of a test cell**

**End-of-chapter test**

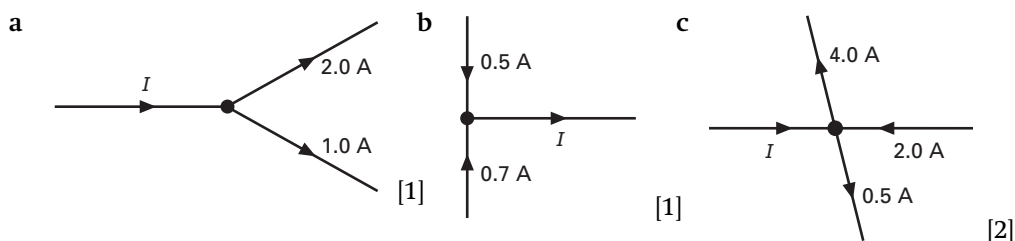
**Marking scheme: Worksheet**

**Marking scheme: End-of-chapter test**

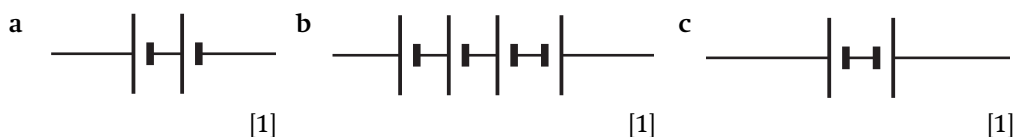
# Worksheet

## Intermediate level

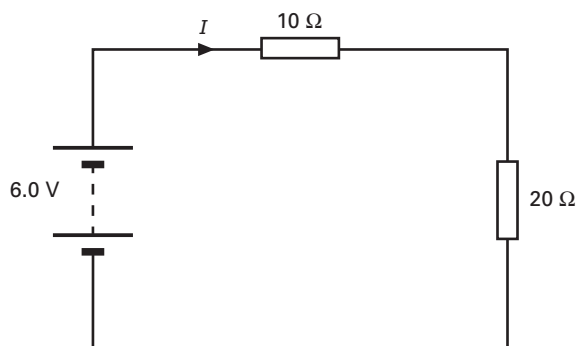
- 1 State Kirchhoff's first law. [1]
- 2 Kirchhoff's first law expresses the conservation of an important physical quantity. Name the quantity that is conserved. [1]
- 3 Determine the current  $I$  in each of the circuits below.



- 4 Several identical cells are used to connect up circuits. Each cell has e.m.f. 1.5 V. Determine the total e.m.f. for the following combinations of cells.



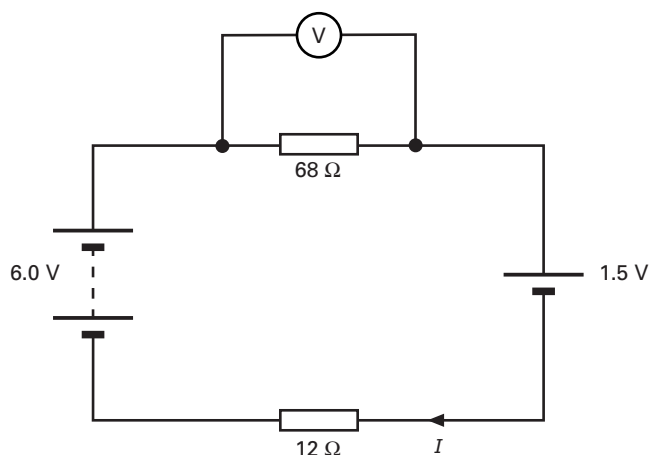
- 5 Use Kirchhoff's second law to calculate the current  $I$  in the circuit shown below. [3]



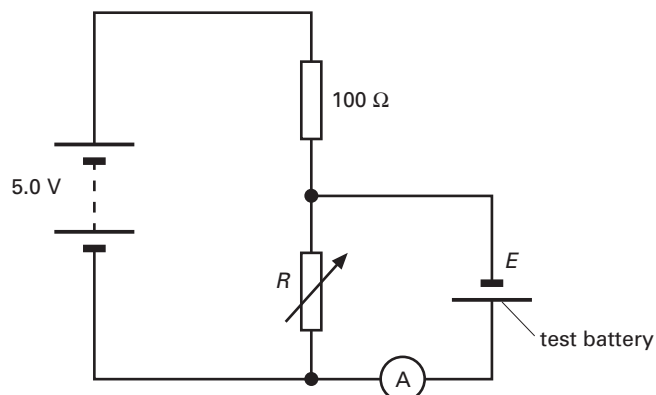
## Higher level

- 6 The diagram shows an electrical circuit. The battery and cell in the circuit may be assumed to have negligible internal resistance. Calculate:

- a the current in the  $12\Omega$  resistor; [3]
- b the p.d. across the  $68\Omega$  resistor. [2]



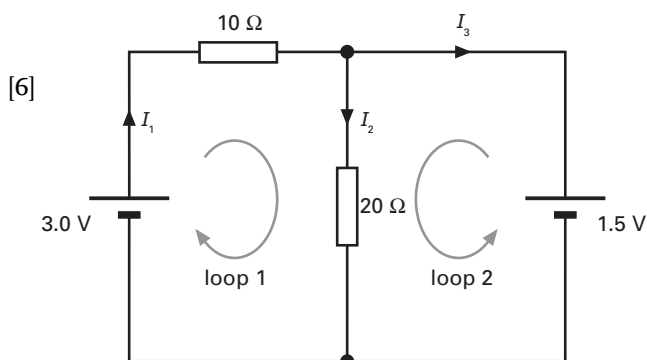
- 7 The arrangement below can be used to determine the electromotive force of a test battery.



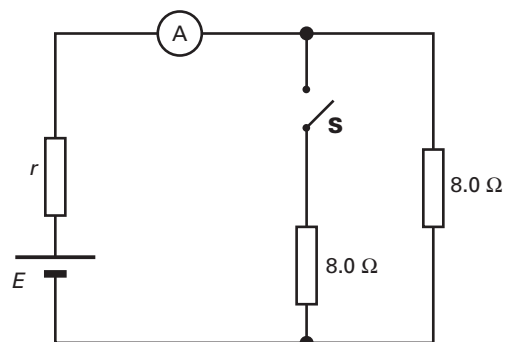
The supply battery may be assumed to have negligible internal resistance. The resistance  $R$  of the variable resistor is adjusted until  $R$  has a value of  $28\ \Omega$  and the current shown by the ammeter is zero. Show that the e.m.f. of the test battery is about  $1.1\text{ V}$ . [3]

### Extension

- 8 Use Kirchhoff's laws to determine the currents  $I_1$ ,  $I_2$  and  $I_3$  in the circuit on the right.



- 9 The current measured by the ammeter in the circuit shown is  $0.25\text{ A}$  when the switch  $S$  is open and  $0.45\text{ A}$  when the switch is closed. Use this information to determine the e.m.f.  $E$  and the internal resistance  $r$  of the cell. [6]

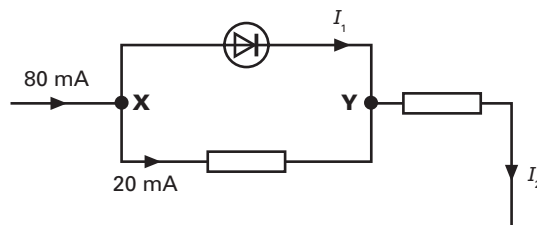


Total:  $\frac{\quad}{32}$  Score: %

# Worked examples

## Example 1

Calculate the currents  $I_1$  and  $I_2$  in the part of the circuit shown.



Applying Kirchhoff's first law to the point X, we have:

$$80 = 20 + I_1$$

$$I_1 = 80 - 20 = 60 \text{ mA}$$

The current  $I_2$  is equal to 80 mA.

The current in a **series** circuit is always the same due to conservation of charge.

### Tip

To determine the current  $I_2$ , you can always apply Kirchhoff's first law again, but this time with reference to point Y.

## Example 2

The diagram shows cells of negligible internal resistance connected in a series circuit.

Determine the circuit current  $I$ .

Applying Kirchhoff's second law, we have:

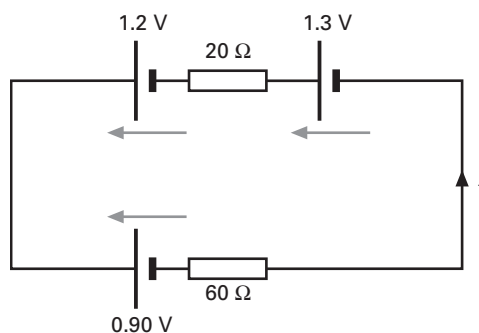
$$\Sigma \text{ e.m.f.} = \Sigma \text{ p.d.}$$

For an anticlockwise 'loop', we have

$$1.3 + 1.2 - 0.90 = (I \times 20) + (I \times 60)$$

$$1.60 = 80I$$

$$I = \frac{1.6}{80} = 0.02 \text{ A}$$



The 0.90 V cell is 'reversed'. Hence its e.m.f. is assigned a **negative** value in this equation.

### Tip

To minimise errors when applying Kirchhoff's second law, always check the polarity of each source of e.m.f. This is indicated by the arrows on the diagram.

# Practical

## Determining the e.m.f. of a test cell

### Safety

Always take sensible precautions when using mains-operated supplies. Teachers and technicians should follow their school and departmental safety policies and should ensure that the employer's risk assessment has been carried out before undertaking any practical work.

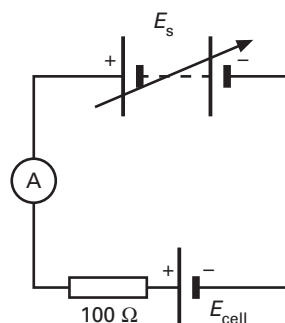
### Apparatus

- chemical cell
- digital d.c. supply
- $100\ \Omega$  resistor
- digital ammeter
- connecting leads

### Introduction

In chapter 13 of *Physics 1*, you met the idea that the e.m.f. of a cell may be determined by connecting a high-resistance voltmeter across its terminals. An alternative method makes use of Kirchhoff's second law.

The arrangement shown here may be used to determine the e.m.f. of a cell by applying Kirchhoff's second law. The direction of the current in the circuit depends on the relative magnitudes of the e.m.f.s. A digital ammeter will show the direction of the current.



According to Kirchhoff's second law:

sum of e.m.f.s in a closed loop = sum of p.d.s in that loop

Therefore:

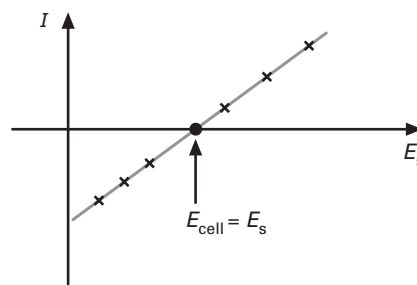
$$\text{e.m.f. of supply} - \text{e.m.f. of cell} = IR$$

$$E_s - E_{\text{cell}} = IR$$

When the e.m.f. of the supply is equal to the e.m.f. of the cell, the current  $I$  in the circuit is equal to zero.

### Procedure

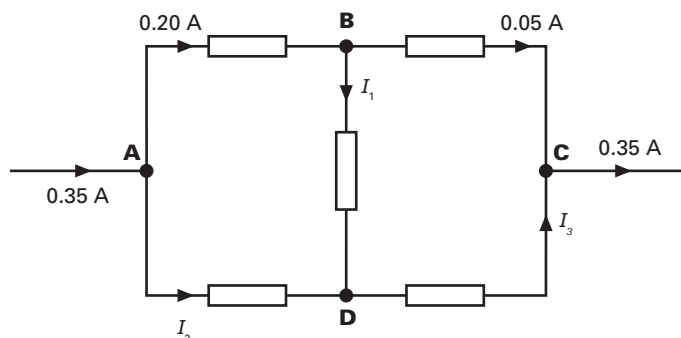
- 1 Set up the circuit as shown in the diagram above.
- 2 Set the e.m.f.  $E_s$  of the supply to zero.
- 3 Measure the current  $I$  in the circuit – make sure that you also take note of the 'sign'.
- 4 Measure the circuit current  $I$  as  $E_s$  is increased in steps of 0.5 V.
- 5 Suitably record your results and plot a graph of  $I$  against  $E_s$ .
- 6 Draw straight a line of best fit through the points (see sketch).
- 7 Use the graph to determine the e.m.f. of the test cell. What is the uncertainty in your value for the e.m.f.?



# End-of-chapter test

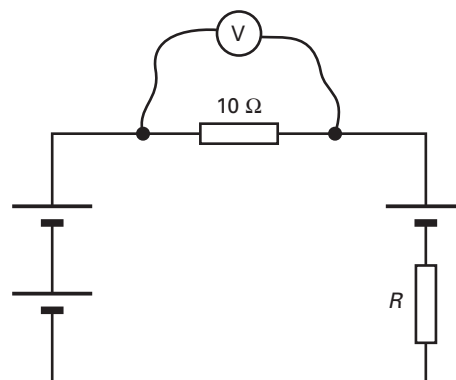
Answer all questions.

- 1 a State Kirchhoff's second law. [1]  
 b Kirchhoff's second law expresses the conservation of an important physical quantity. Name the quantity that is conserved. [1]
- 2 Determine the currents  $I_1$ ,  $I_2$  and  $I_3$  in the circuit below. [3]

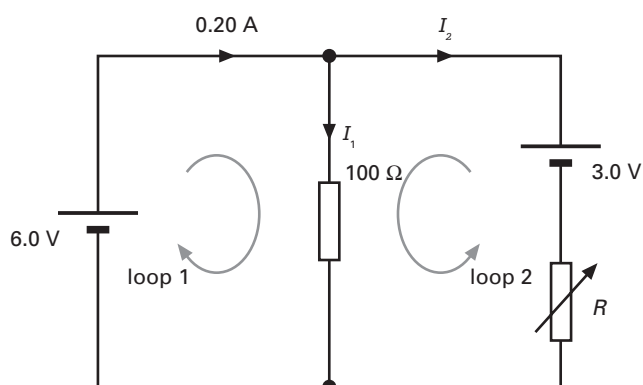


- 3 In the electrical circuit shown, each cell has e.m.f. 1.5 V and has negligible internal resistance. The voltmeter reading is 1.0 V.

- a State the direction of the current in the circuit. [1]
- b Calculate the value of the circuit current  $I$ . [1]
- c Calculate the resistance  $R$  of the resistor. [3]



- 4 This question is about applying Kirchhoff's laws to predict results for the circuit shown below.



The sources of e.m.f. have negligible internal resistance.

- a Use 'loop 1' to determine the current  $I_1$  in the  $100\Omega$  resistor. [2]
- b State the current  $I_2$ . [1]
- c Use 'loop 2' to determine the resistance  $R$  of the variable resistor. [3]

Total:  $\frac{\quad}{16}$  Score: %

# Marking scheme

## Worksheet

1 The sum of the currents into a point = sum of currents out of the same point. [1]

2 Charge is conserved. [1]

3 a  $I = 2.0 + 1.0 = 3.0 \text{ A}$  [1];

b  $0.7 + 0.5 = I$  therefore  $I = 1.2 \text{ A}$  [1];

c  $2.0 + I = 4.0 + 0.5$  [1];  $I = 4.5 - 2.0 = 2.5 \text{ A}$  [1]

4 a  $E = 1.5 + 1.5 = 3.0 \text{ V}$  [1];

b  $E = 1.5 + 1.5 + 1.5 - 1.5 = 3.0 \text{ V}$  [1];

c  $E = 1.5 - 1.5 = 0 \text{ V}$  [1]

5  $\sum \text{e.m.f.} = \sum \text{p.d.}$  [1]

$6.0 = (I \times 10) + (I \times 20)$  (clockwise 'loop') [1]

$30I = 6.0$  so  $I = \frac{6.0}{30} = 0.20 \text{ A}$  [1]

6 a  $\sum \text{e.m.f.} = \sum \text{p.d.}$

$6.0 - 1.5 = (I \times 68) + (I \times 12)$  (clockwise 'loop') [1]

$80I = 4.5$  [1]; so  $I = \frac{4.5}{80} = 5.63 \times 10^{-2} \text{ A}$ ;  $I \approx 5.6 \times 10^{-2} \text{ A}$  [1]

b  $V = IR = 5.63 \times 10^{-2} \times 68$  (the current is the **same** in a series circuit) [1]

$V \approx 3.8 \text{ V}$  [1]

7 If the ammeter reading is zero, then the e.m.f.  $E$  of the test cell is equal to the p.d. across the variable resistor ( $\sum \text{e.m.f.} = \sum \text{p.d.}$ ). [1]

$$E = \frac{R_b}{R_a + R_b} \times V_{\text{in}} \text{ [1]}$$

$$E = \frac{28}{100 + 28} \times 5.0 = 1.09 \text{ V} \approx 1.1 \text{ V} \text{ [1]}$$

8 Loop 1:  $\sum \text{e.m.f.} = \sum \text{p.d.}$

$$3.0 = 10I_1 + 20I_2 \text{ (equation 1) [1]}$$

Loop 2:  $\sum \text{e.m.f.} = \sum \text{p.d.}$

$$1.5 = 20I_2 \text{ (equation 2) [1]}$$

$$I_2 = \frac{1.5}{20} = 0.075 \text{ A [1]}$$

Substituting the value for  $I_2$  into equation 1, we have:

$$3.0 = 10I_1 + (20 \times 0.075) \text{ [1]}$$

$$I_1 = \frac{1.5}{10} = 0.15 \text{ A [1]}$$

Finally, using Kirchhoff's first law, we have:

$$0.15 = 0.075 + I_3$$

$$I_3 = 0.075 \text{ A [1]}$$

**9** With the switch open:

$$E = 0.25 (8.0 + r) \quad \text{or} \quad E = 2.0 + 0.25r \quad (\text{equation 1}) [1]$$

With the switch closed:

$$\text{total resistance of the parallel combination} = \frac{8.0}{2} = 4.0 \, \Omega$$

$$E = 0.45 (4.0 + r) \quad \text{or} \quad E = 1.8 + 0.45r \quad (\text{equation 2}) [1]$$

We have two equations and two unknowns. Substituting for  $E$  gives:

$$2.0 + 0.25r = 1.8 + 0.45r [1]$$

$$0.20r = 0.2 \quad \text{so} \quad r = 1.0 \, \Omega [1]$$

Substituting this value into one of the equations for e.m.f. (*equation 1*) gives:

$$E = 2.0 + (0.25 \times 1.0) [1]$$

$$E = 2.25 \, \text{V} [1]$$

(You can check by substituting into *equation 2*.)



# Marking scheme

## End-of-chapter test

- 1**   **a**   The sum of the e.m.f.s around a circuit loop = sum of the p.d.s around that loop. [1]  
      **b**   Energy is conserved [1]
- 2**   Point B  $\Rightarrow I_1 + 0.05 = 0.20$       therefore       $I_1 = 0.15 \text{ A}$  [1]  
      Point A  $\Rightarrow 0.35 = 0.20 + I_2$       therefore       $I_2 = 0.15 \text{ A}$  [1]  
      Point C  $\Rightarrow I_3 + 0.05 = 0.35$       therefore       $I_3 = 0.30 \text{ A}$  [1]
- 3**   **a**   Clockwise [1];  
      **b**    $I = \frac{V}{R} = \frac{1.0}{10} = 0.10 \text{ A}$  [1];  
      **c**    $\sum \text{e.m.f.} = \sum \text{p.d.}$  [1]  
           $1.5 + 1.5 - 1.5 = 1.0 + (0.1 \times R)$       (clockwise 'loop') [1]  
           $R = \frac{1.5 - 1.0}{0.1} = 5.0 \Omega$  [1]
- 4**   **a**    $\sum \text{e.m.f.} = \sum \text{p.d.}$   
           $6.0 = 100I_1$       (clockwise 'loop') [1]  
           $I_1 = 0.06 \text{ A}$  [1]  
      **b**    $I_2 = 0.20 - 0.06 = 0.14 \text{ A}$  [1]  
      **c**    $\sum \text{e.m.f.} = \sum \text{p.d.}$   
           $3.0 = 6.0 - (0.14 \times R)$       (anticlockwise 'loop') [1]  
           $R = \frac{3.0 - 6.0}{-0.14}$  [1];    $R = 21.4 \Omega \approx 21 \Omega$  [1]