

Logarithm Functions

1. In 2009 the population of Brockton was 89,000 and growing at a rate of 1.3%, in the same year, New Bedford's population was 78,000 and growing at 3.5%. In which year will the populations be the same?

$$89000(1.013)^x = 78000(1.035)^x$$

$$\frac{89000}{78000} = \left(\frac{1.035}{1.013}\right)^x \quad \log \frac{1.035}{1.013} = \frac{\log \frac{89000}{78000}}{x}$$

$$x = 6$$

$$\boxed{2015}$$

2. There were 50 elephants on a reservation in 2014 and growing at a rate of 2.4% per year, in Africa there were 500 elephants but their population is decreasing at rate of 7.8%, in what year will the population be the same?

$$50(1.024)^x = 500(.922)^x$$

$$\frac{50}{500} = \left(\frac{.922}{1.024}\right)^x \quad \log \frac{.922}{1.024} = \frac{\log \frac{50}{500}}{x} \approx 22 \text{ years}$$

$$\boxed{2036}$$

3. In 1980 the minimum wage was \$4.75 and increasing at a rate of 1.1% per year and in Europe the minimum wage was \$3.50 and increasing at a rate of 2.2% in which year will the wages be the same?

$$4.75(1.011)^x = 3.5(1.022)^x$$

$$\frac{4.75}{3.5} = \left(\frac{1.022}{1.011}\right)^x \quad \log \frac{1.022}{1.011} = \frac{\log \frac{4.75}{3.5}}{x}$$

$$\approx 28$$

$$\boxed{2008}$$

4. You have two investment options, they are both CD's compounded monthly. The first you can deposit \$10,000 at a rate of 4.8% and the other you can only deposit \$5,000 but the rate is 10.8%, how many year before the investments are equal?

$$10000(1.004)^{12t} = 5000(1.009)^{12t} \quad 139.5 \text{ months}$$

$$\frac{10000}{5000} = \left(\frac{1.009}{1.004}\right)^{12t}$$

$$\log \frac{1.009}{1.004} \quad \frac{10000}{5000}$$

$$\boxed{11.6 \text{ years}}$$

5. You have an investment option a CD compounded monthly at a rate of 3.6% in which you invest \$12,000 and you have a taken a loan which is compounded monthly at a rate of 7.2% and you borrow \$8,000, when will your investment equal your loan?

$$12000(1.003)^{12t} = 8000(1.006)^{12t} \quad 135.76$$

$$\frac{12000}{8000} = \left(\frac{1.006}{1.003}\right)^{12t}$$

$$\log \frac{1.006}{1.003} \quad \frac{12000}{8000}$$

$$\approx 11.3 \text{ years}$$

6. If a population of 2.2 million is growing continuously at a rate of 3.2% and another population of 1.8 million is growing continuously at a rate of 3.8% per year, how many years before they are equal?

$$2.2e^{.032t} = 1.8e^{.038t}$$

$$\frac{2.2}{1.8} = \frac{e^{.038t}}{e^{.032t}}$$

$$\frac{2.2}{1.8} = \left(\frac{e^{.038}}{e^{.032}}\right)^t$$

$$\log \frac{2.2}{1.8} = (e^{.006})^t$$

$$33.445$$

$$\ln(2.2/1.8) = .006t$$