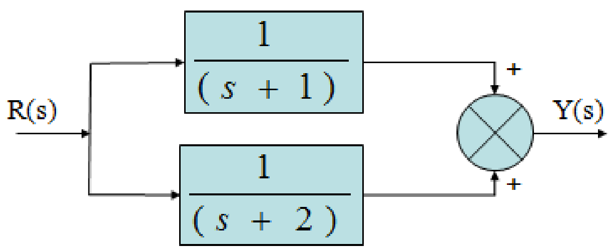
■ Digital Control Tutorial No.2 ■

Class: Fourth

Division: **Production Engineering / University of Technology**

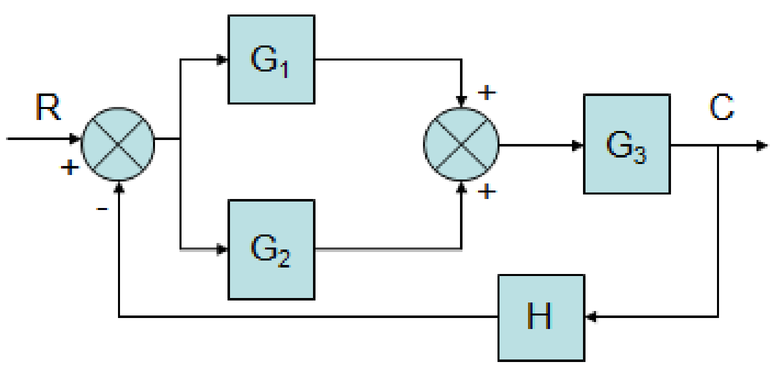
Question (1): Find the transfer function Y(s)/R(s) for the block diagram, then find the complete solution as Y(t) when input signal is the function ($f(t)=e^{-3t}$)



Answer:

$$\boxed{\frac{Y(s)}{R(s)} = \frac{2s+3}{s^2+3s+2}}$$

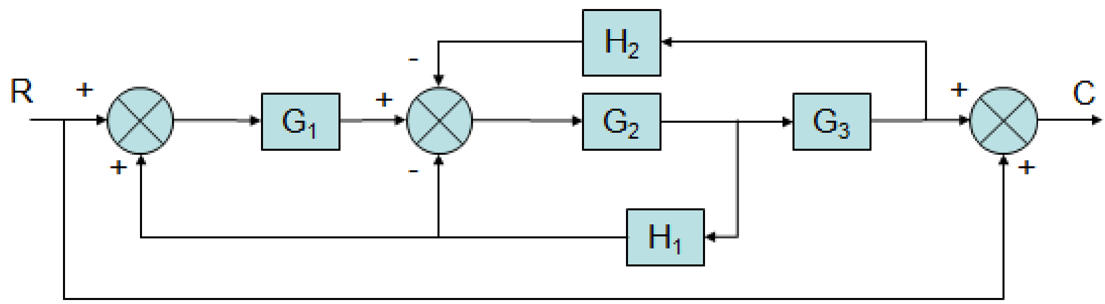
Question (2): Find the transfer function C(s)/R(s) for the block diagram.



Answer:

$$\boxed{\frac{C}{R} = \frac{(G_1+G_2)G_3}{1+G_2G_3H+G_1G_3H}}$$

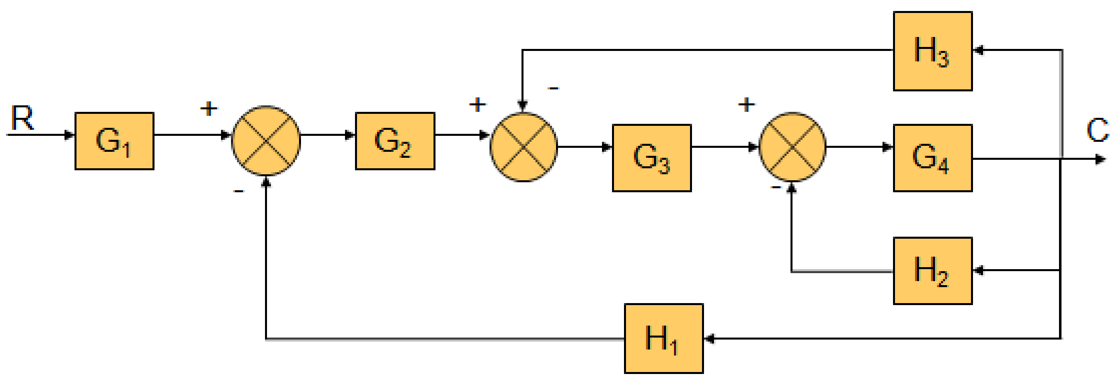
Question (3): Find the transfer function C(s)/R(s) for the block diagram.



Answer:

$$\boxed{\frac{C}{R}=1+\frac{G_1G_2G_3}{1+G_2H_1+H_2G_2G_3-H_1G_2G_1}}$$

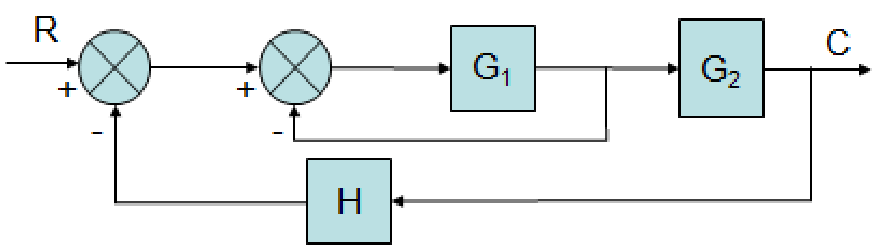
Question (4): Find the transfer function C(s)/R(s) for the block diagram.



Answer:

$$\boxed{\frac{C(s)}{R(s)}=\frac{G_1G_2G_3G_4}{1+G_4H_2+G_3G_4H_3+G_2G_3G_4H_1}}$$

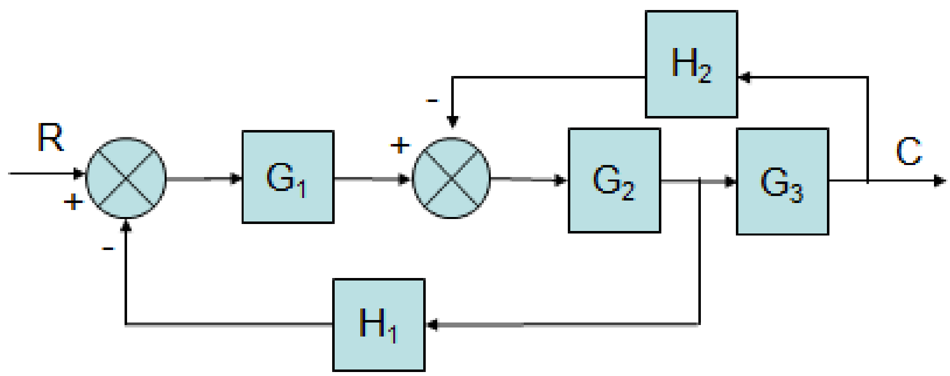
Question (5): Simplify the block diagram shown below to determine the transfer function.



Answer:

$$\boxed{\frac{C}{R} = \frac{G_2 G_1}{1 + G_1 + G_1 G_2 H}}$$

Question (6): Simplify the block diagram shown below to determine the transfer function.



Answer:

$$\boxed{\frac{C}{R} = \frac{G_2 G_1 G_3}{1 + G_2 G_3 H_2 + G_1 G_2 H_1}}$$

Question (7): A laser jet printer uses a laser beam to print copy rapidly for a computer. The laser is positioned by a control input, r(t), so that we have:

$$Y(s) = \frac{5(s+100)}{s^2 + 60s + 500} R(s)$$

The input r(t) represents the desired position of the laser beam. If r(t) is unit step input, find the output y(t).

$answer: y(t) = 1 - 0.125e^{-50t} - 1.125e^{-10t}$

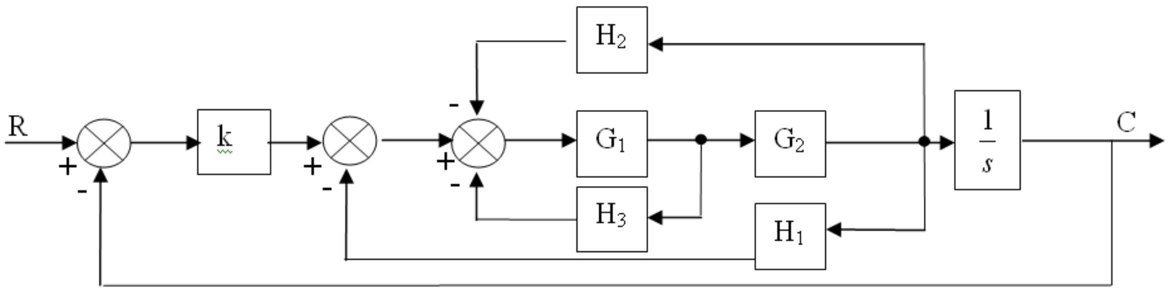
Question (8): The transfer function of a system is:

$$\frac{Y(s)}{R(s)} = \frac{10(s+2)}{s^2 + 8s + 15}$$

Determine y(t) when r(t) is a unit step input.

$Answer: y(t) = 1.33 + 1.67e^{-3t} - 3e^{-5t}$

Question (9): Determine the transfer function for the following configuration. Represent the final transfer function by a single block.



$$Answer: \frac{Y(s)}{R(s)} = \frac{KG_1G_2/s}{1+G_1H_3+G_1G_2[H_1+H_2]+KG_1G_2/s}$$

Question (10): Consider the differential equation:

$$\frac{d^3y(t)}{dt^3}+5\frac{d^2y(t)}{dt^2}+\frac{dy(t)}{dt}+2y(t)=u(t)$$

Write the state equation for this equation.

Answer:

state variables:

$$x_1(t)=y(t)$$

$$x_2(t)=\frac{dy(t)}{dt}$$

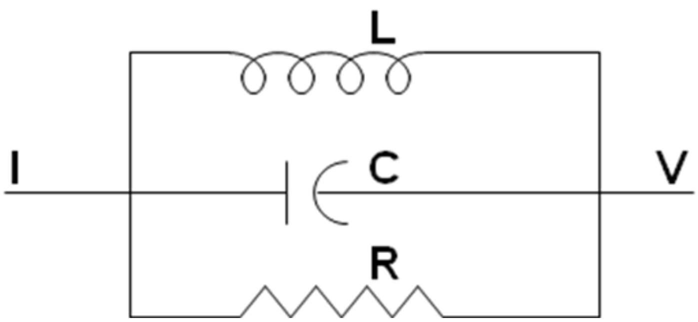
$$x_3(t)=\frac{d^2y(t)}{dt^2}$$

$$A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -1 & -5 \end{bmatrix}$$

$$B=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Output Equation: $y(t)=x_1(t)=[1 \quad 0]x(t)$

Question (11): Find the transfer function of the shown system



Solution:

$\because$ Parallel Resistor–Inductor–Capacitor circuit.

$\therefore I = I_L + I_R + I_C$

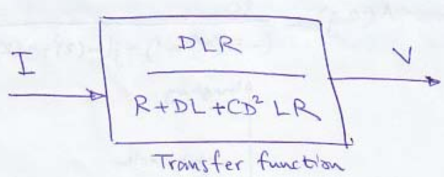
$\because L = \dfrac{V_L}{\frac{dI_L}{dt}} = \dfrac{V_L}{D I_L}$, $R = \dfrac{V_R}{I_R}$, $C = \dfrac{I_c}{\frac{dV_c}{dt}} = \dfrac{I_c}{D V_c}$

$I_L = \dfrac{V_L}{D L}$, $I_R = \dfrac{V_R}{R}$, $I_C = C D V_C$

$\because$ parallel system $\Rightarrow V = V_L = V_R = V_C$

$\therefore I = \dfrac{V}{DL} + \dfrac{V}{R} + CDV = V\left(\dfrac{1}{DL} + \dfrac{1}{R} + CD\right)$

$$\therefore \frac{V}{I} = \frac{\text{output signal}}{\text{input signal}} = \frac{DLR}{R + DL + CD^2 LR}$$



Question (12): Find the temperature difference equation for the thermometer system shown in the network representation.

Solution.

Q = flow of heat = $\dfrac{T_1 - T}{R_T}$ = $\dfrac{\text{Temp. difference}}{\text{Thermal resistance}}$

$Q = M\,c\,\dfrac{dT}{dt}$ = Mass * specific heat rate * Temp. difference with time

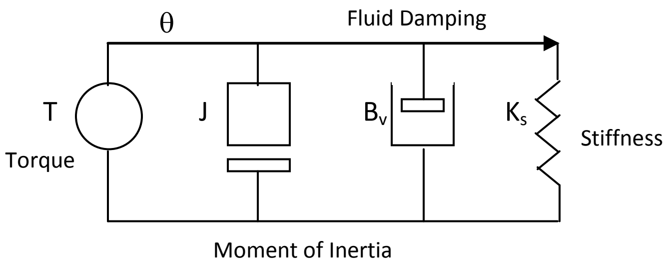
$\because$ C_T = Thermal capacitance $= M\,c$, $D = \dfrac{d}{dt}$

$\therefore Q = C_T D T$

$\therefore \dfrac{T_1 - T}{R_T} = C_T D T$

$\therefore$ Temp. difference $= R_T C_T D T = T_1 - T$

Question (13): Find the torque equation for the mechanical rotational system shown in the network representation.



Solution

$T_s = \text{torque} = K_s\theta = \text{stiffness} * \text{Angular displacement}$

$T_d = \text{torque required to resiste damping} = B_v\,\omega = \text{fluid damping} * \text{Angular velocity}$

$= B_v \dfrac{d\theta}{dt} = B_v D\theta$

$\sum T_e = T - T_s - T_d = \text{Total effective torques} = J D^2 \theta$

$= T - (K_s\theta) - (B_v D\theta) = J D^2 \theta$

$\therefore\ T = J D^2\theta + K_s\theta + B_v D\theta = (J D^2 + B_v D + K_s)\theta$

Question (14): Write the state model for the system described by transfer function: $C(s)=f(s)/(s^2+10s+25)$

solution.

let $c(s)=x_1$, $x_1=\frac{f(s)}{s^2+10s+25}$

$\therefore f(s)=\ddot{x}_1+10\dot{x}_1+25x_1$

$\because \dot{x}_1=x_2$

$\therefore \ddot{x}_1=-10\dot{x}_1-25x_1+f(s)$

$\ddot{x}_1=-10x_2-25x_1+f(s)$

$\ddot{x}_1=-25x_1-10x_2+f(s)$

$\therefore \dot{x}_1=0x_1+x_2+0f(s)$

$\ddot{x}_1=\dot{x}_2=-25x_1-10x_2+f(s)$

$\therefore x=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$,

state equation

$$\boxed{\dot{x}=\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}=\underbrace{\begin{bmatrix} 0 & 1 \\ -25 & -10 \end{bmatrix}}_{\text{system Matrix}}\underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\text{state vector}}+\underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\text{control vector}}f(s)}$$

output equation $=C(s)=\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=x_1$

Question (15): Write the state model for the system described by transfer function: C(t)/f(t)=(D+6)/(D^2+5D+6)

solution

$\because \quad C(t) = \dfrac{(D+6)\,f(t)}{D^2+5D+6}$, let $\boxed{C(t) = (D+6)\,x_1}$

$x_1 = \dfrac{1}{(D^2+5D+6)} f(t)$, $\quad C(t) = \dot{x}_1 + 6x_1$

let $\dot{x}_1 = x_2 \Rightarrow C(t) = x_2 + 6x_1$

$\because x_1 = \dfrac{1}{(D^2+5D+6)} f(t) \Rightarrow f(t) = \ddot{x}_1 + 5\dot{x}_1 + 6x_1$

$\because \dot{x}_1 = x_2$, $\quad \ddot{x}_1 = \dot{x}_2$

$\therefore f(t) = \dot{x}_2 + 5x_2 + 6x_1$

$\therefore \dot{x}_2 = -6x_1 - 5x_2 + f(t)$

$\because C(t) = \dot{x}_1 + 6x_1 = x_2 + 6x_1 = 6x_1 + x_2$

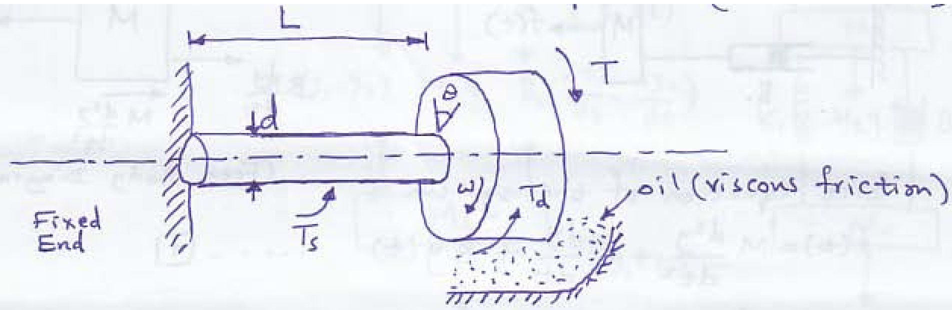
$\therefore \dot{x}_1 = 0x_1 + x_2 + 0f(t)$

$\dot{x}_2 = -6x_1 - 5x_2 + f(t)$

$$\boxed{\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1\\ -6 & -5\end{bmatrix}}_{\text{System Matrix}} \underbrace{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}_{\text{State vector}} + \underbrace{\begin{bmatrix}0\\ 1\end{bmatrix} f(t)}_{\text{control vector}}}$$ State equation

$C(t) = \text{output equation} = \begin{bmatrix}6 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$

Question (16): Rotational Mechanical Component (like shaft)



T_s = torsion moment (torque) $= K_s\,\theta$

K_s = Torsion spring ratio

θ = spring angular displacement

$$K_s = \frac{\pi d^4 G}{32\,L}$$

G = shear moduls

d = shaft diameter

L = shaft length

T_d = viscous friction torque $= B_2\,\omega = B_2 \dfrac{d\theta}{dt} = B_2 D\theta$

$D = \dfrac{d}{dt}$, B_2 = viscous friction factor

ω = angular velocity (rad/sec) $= \dfrac{2N\pi}{60}$, N = speed (rpm)

$$\sum \text{Torque} = T - T_s - T_d$$

$$JD^2\theta = T - K_s\theta - B_2 D\theta$$

$$T = JD^2\theta + K_s\theta + B_2 D\theta$$

J = Polar Moment of area $= \dfrac{\pi d^4}{32}$

$$\therefore\ T = (JD^2 + K_s + B_2 D)\,\theta$$

Transfer function

$$T \xrightarrow{\text{input signal}} \boxed{\frac{1}{JD^2 + K_s + B_2 D}} \xrightarrow{\text{output signal}} \theta$$