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#  **THE MATHEMETICS OF**

 **EVOLUTIONARY**

 **PROCESSES**

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##  **BY**

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Even the most cursory glance at the broad spectrum of living things both plant and animal cannot help but bring to the attention of the observer the huge variety of species which exist in the world today. Moreover, examination of the fossil record reveals an even greater variety of life forms which existed in the past. Indeed it is true to say that so many life forms have existed in the past and have become extinct that a statistician could say hand on heart, that to a first approximation, all life forms have already become extinct such is the vast number of varieties which have in the past and do now exist.

Clearly life forms did not begin with an infinite variety, they began as a few simple structures which evolved over time into many complex structures. Therefore we can generalise by hypothesising that life forms occupied a small volume of space in the past and evolved to occupy a larger volume space in the future.

We can then extend our hypothesis further and say that evolutionary processes appear to have filled every possible available niche.

Generally we can say that the rate of evolutionary processes increases with time. That is to say the possibility space available to living species to undergo evolutionary change increases with time up to a maximum available possibility space which is that figure made available by the age of the universe itself. It follows that evolutionary processes are dependent on two main criteria, these being the total possibility space and the passage of time.

Thus we can write that the possibility space is proportional to time i.e:-

# *Ps ∝ T* where *Ps = possibility space* and *T = time*.

From this it follows that *Ps* represents the possibility space occupied up to and including the present epoch.

The rate of change of evolutionary processes i.e.  can be written as :-

 Equ.1

where  is the elapsed time of the evolutionary process to date and  is the total time available to the evolutionary process in question (clearly  cannot exceed the present age of the universe!) and thus we can write:-



where  and  are the final and initial states respectively of evolutionary processes.

Since the processes of evolutionary change are irreversible (if the processes were reversible, this would amount to a reversal of causality), the multiplicative qualities of possibility space can be converted into the additive process of evolutionary change by becoming a function of the natural logarithm of the possibility space in the following manner :-



i.e. the possibility space increases over time.

The foregoing then is a description of any naturally occurring system which is subject to evolutionary processes and we assume of course, that these processes are an inherent part of the general structure of nature.

We now ask the question ‘What is the effect on these natural processes if the are curtailed or interrupted in any way?’ A broad assumption could be that any decision made at any particular given point in time would affect the outcome of the process at any future given point in time. So how do we quantify the effects of ‘decisions’ taken today on future events?

In any probabilistic situation the set of all possible outcomes is the possibility space. Possibility space can best be illustrated by the use of branching models. In the case of random decisions all possibilities are equally likely. For example in a system where there are two alternatives presented at every decision node, the tree diagram is structured as below where each decision node is numbered 1,2 or 3 etc:-

 2

 1 3

 3

1

 3 FIG.1

2

i.e. at the first decision node there is a possibility space of 2. At the second decision node there is a possibility space of 4 and at the third decision node the possibility space is 8 and so on. Thus we can say that the possibility space of a branching process is multiplicative and the process can be described as a progression of the form:-

1+2+4+8……..∞

In the foregoing case the probability of an event occurring is always the inverse of the total possibility space. For example in the case of two alternatives the probable outcome results in a geometric progression of the form:-

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}…………..=\sum\_{r=0}^{\infty }\left(\frac{1}{2}\right)^{r}$$

In this case the first term is 1 or  so that the first value that *‘r’* takes is zero in every case whatever the bias of the system.

Thus the branching system is a geometric progression and is multiplicitive in nature. This being the case it now remains for us to convert this multiplicitive process into an additive process of cumulative behavioural change.

It is here that the significance of the “half-life” of the decision making process becomes apparent because the “half-life” would be the time taken to reverse any particular decision which in turn would quantify the consequences of reversing that decision.

The change in probability when *‘N’* behaviour patterns undergo a change of possibility space from  to  is:-



Where  and  are the final and initial states of the behaviour pattern in question and  and  are the initial and final volumes of possibility space respectively.

For the sake of simplicity we take the branching model where two alternatives are present as in FIG. 1. (although the comments which follow apply equally to any other model of three or more alternatives). By eliminating one alternative, half of the future possibility space is eliminated and all possibilities of choices being made in that ‘volume’ of possibility space vanish.

Any further elimination of alternatives further along the decision chain will result in a further reduction of the volume of possibility space and so a clear pattern emerges and we can say that firstly, the volume of possibility space is always halved at every point or node of elimination. Secondly we can say that the probability of evolutionary change is thus delayed because the aforementioned geometric progression commences at a later point in time. For example, in the ‘two’ alternative system just described, the probability of evolutionary change is now zero at node *1* whereas it was previously *1/2* and is now *1/2* at the point where it was previously *1/4*. Thus the whole of the geometric progression is moved forward in time and simultaneously the available possibility space has been halved. Clearly if this process of the elimination of alternatives and its attendant halving of possibility space is continued and is carried on to its logical conclusion, the effects on evolutionary processes over time must become apparent and these effects are manifested in the following manner.

##### As a general rule, we can write :-

 (Here $∆S$ is the change in possibility space

where the change in possibility space is positive and *N* is the total number of possibilities.



where the change in possibility space is negative and *N* is the total number of possibilities.

From the foregoing we can deduce that if  and  converge, that is to say if  is reduced in ‘volume’ then *ΔS* will tend to zero at which point evolutionary processes will cease.

Thus from Equ.1 we can write:-



from which we can conclude that  also tends to zero. Here  is a property of .

This then is the general model of evolutionary processes. The model can equally well be used to describe any one species as the totality of species. This being the case we can apply the model equally to a single species and thus calculate the consequences of any interruption to or interference with the evolutionary processes of that species.

It is possible to adapt the model to any population of a particular species provided only that the numerical population of the species is known and also the reproductive cycle of the said species.

The adaptation of the model proceeds as follows:-

The rate of reduction of possibility space at any given time is proportional to the number *N* of possibility spaces available at that time i.e. *Ps ∝ N* and we can write :-

 and since  is negative it follows that *Ps* decreases as *T* increases.

The introduction of a constant *λ* ( which we call the *‘space reduction* *constant’*) gives:-

 Equ. (2)

Now if there are  possibility spaces existing at *T = 0* and a smaller number *N* at a later time *T* then we can integrate (2) to give:-



In order to calculate the value of *λ* we relate it’s value to the reproductive cycle of any particular species. In the case of humans, the reproductive cycle is *≈ 25* years.

Applying this principle to a model of possibility space we can say that the half life of a species is the time taken to reduce the number the number of original possibility spaces  to one half of their original value thus;-



# When  then

 and 



From all the foregoing and substituting for  *= 25* years we can conclude that :-



and here we note that *λ* is a dimensionless number.

The process of decision making falls into two main categories. The first category concerns the simple question of a yes/no alternative or zero-sum game and the long term outcomes of those decisions. The second category concerns the question of evaluating the opportunities which occur (or do not occur) for those decisions to be made.

As a preliminary to analysing the reasons for the outcomes of various decisions made by societies it is appropriate to examine the way in which collective decisions are made and the underlying forces which guide those processes.

The first process to be examined is that of simple decision making where two choices present themselves i.e. a zero sum game. To this end we can utilise some of those mathematical principles which underlie the some of the general processes to be found in nature. The first of these processes will be an adaptation of the Hardy-Weinberg theorem which can be interpreted as a probability theorem more commonly applied to the understanding of genetic processes.

For the purposes of this exercise I will describe the majority of the decision making population as the Dominant (D) tendency. That is to say that that percentage of the population outnumbers the percentage of the population which may be subject to the likelihood or possibility to change and I will describe that proportion of the population as the Recessive (r) tendency. These characteristics *'D'* and *'r'* occur in the following combinations.

First Second Frequency of

tendency tendency occurrence

*D D D x D = *

*D r D* x *r*

 *=2Dr*

*r D r x D*

*r r r x r = *

Thus for example let us say that in any population of decision makers there is a 1/4 (25%) chance that an individual member of that population will belong to the *'DD'* dominant group which occurs with a probability of . Likewise there is 1/4 (25%) chance that an individual member of the population will make belong to the *'rr'* recessive group which occurs with a frequency of . It follows that 1/2(50%) of the population will be a mixture of *'D'* and *'r'* tendencies whose members will occur with a frequency of *2Dr.* The frequency of occurrence is in fact the same expression as that for the joint probability that any one choice tendency will dominate and the sum of these characteristic choice tendencies (or the joint probability) is then :-

*DD + 2Dr + rr = 1 (100%)* or 

This equation can be used to establish the probability of the tendency to change in any large population. For example take any population of 20000 individuals. Let us say that the number of population members who belong to the recessive tendency  i.e. 1/20,000 or 0.00005 and the frequency of this group is$\sqrt{0.00005}$ $≈ .007$

Since all individuals must belong to one group or another then the frequency (probability) of both tendencies must be *1* since *D+r = 1.* From this we can calculate the frequency of the dominant tendency *'D'* as follows i.e:-

  *D + r = 1*

 *D = 1 - r*

 *D = 1 - .007*

 *D = .993*

The probability of the occurrence of the recessive tendency in the 'undecided' population i.e:-

*2Dr = 2 x .993 x .007 = .014*

That is to say 140 in 20,000 will have a probability of tendency towards recession while only 1 in 20,000 will actively participate in the same recessive tendency.

Thus as soon as the proportion of the population aspiring to change reaches a total of $≈$25% of the population, in this illustration 5000/25000 ( i.e. the value of *r* = .25 ), then the proportion of the population possessing a tendency towards evolutionary change exceeds 50% of the population i.e:-

 2Dr = 2 x .553 x .447 = 51.3%

Therefore at this point the tendency towards change exceeds dominant tendencies and it becomes very apparent how it is that the frequency of extremely small minorities can come to dominate the tendencies of the majority. Furthermore it is remarkable to note that once the value of ‘r’ reaches 0.50 in a population of 20000 it is probable (not just possible) that the majority of the population will tend towards a change to a recessive tendency.

Thus as a general rule we can write :-

*Q = 1 - P - R*  where . Thus as *2Dr* increases over time $D^{2}$ decreases while $r^{2}$ increases proportionately.

Thus for any value of ‘*R’* we can calculate the probability of the tendency towards behavioural change in any given large population.

Having established that it is possible to predict with some accuracy the tendency for certain tendencies to change from a dominant to a recessive tendency in the total population, we can now establish that the underlying cause of this phenomenon is purely statistical in its nature.

Thus we note that the existence of any tendency towards evolutionary change must produce a greater proportion of potential evolutionary change in the population at large and it is remarkable to note that once the value of ‘*r*’ reaches .50 it is probable (not just possible) that the majority of the population will tend towards behavioural change. The probability function *Q = 1 - P - R* can only exist if the opportunity for change exists. More particularly we can say that, if the opportunity for evolutionary change is removed, then the probability towards change will cease to exist. But as soon as the opportunity for evolutionary change is present for the whole of the population, then the tendency towards evolutionary change for the whole of the population must increase dramatically and the most probable evolutionary state is the one for which the number of possibilities is at a maximum.

 END

