GOLDBACH'S CONJECTURE

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Goldbach's Conjecture

THEOREM

"Every even number greater than 2 is the sum of two primes"

AXIOM 1-- There is an infinite number of integers.

Proof

Suppose that to the contrary there exists an ultimate integer "p". Let "n" equal the sum of all the integers including "p" as follows:-

$$n = 1 + 2 + 3 + 4 + 5 + ... + p$$

Since 1 can always be added to any number, there must exist a number greater than "p" as follows:-

$$n = (1 + 2 + 3 + 4 + 5 + ... + p) + 1$$

Therefore the proposition is true.

AXIOM 2-- Contained in the infinite set of integers there is an infinite set of even numbers.

Proof

Suppose to the contrary there exists an ultimate even number "p". Let "n" equal the sum of all the even numbers including "p" as follows:-

$$n = 2 + 4 + 6 + 8 + 10 = ... + p$$

Since 2 can always be added to any even number, there must exist a number greater that "p" as follows;-

$$n = (2 + 4 + 6 + 8 + 10 + ... + p) + 2$$

Therefore the proposition is true.

AXIOM 3-- Also contained in the infinite set of integers, there is an infinite set of odd numbers.

Proof

Suppose to the contrary that there exists an ultimate odd number “p”. Let “n” equal the sum of all the odd numbers including “p” as follows:-

$$n = 1 + 3 + 5 + 7 + 9 + ... + p$$

Since 2 can always be added to any odd number, there must exist a number greater than “p” as follows:-

$$n = (1 + 3 + 5 + 7 + 9 + ... + p) + 2$$

Therefore the proposition is true.

AXIOM 4-- There are infinitely many prime numbers.

Euclid’s Proof (Brown 1999)

Suppose that to the contrary there is only a finite number of primes, therefore there will be a largest prime which is “p”. We now define a number “n” which is 1 plus the product of all the primes:-

$$n = (2x3x5x7x11x.........xp) + 1$$

Is “n” itself prime or composite?
If it is prime then the original supposition is false since “n” is larger than the supposed largest prime “p”. If “n” is composite it must be divisible without remainder by prime numbers. However none of the primes up to “p” will divide “n” (since there would always be a remainder 1), thus any number which does divide “n” must be greater than “p”. Therefore, whether “n” is prime or “n” is composite, the supposition that there is a largest prime number is false

Therefore the set of prime numbers is infinite.

AXIOM 5-- The sum of any two odd numbers is always an even number.

Proof

Suppose to the contrary that there is a pair of odd numbers which does not sum to an even number. Designate that pair as “p”

Let “n” be the product of all the odd numbers including “p”.

$n = 1x3x5x7x.........xp$

The product of a series of odd numbers is always itself an odd number and is always divisible by an odd number.
In the series above “p” would be equal to an odd number plus 1 and therefore “p” itself would be an even number, therefore “n” itself would be even and not odd.
Therefore in the infinite series of odd numbers, the sum of any two odd numbers is always an even number for example;-

1+3=4, 3+5=8, 5+7=12, 7+9=16 etc.

Therefore the proposition is true.

Thus from (5) above we can infer that there exists an infinite set of odd numbers (p) which contains an infinite sub-set of the sum of any two odd numbers (r). Since the sum of any two odd numbers is always an even number, then the infinite sub-set of the sum of any two odd numbers (r) contains a further infinite sub-set of any two prime numbers (q). Here we note that (q) is always an even number because the infinite set of odd numbers (p) contains an infinite set of even numbers (r) i.e. (r) and (p) are equivalent because any two odd numbers always sum to an even number as already stated)

The foregoing can be written in terms of formal logic (i.e. a necessary truth) as follows;-

2/ $(p \supset r) \supset [(q \supset r) \supset \{(p \vee q) \supset r\}]$

Which can be read as (if p then r), then [if (if q then r) then {if (either p or q) then r }].

Therefore the sum of any two primes will always sum to an even number.

Therefore Goldbach’s conjecture is true.

Q.E.D