Logarithmic Scales (4.6)

Planet	Distance (millions of km	log distance
Mercury	58	1.76
Venus	108	2.03
Earth	150	2.18
Mars	228	2.36
Jupiter	779	2.89
Saturn	1430	3.16
Uranus	2280	3.46
Neptune	4500	3.65

<u>Order of Magnitude</u>: when one quantity is roughly ten times larger than another, ~~the~~ we say it is <u>one</u> order of magnitude larger.

Notice, the order of magnitude can be found as the common log of the ratio of the quantities.

⇒ Given two values A and B, determine how many orders of magnitude A is greater than B,

Difference in orders of magnitude

$$\log\left(\frac{A}{B}\right) =$$

Example 1

Launching of Space Shuttle = 2,000,000,000
Softest sound human can hear = 20

Difference in order of magnitude = 8

Moment Magnitude scale

$$M = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$$ where $S_0 = 10^{16}$ is a baseline measure for the seismic moment

Example 2

If one earthquake has a MMS magnitude of 6.0 and another has a magnitude of 8.0, how much more powerful (in terms of earth movement) is the second earth quake.

$$M = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$$

$$6.0 = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$$

* multiply by 3/2 to eliminate 2/3

$$9 = \log\left(\frac{S}{S_0}\right)$$

$$10^9 = \frac{S}{S_0}$$

* convert to exponential form

$$\boxed{S = 10^9 S_0}$$

* Isolate S by multiplying both sides by S_0.

$$8.0 = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$$

$$12 = \log\left(\frac{S}{S_0}\right)$$

$$10^{12} = \frac{S}{S_0}$$

$$S = 10^{12} S_0$$

Quake 1

$$S = 10^9 S_0$$

Quake 2

$$S = 10^{12} S_0$$

$$S = \frac{10^{12} \cancel{S_0}}{10^9 \cancel{S_0}}$$

$= 10^3$ or 1000 times as large

Example 3

One earthquake has a magnitude of 3.0 If a 2nd has twice as much earth movement as the first, find the magnitude of the second quake.

$$3.0 = \frac{2}{3}\log\left(\frac{S}{S_0}\right) \qquad 10^{4.5} = \frac{S}{S_0}$$

$$4.5 = \log\left(\frac{S}{S_0}\right) \qquad S = 10^{4.5} S_0$$

$$M = \frac{2}{3}\log\left(\frac{2 \cdot 10^{4.5} \cancel{S_0}}{\cancel{S_0}}\right) \rightarrow M = \frac{2}{3}\log\left(2 \cdot 10^{4.5}\right)$$

$$M = 3.201$$

$$7.9 = \frac{2}{3}\log\left(\frac{S}{S_0}\right) \qquad 4.6 = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$$

$$11.85 = \log\frac{S}{S_0} \qquad 6.9 = \log\frac{S}{S_0}$$

$$10^{11.85} = \frac{S}{S_0} \qquad 10^{6.9} = \frac{S}{S_0}$$

$$10^{11.85} S_0 \qquad 10^{6.9} S_0$$

$$\frac{10^{11.85} \cancel{S_0}}{10^{6.9} \cancel{S_0}} = 10^{4.95}$$

$$4.3 = \frac{2}{3}\log\left(\frac{S}{S_0}\right) \qquad M = \frac{2}{3}\log\left(\frac{180 \cdot 10^{6.45} \cancel{S_0}}{\cancel{S_0}}\right)$$

$$6.45 = \log\left(\frac{S}{S_0}\right)$$

$$10^{6.45} S_0 = S$$