Gauss Jordan Method

Mr. Edem K. Bankas

Online portal

- http://computational-mathematics-matlab.wikispaces.com

Gauss Jordan Method

- The Gauss-Jordan Method is similar to the Gaussian Elimination, just that we do more
- Elimination --- use ERO to convert the matrix into an identity matrix

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} = \begin{Bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{Bmatrix}$$

The Gauss-Jordan method changes the matrix into the identity matrix.

$$\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} = \begin{Bmatrix} r_1' \\ r_2' \\ \vdots \\ r_n' \end{Bmatrix}$$

Example

X + y = 0

Y = -1

Z = 2

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

R1 –R2 ⟹ R1

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

x = 1, y= -1 , z = 2

Example

- Solve the system below using Gauss Jordan

$$x + y - z = -2$$

$$2x - y + z = 5$$

$$-x + 2y + 2z = 1$$

- **Solution:**
  We begin by writing the system as an augmented matrix

We have, $$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 2 & -1 & 1 & 5 \\ -1 & 2 & 2 & 1 \end{array}\right] \Rightarrow$$

-2R1 +R2 $$\begin{pmatrix} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ -1 & 2 & 2 & 1 \end{pmatrix}$$

R1 +R3 $$\begin{pmatrix} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 3 & 1 & -1 \end{pmatrix}$$

- We now move to column 2, to get 1 in the diagonal position

R2 /-3 $$\begin{pmatrix} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 3 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & 3 & 1 & -1 \end{pmatrix}$$

To make 3 below 1, 0

$$\begin{pmatrix} 1 & 1 & -1 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & -4 & -8 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & -1 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & -4 & -8 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

R3 +R1 ➡ R1

R2+ R3 ➡ R2

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \qquad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

We can now "read" our solution from this last matrix. We have

x = 1,

y = -1

z = 2.

(x,y,z)' = (1, -1, 2)'

Example 2

- Solve the following set of simultaneous equations by Gauss-Jordan method

$$4x_1 + 2x_2 - 3x_3 = 2$$
$$3x_1 + x_2 - 2x_3 = 1$$
$$2x_1 - 2x_2 + x_3 = 2$$

$(x_1, x_2, x_3)' = (2, 3, 4)'$