THE TREATMENT OF SPACE AS A SOLID STRUCTURE

A METHOD FOR THE QUANTISATION OF GRAVITY

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INTRODUCTION

In my previous paper 'The Large Scale Structure of the Universe' I outlined a proposition that the expansion of the Universe was the result of gravitational repulsion, this being a product of the so-called cosmological constant or Λ factor. In that paper I went on to relate the maximum radius of the universe,via the use of Hubble's constant, to all the other principle constants of physics,thus enabling all the constants to be written in terms of the Gravitational constant. It will be recalled that this is one of the principle requirements for the quantisation of gravity.

If one accepts the validity of the Λ proposition, one can then envision that all of space is confined inside two parameters, these being the Compton Length at the smallest end of the scale and the Radius of the Universe at the largest end of the scale. If, in fact, the universe is not expanding in terms of it's volume but is held in a steady and finite state by cosmic repulsion, then it is clear that that space and time are both 'trapped' between these two parameters which in turn act as impenetrable potentials.

Since the space which we occupy is, itself, the product of the gravitational Λ field it follows that this paper is the description of that quantised gravitational field and the quantised space confined by the two potentials.

In my previous paper 'The Large Scale Structure of the Universe', I advanced a 'steady state' model of the universe and showed how the large scale structure of the universe was related to the very smallest structures of matter. I now intend to develop this model further and to show how it's structure leads to the quantization of gravity.

In the steady state model the dimensions of the universe are known and are fixed. This enables us to imagine that the structure of space is, as it were, rigid. In other words we can treat space similarly to the way we would treat a solid and we can treat a galaxy moving through that 'solid' as if it was a quantum particle moving through the potential zones in a solid in a similar manner to the well known models of solid state physics.

The wave description of matter defines a natural scale for a particle through it's Compton wavelength i.e. $\lambda = h/mc$. This in turn leads to the conclusion that the Planck length $10^{-35}m.$ is a limiting state of matter, that is to say it is a boundary condition at one end of the scale of structure of the universe. At the other end of the scale, the boundary condition is the limiting distance of the 'event horizon' of the universe i.e its radius of $1/2c^2$ light years. The establishment of these two boundary conditions enables us to describe the properties of a galaxy or any other particle moving in a quantised gravitational field, because, as described in my previous paper, the galaxies are 'falling' in a repulsive gravitational field. We can say that both boundary conditions take the form of potential barriers and this enables us to describe the wave function of any particle (or of any galaxy) in the universe by imagining that the particle (or galaxy) is moving through a quantised field. We can call this field the Quantum Gravity Field.

To recap, the description of the gravitationally repulsive repulsive field or Lambda field is one where the field strength inside the mass of the universe 'M' is given by $\bar{g} = \frac{Mr}{4\pi G R_0^3}$ and the potential difference inside the mass integrating along a radial path is given by:-

$$V_b - V_a = -\int_{r_a}^{r_b} \left(\frac{Mr}{4\pi G R_0^3} \hat{r} \right) . \left(dr \hat{r} \right)$$

$$= \frac{M}{8\pi G R_0^3} (r_a^2 - r_b^2)$$

Here r_a corresponds to a point inside a spherical mass i.e:- $(r_a = R, V_a = V(R))$ and r_b correspondscorresponds to a point onthe surface of the spherical mass i.e:- $(r_b = R_0, V_b = V_0)$ .Thus the potential varies within the spherical mass and any particle can be said to be lying on an equipotential surface within the mass.

As with any oscillator or wave function it is the boundary conditions which lead to a set of quantised energy levels. The particle cannot have zero energy. The lowest energy value occurs at $n = 1$, known as the *zero point* energy and this is true for any particle which is confined to a region of space by the presence of boundary conditions. This is, of course, already well known but it is worth reminding ourselves of how boundary conditions lead to quantisation of the wave function by referring to the familiar 'particle in a box' model, as this will help to lead us into a more full description of the quantised gravity field.

Now a particle in a box is confined by two impenetrable walls (or potentials) at x and L. Since the particle (or galaxy) cannot penetrate the walls then $\Psi = 0$ for $x < 0$ and $x > L$. The requirement that the wave function be continuous leads to the boundary condition $\Psi = 0$ at $x = 0$ and $x = L$. With $\Psi = 0$, the Schroedinger wave equation becomes :-

$\frac{\partial^2 \Psi}{\partial x^2} = k^2 \Psi = 0$ *where* $k = \sqrt{\frac{2mE}{\hbar}}$ the solution to this equation is $\Psi(x) = A\sin(kx = \theta)$. The boundary conditions are $\Psi = 0$ at $x = 0$ and from the condition that $\Psi = 0$ at $x = L$ we find that $(kL) = 0$ which means that $kL = n\pi$ where n is an integer. Thus we have a wave function which satisfies the boundary conditions in the form of a standing wave i.e :- $\Psi(x) = \left(\frac{n\pi x}{L}\right)$ $n = 1,2,3.....$. Since $k = \frac{2\pi}{\lambda} = \frac{n\pi}{L}$ the wavelength of the n^{th} standing wave is $\lambda = \frac{2L}{n}$. When this is equated to de Broglies equation $\lambda = \frac{h}{mv}$ we find $v = \frac{nh}{2mL}$. Since *'n'* takes on only integer values, the speed is quantised. The particle's (or galaxy's) energy, which is purely kinetic is $1/2mv^2$, is thus also quantised. The energy of any particle moving within fixed boundary conditions is therefore quantised. So much is elementary, and we can go on to expand this fact to illustrate more precisely the nature of the quantised gravitational field.

Since it is the boundary conditions of a state which produce its quantised properties we can say that any galaxy is moving in a quantised gravitational field (because our gravitational field is limited by the boundary conditions already described). The field itself can be described by imagining a series of spherical potential field lines whose centre is located at the observer's position. The precise position in space of the point of the observers position is not relevant because the cosmological principle tells us that any point in the universe is equally valid as a central reference frame. In other words, any arbitrary point can be considered as the centre of the universe for the purposes of this discussion. Thus we can imagine each potential field line to be quantised in terms of $\hbar$ and an integer where integer *1* occurs at the point of the strongest field (i.e. on the outermost circumference of the sphere). Thus the galaxy is moving from a position of high potential to a position of low potential . We can do better than this and say that the universe is seeking its own lowest energy level.

Furthermore we can say that the field lines (or equipotential lines) are arranged in a manner which exhibit spatial periodicity and the potential gaps can be described in a manner similar to the Kronig-Penney model of a solid. This periodicity has an effect on the motion of the galaxys moving through the field. The periodicity is built into the potential for which we require that *V(x + a) = V(x)*. Since the kinetic term $-\frac{h^2}{2m}\frac{d^2}{dx^2}$ is unaltered by the change $x \to x + a$, the whole Hamiltonian is unaltered by *'a'*. For the case of zero potential when the solution corresponding to a given energy $E = \frac{\hbar^2 k^2}{2m}$ *is* $\Psi(x) = e^{ikx}$ the displacement yields $\Psi(x+a) = e^{ik(x+a)} = e^{ika}\Psi(x)$ that is the original solution multiplied by a phase factor so that $|\Psi(x+a)|^2 = |\Psi(x)|^2$ .The observables will therefore be the same at *'x'* as at *x + a*. In our example we shall also insist that *Ψ(x)* and *Ψ(x + a)* differ only by a phase factor which need not, however be of the form e^{ika} . If we take a series of repulsive delta function potentials $V(x) = \frac{\hbar^2}{2m}\frac{\lambda}{a} \sum_{n=-\infty}^{\infty} \delta(x - na)$, then away from the points *x = na*, the solution will be that of a free particle equation, that is in a linear combination of *sin kx* and *cos kx*. Let us assume that in the region *Rn* defined by *(n - 1)a ≤ x ≤ na* we have :-

(1) *Ψ(x) = An sin k(x - na) = Bn cos k(x - na)*

and in the region *Rn + 1* defined by *na ≤ x ≤ (n + 1)* we have :-

(2) *Ψ(x) = A(n + 1) sin 〈x - (n + 1)a〉 + Bn + 1 cos k 〈x - (n + 1)a〉.*

Continuity of the wave function implies that :-

x = na, -An + 1 sin ka + Bn + 1 cos ka = Bn and the discontinuity codition implies that :-

kAn + 1 cos ka + kBn + 1 sin ka - kAn = $\frac{\lambda}{a} Bn$. Letting $g = \frac{\lambda}{ka}$ it follows that *An + 1 = An cos ka + (g cos ka - sin ka)Bn* which leads to :-

(3) *Bn + 1 = (g sin ka + cos ka)Bn + An sin ka*.

The rquirement that the wave functions (1) and (2) be related by $\Psi(Rn+1) = e^{i\theta}\Psi(Rn)$ is satisfied if :-

(4) $An + 1 = e^{i\theta} An \quad and \quad Bn + 1 = e^{i\theta} Bn$

When this is inserted into (3) we find a consistency condition which reads

$\left(e^{i\theta}-\cos ka\right)\left(e^{i\theta}-g\sin ka-\cos ka\right)=\sin ka\left(g\cos ka-\sin ka\right)$ .That is :-

$e^{2i\theta}-e^{i\theta}\left(2\cos ka+g\sin ka\right)+1=0$ .Multiplication by $e^{-i\theta}$ gives :-

(5) $$\cos\theta=\cos ka+\frac{1}{2}g\sin ka.$$

If we take periodic boundary conditions so that *Ψ(Rn + N) = Ψ(Rn)* then it follows from (4) that $e^{iN\theta}=1,\ that\, is\ \theta=\frac{2\pi}{N}m\quad m=0\pm1,\pm2....$. We denote θ by *qa* where *q* is the wave number of a galaxy in a 'box' of length *Na* with periodic boundary conditions and without any potential. Thus (5) should be re-written in the form $\cos qa=\cos ka+\frac{1}{2}\lambda\frac{\sin ka}{ka}$. Now because the left side is always bounded by *1*, there are restrictions on the possible ranges of the energy $E=\frac{\hbar^2k^2}{2m}$ that depend on the parameters of the field.

Fig.1. where marked yellow shows a plot of the function $\cos x+\lambda\frac{\sin x}{2x}$ as a function of *x = ka*. The horizontal line represents the bounds on *cos qa* and the regions of *x* for which the curve lies outside the strip are FORBIDDEN regions. Thus there are allowed energy bands separated by regions which are forbidden. Note that the onset of a a forbidden band corresponds to the condition $ka=n\pi,\quad n=\pm1,\ \pm2,\ \pm3.....$ which is the condition for Bragg reflection with normal incidence.

Referring again to Fig.1. but this time noting the pink shaded region. this region indicates a region of space which is forbidden to a wave moving from left to right but which is occupied by a matter wave moving from right to left. This matter wave is of opposite parity to the wave moving from left to right. Clearly this indicates a condition of broken symmetry which occurs occurs in the following way.

Schroedinger's equation can be written as :-

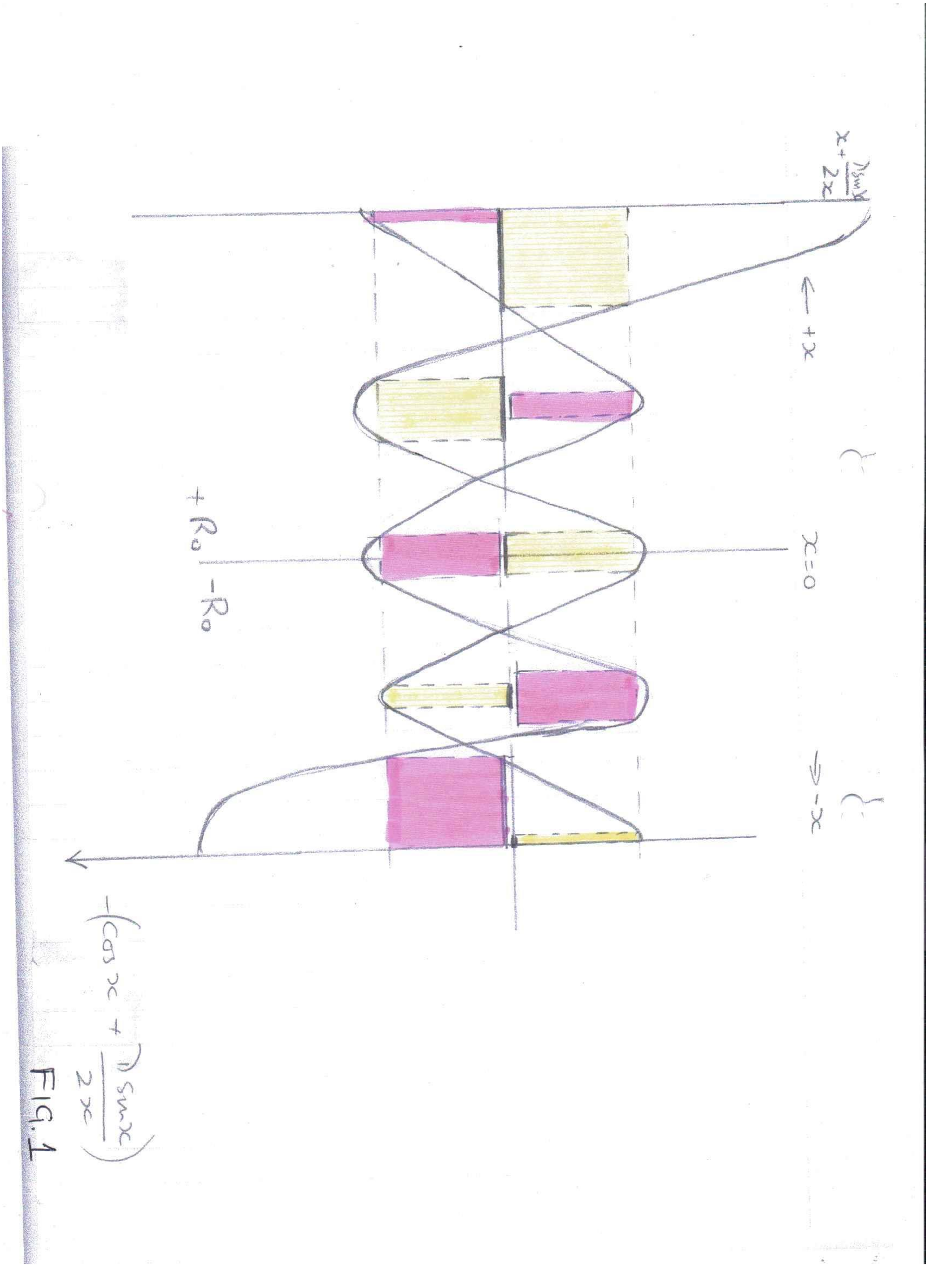

FIG. 1

(6) $-\frac{\hbar^2}{2m}\nabla^2\Psi - V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$ where in cartesian co-ordinates the wave function of the galaxy is given as :- *Ψ= (x, y, z, t)* and,

(7) $\nabla^2\Psi = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial\Psi^2}{\partial y^2} + \frac{\partial\Psi^2}{\partial z^2}$. In the foregoing the potential energy of the galaxy is given by *V = (x, y, z, t)* and $i = \sqrt{-1}$. If *V* is independent of time,we can separate space and time variables by setting *Ψ= Ψ(x, y, z,) T*(t). Substituting into (6) and dividing by *ΨT* we find :-

(8) $-\frac{\hbar^2}{2m}\frac{\nabla^2\Psi}{\Psi} + V = \frac{i\hbar}{\mathrm{T}}\frac{d\mathrm{T}}{dt}$.

From the R.H.S. of (8) we then obtain $\mathrm{T} = Ce^{-i(E/\hbar)t}$ and the L.H.S of (8) can be written as :-

(9) $\frac{-\hbar^2}{2m_u}\nabla^2\Psi + V\Psi = E\Psi$.

Now from Equs. (7) and (9) we can see that the substitutions $x \rightarrow -x$, $y \rightarrow -y$, $z \rightarrow -z$, (abbreviated by $\bar{r} \rightarrow -\bar{r}$ below will not alter the solution of Schroedinger's equation if :-

(10) $V(-x,-y,-z) = V(x,y,z)$.

The substitution $\bar{r} \rightarrow -\bar{r}$ is called the parity operation, and a potential which has the property expressed in Equ.(10) is said to be conservative under the parity operation, or to conserve parity. For a potential of the form of (10), the wave function *Ψ* in Equ.(9) must have the property :-

(11) $\Psi\left(-\bar{r}\right) = +\Psi\left(\bar{r}\right)$

or

(12) $\Psi\left(-\bar{r}\right) = -\Psi(r)$

The wave function (11) is said to possess even parity, the other wave function (12) is said to possess odd parity. Further if any system, however complicated, has a wave function of a given type it can never change over to the wave function of the other type as long as all the interactions in the system remain parity conserving.

What has this to do with the manifest a-symmetry we see in the direction of the arrow of time? This a-symmetry can be explained by returning to the 'particle in a box' model.

We recall again the Cosmological principle which permits us to use the concept that any point in the universe can be described as the 'central reference frame' of the universe. With this in mind we now take the somewhat unconventional step of treating a galaxy in our spherical universe in the same way as we would treat a particle in a closed cubical box. now if the universe was indeed a closed cubical box the parity of the wave function given by $\Psi = \left(\frac{2}{L}\right)^{\frac{3}{2}} \frac{\sin n_x \pi_x}{L} \; \frac{\sin n_y \pi_y}{L} \; \frac{\sin n_z}{L}$ is not a definate quality (since $\Psi = 0$ outside the box we can see that $\Psi(x) \neq \Psi(-x)$ *for* $0 \langle \, /x/ \langle L$ *)*. This occurs because the location of the box with respect to the origin causes 'V' NOT to have the property as in (10) BECAUSE THE ORIGIN STARTS AT THE END OF THE BOX. But if the origin is moved to the centre of the box as permitted in our universal model (because the Cosmological principle allows any point to be considered as a central reference frame) then *'V'* WILL have the property as in (10) and the wave function then has the form :-

$$\Psi = \left(\frac{2}{L}\right)^{\frac{3}{2}} \sin\left(\frac{n_x \pi x'}{L} + \frac{n_x \pi}{2}\right) \sin\left(\frac{n_y \pi y'}{L} + \frac{n_y \pi}{2}\right) \sin\left(\frac{n_z \pi z'}{L} + \frac{n_z \pi}{2}\right)$$

where x', y', z', are the coordinates measured with respect to the centre of the box $\left(x' = x - \frac{L}{2} \; etc.\right)$. For any odd value of n_x , the first sine function beecomes $\pm \cos \frac{n_x \pi x'}{L}$ which has even parity. For any even value of n_x the first sine function becomes $\pm \sin \frac{n_x \pi x'}{L}$. Hence the overall parity of the wave function is even or odd depending on whether or not $(n_x + n_y + n_z)$ is an odd or even integer. From the above it follows that FIG.1. is describing a quantised gravitational field, brought about principally because it describes a matter wave moving from an area of high gravitational potential to an area of low gravitational potential under the influence of a gravitational field. Further it describes the wave function for those areas of space which are of opposite parity (i.e. that part of the universe which consists of anti - matter and negative energy). Thus in our model of a steady state universe we have described space as functioning in a similar manner to a solid with a matter wave (the galaxy) travelling through it. The next obvious question to ask is 'What happens to the matter wave which has travelled furthest from its central reference frame and is at the point where it has gained it's maximum velocity?' Firstly let us remind ourselves of the relatvistic addition of velocities. Here we seek to add the velocities of two galaxies, A and B, receding from each other . Their combined velocity is given by :- $V_{AB} = \frac{V_A + V_B}{1 + \frac{V_A V_B}{c^2}}$. Thus the test galaxy has exchanged all its gravitational potential energy for the kinetic energy of motion and since the galaxy cannot escape from the universe, clearly something must happen to the wave function. I have already mentioned that the wave function reaches a point which resembles the condition for Bragg reflection and indeed this is exactly what happens to the matter wave. It is reflected back into space at the point of the impenetrable potential barrier at the 'edge of the sphere, but the reflected wave is of opposite parity to the incident wave. This being the case we can write the wave equation of the reflected wave in gravitational terms as follows.

The potential *V(x)* on the left of the diagram can be said to represent the high potential at the centre of the sphere and the potential *V(o)* at the right is the potential of the energy barrier so we can write :-

$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + \left(mV(x) - mgx\right)\Psi = E\Psi$ where *mV(x)* is the energy due to the motion of the galaxy and *'mgx'* is the energy due to the gravitational field. the slope of the potential barrier is given by $mg = -\frac{E}{a}$ or $a = \frac{V}{g}$ where E represents the height of the energy gap in electron volts (i.e. $2mc^2$ expressed in electron volts and *'a'* is the width of the energy gap in metres). Normally a particle will tunnel when its wavelength is equal to or shorter than *'a'*. But in our case, since the energy gap or potential is of infinite width, the particle cannot tunnel and is therefore totally internally reflected. Thus we can say that the matter wave at R_0 undergoes a total internal reflection which can be described in the following manner.

Essentially what is happening is that the potential is generating a rotation in internal symmetry space. To generate this rotation we define the potential in the language of a Rotation group. A three dimensional rotation R(θ) of a wave function is written as $R(\theta)\Psi = e^{-i\theta L}\Psi$ where θ is the angle of rotation and L is the angular momentum operator. This rotation is comparable with the phase change of a wave function after a gauge transformation. The rotation has the same mathematical form as the phase factor of the wave function. But this does not mean that the potential itself is a rotation operator like R(θ). The amount of the phase change must be proportional to the potential to ensure that the Schroedinger equation remains gauge invariant. To satisfy this condition the potential must be proportional to the angular momentum operator L. The most general form of the Yang Mills potential to which the 'barrier potential' is exactly similar, is a linear combination of the angular momentum operators :-

(13) $$A_\mu = \sum_i A_\mu^i(x) L_i$$

where the coefficients $A_\mu^i(x)$ depend on the space-time position. This relation indicates that the potential is not a rotation but is the generator of a rotation.

The relation in Equ.13. displays the dual role if the potential is both a field in spacetime and an operator in the isotpic spin space. The potential acts like a raising operator L_x and can, for example, transform a down state into an up state. Thus a total internal reflection has induced a phase change in space-time. This is because the phase of a wave function can be described as a new local variable. Instead of a change of scale a gauge transformation can be reinterpreted as a change in the phase of a wave function i.e:-

(14) $$\Psi \rightarrow \Psi e^{-ie\lambda}$$

and the familiar gauge transformation for the potential A_μ becomes :-

(15) $$A_\mu \rightarrow A_\mu - \partial_\mu \lambda$$

Thus the wave equation is left unchanged after the two transformations in Equs.(14) and (15) are applied.

The non relativistic wave equation for the test galaxy can be written as :- $\left[\frac{1}{2m}\left(-i\hbar\nabla - gA\right)^2 + g\phi + V\right]\Psi = i\frac{\hbar d\psi}{dt}$ where the canonical momentum now appears as the quantum operator for $-i\hbar\nabla - gA$. After the phase change in Equ.14. there will be a new term proportional to $e\,\nabla\lambda$ from the operator $-i\hbar\lambda$ acting on the transformation wave function. This new term will be cancelled exactly by the gauge transformation of the potential according to Equ.15.

This then is the broad description of a spacetime which is trapped between two potentials. The form of this entrapment means that the galaxies can be likened to particles which are moving against the back ground of a field which is omnipresent in nature and which can be likened to the aether filed which was long since abandoned.

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