THE

 **ABC**

 CONJECTURE

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 **SECOND DRAFT**

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The ABC conjecture states that there will only be finitely many examples where C counts as much greater then *rad(abc).* Here we state that A + B = C are three co-prime integers and that *rad(abc)* represents the multiplication of all the distinct primes which divide any of A,B and C (see unsolved problems.org). The problem is to prove or disprove the conjecture. We begin by defining the axioms of this proof—they are:-

1/ There exists an infinite number of even integers.

2/ There exists an infinite number of odd integers.

3/ There exists an infinite number of prime numbers (Euclid’s proof).

4/ There are fewer prime numbers than there are ordinal numbers

5 The frequency of prime numbers in the number line decreases with distance from the origin

6/ Each equation of the form A + B = C contains at least one even number because even plus even equals even. Odd plus odd equals even and odd plus even equals odd.

We are examining here five separate series of numbers which are:-

1/ 2 + 2 = 4

2/ 2 + 4 = 6

3/ 1 + 2 = 3

4/ 2 + 3 = 5

5/ 3 + 5 = 8

The following are the axioms of the further discussion and we will deal with each of the series from (1) through (5) as follows:-

**1/ The series beginning 2 + 2 = 4**

1x 2 + 2 = 4 *rad (abc)*

2x 4 + 4 = 8

3x 6 6 = 12

4x 8 + 8 = 16

5x 10 + 10 = 20

6x 12 + 12 = 24

7x 14 + 14 = 28

and so on….

In this series it will be noted that equations 1 to 4 inclusive and equation 6 all have prime factors of 2 and 3 and in each of these cases *rad(abc)*has prime factors of 2 and/or 3. In the case of equations 5 and 7 it will be noted that *rad(abc)* contains the prime factor of 2 in each case and prime factors of 5 and 7 respectively. Thus we can conclude that this pattern continues throughout the series through to infinity and we can also note that in each case A + B = C is always greater than *rad(abc)* and these results will stand for every integer where A = B and both are even integers.

**2/ The series beginning 2 + 4 = 6**

1x 2 + 4 = 6 *rad(abc)*

 = 6

2x 4 + 8 = 12

= 6

3x 6 + 12 = 18

 = 6

4x 8 + 16 = 24

= 6

5x 10 + 20 = 30

= 30

6x 12 + 24 = 36

=6

7x 14 + 28 = 42

and so on….

In a similar manner to series(1) we note that equations 1 to 5 inclusive all have prime factors of 2 and 3 as does equation 6. In the case of equations 5 and 7, *rad(abc)* contains the prime factors 2,5 and 7. Again this pattern continues through to infinity with the prime factors greater than3 always corresponding with the same prime factor in the number line. We also note that I each case

 A + B = C is always equal to or greater than *rad(abc)* and this continues through to infinity.

**3/ The series beginning 1 + 2 = 3**

1x 1 + 2 = 3 *rad(abc)*

2x 2 + 3 = 5

3x 3 + 4 = 7

4x 4 + 5 = 9

5x 5 + 6 = 11

6x 6 + 7 = 13

7x 7 + 8 = 15

and so on….

Again in a similar manner to series (1) and (2) equations 2 to 5 inclusive each have the prime factors 2,3 and 5. In the case of equations 5,6 and 7 these contain the prime factor equivalent to their position in the number line and again we can say that in all cases listed so far rad(abc) is always greater than

 A + B = C.

**4/ The series beginning 2 + 3 = 5**

1x 2 + 3 = 5 *rad(abc)*

2x 4 + 6 = 10

3x 6 + 9 = 15

4x 8 + 12 = 20

5x 10 + 15 = 25

6x 12 + 18 = 30

7x 14 + 21 = 35

8x 16 + 24 = 40

9x 18 + 27 = 45

10x 20 + 30 = 50

11x 22 + 33 = 55

12x 24 + 36 = 60

13x 26 + 39 = 65

14x 28 + 42 = 70

In this case the series is fundamentally different from the previous three series. equations 1 to 6 inclusive we find that A + B = C is smaller than or equal to *rad(abc)* but from equation 7 onwards A + B = C is always greater than *rad(abc)* except where the position in the number line equates to the prime factor which is introduced at that position. Since there are less prime numbers than ordinal numbers and the frequency of prime numbers decreases with distance in the number line , the value of *rad(abc)* will reduce with distance from the origin and the cumulative value of A + B = C will increase with distance from the origin.

**5/ The series beginning 3 + 5 = 8**

1x 3 + 5 = 8 *rad(abc)*

2x 6 + 10 = 16

3x 9 + 15 = 24

4x 12 + 20 = 32

5x 15 + 25 = 40

6x 18 + 30 = 48

7x 21 + 35 = 56

8x 24 + 40 = 64

9x 27 + 45 = 72

10x 30 + 50 = 80

11x 33 + 55 = 88

12x 36 + 60 = 96

13x 39 + 65 = 104

14x 42 + 70 = 112

and so on….

In a similar manner to (5) we find in this series that except for equations 1,2 and 3 and all those subsequent equations that A + B = C is always greater than *rad(abc)* and each equation contains the same prime factor which equates to its position in the number line.

Now each equation of the form A+B=C can be considered as being the origin of an infinite arithmetic progression of the form:-

where is the expression for the term of the series (but note that this expression is not relevant as each series is to be considered to be infinite). More concisely we can use the symbol for “the sum of terms such

as…” to write :-

 for the infinite series or :-

 for any particular point on the number line.

As an illustration, listed below is the series commencing 4 + 5 = 9.

1x 4 + 5 = 9 *rad(abc)*

2x 8 + 10 = 18

3x 12 + 15 = 27

4x 16 + 20 = 36

5x 20 + 25 = 45

 **Table 1**

6x 24 + 30 = 54

7x 28 + 35 = 63

8x 32 + 40 = 72

9x 36 + 45 = 81 **Table 1cntd.**

10x 40 + 50 = 90

11x 44 + 55 = 99

12x 48 + 60 = 108

13x 52 + 65 = 117

14x 56 + 70 = 126

15x 60 + 75 = 135

16x 64 + 80 = 144

17x 68 + 85 = 153

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and so on…

As can be seen the sequence commences with the equation 4+5=9 which occupies the first position in the number line and each subsequent equation occupies its own unique position in the number line i.e. .

The prime factorisation of equation 1 forms the basis of the prime factorisation of the entire series and we can see that the underlying prime factorisation has a multiplicative value of 30 and sums to this figure throughout the entire series. The exceptions to this rule occur at all points on the number line which are denoted by prime numbers i.e. positions 7,11,17… etc. and this pattern continues throughout the series.

It is interesting to note also that when a prime factor constitutes a part of either A,B or C then the series *rad(abc)* “spikes” at that point. Examples are positions 21 and 22 (not listed):-

21x 84 + 105 = 189

22x 88 + 110 = 198

and this pattern continues throughout the series.

From this it follows that *rad(abc)* multiplicative totals also form a series. This series is not progressive in the same way as the arithmetic progression

A + B = C. Instead it is a series that “flat-lines” until it spikes at the point of insertion of a new prime factor. As an illustration we again examine the A,B,C sequence 4 + 5 = 9 but this time from the point of view of the prime factorisation of the series which has the underlying value of 30 (see table 1) and symbolised by ‘p’:-

and in general we can write:-

where ‘n’ represents the relevant position on the number line.

Reminding ourselves that the objective of this exercise is to prove or disprove that are only finitely many examples where C counts as much bigger than *rad(abc)* we can write:-

where represents a position on the number line greater than position 1.

Turning again to Table 1 we note that there is a point of convergence at position 4 where A + B = C exceeds the underlying multiplicative total of *rad(abc)* i.e:- ABC *rad(abc).* This is true in all cases where A + B = C through to infinity and some further examples are:-

3 + 4 = 7 converges at position 6

5 + 6 = 11 “ “ “ 30

5 + 7 = 12 “ “ “ 18

6 + 7 = 13 “ “ “ 42

After the position of convergence A + B = C continues its arithmetic progression and grows ever larger whereas *rad(abc)* maintains its under lying flat line sequence except in those positions where a new prime factor occurs or ABC contains a prime factor which has previously been employed at an earlier position in the number line . Since it is known that the frequency prime numbers in the number line decreases with distance from the origin, we can say with confidence that it is not true that there are finitely many examples where C counts as much bigger than *rad(abc)* and therefore the conjecture is dis-proved.

All of the foregoing can be summarised in a theorem in elementary logic which forms a necessary truth as follows:-

which can be written in a longer form as:-

*If (if p then r), then [if (if q then r) then (if (either p or q) then r)]*

Accordingly the rule for Sentential Variables can be applied to the theorem allowing us to make the following substitutions:-

Let p represent the number line 1,2,3…

Let r represent the series of equations A + B = C…

Let q represent the series *rad(abc)*…

Then in words we can write :-

*If ( if there exists a number line 1,2,3… then there exists a series of equations A+B=C… ),then [if( if there exists a series rad(abc)… then there exists a series of equations A + B = C… ) then (if ( either there exists a number line 1,2,3… or there exists a series rad(abc)…) then there exists a series of equations A + B = C…)].*

Thus since all components of the theorem are infinite and that this is then shown to be a necessary truth and we have shown that A + B = C is always, or has the potential to be, greater than *rad(abc).*

To re-inforce the foregoing there is another method of expressing the theorem of a necessary truth.

Any ABC equation described within the sum :-

and which originates the series can be considered as either composite or

singular, meaning that if it is composite the factorisation has the potential to be either smaller or larger than ABC and similarly ABC has the potential to be either smaller or larger than *rad(abc),* the only certainty being that ABC “contains” both its own value and the value of its own prime factorisation and at some point the values of ABC and *rad(abc)* will converge and then diverge as previously described. This then allows us to use the Rule of Substitution for Sentential Variables. Applying this rule to and substituting ‘y is the greater value for p’, ‘y is composite for q’ and ‘x is not the greater value for r’ and noting that ABC is represented by y and *rad(abc)* is represented by x we obtain the following:-

If ( if y is the greater value then x is not the greater value),then [if (if y is composite then x is not the greater value) then (if( either y is the greater value or y is composite ) then x is not the greater value )].

 Therefore the conjecture is disproved.

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