THE LARGE SCALE STRUCTURE OF THE UNIVERSE

THE RELATIONSHIP BETWEEN THE VERY LARGEST AND VERY SMALLEST STRUCTURES AND THE COSMOLOGICAL CONSTANT

BY

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INTRODUCTION

In this paper I have set out to describe those conditions which are necessary to lead us some way down the path towards the quantisation of gravity. Before addressing this question, there is one over-riding condition which must be established and that is the description of a state where the Weyl tensor equates to zero, that is to say there must be a description of a gravitational field where tidal distortion is non-existent and the field is symmetrical at all points in space. The description of this state leads to the conclusion that the use of the cosmological constant (the Λ term) is the correct description of the large scale structure of the Universe. Furthermore, by utilising this description I am led to conclude that the numerical relationship between the largest and smallest structures is of a very intimate nature, very much along the lines already proposed by Sir Arthur Eddington and Professor P.A.M. Dirac many years ago. Indeed I have drawn very heavily on their work in preparation of this paper. Similarly, I have drawn heavily on 'The Emperors New Mind' by Professor Sir Roger Penrose and 'The Accidental Universe by Professor P.C.W. Davies.

A list of reference sources appears at the end of this paper and it is my sincere belief that I have fully acknowledged the use of those sources in my text. If I have not then I apologise for any omissions of acknowledgment which may have occurred.

There are three topics in Physics and Cosmology which have been the subject of debate in the scientific press recently. These topics are the Critical Mass Density of the universe, the value of Hubbles constant and Einstein's lambda term (otherwise known as the cosmological constant). It seems to me that these three topics are all intimately connected with each other and more particularly they are also connected with the vexed topic of the quantization of gravity. In this paper I set out an attempt to unify all these topics by examining each of their places in both the large and small scale structure of the universe.

Firstly in order to define quantum gravity it is necessary to establish those phenomena which the theory should describe and I list below those conditions which I consider to be necessary conclusions of the theory. They are :-

1/ To define the fine structure constant and the value of the electric charge in terms of the gravitational constant.

2/ To deduce the masses of the fundamental particles in terms of the gravitational constant.

3/ To describe the conditions where the tensor Weyl = 0 and the tensor Ricci =∞ (Ref. 25 p.453)

The full mathematical expression for the curvature of four dimensional space-time is written in terms of the Reimann curvature tensor which is itself split into two parts, the Weyl tensor and the Ricci tensor already mentioned above. The Weyl tensor measures the tidal distortion and the Ricci tensor measures the change in volume of a body in a gravitational field. The tidal effect is produced because the gravitational field is directional, that is to say the field is operating in one direction only (Ref. 25 p. 271).Thus for the Weyl tensor to equal zero the field must be one which operates uniformly in all directions at the same time. The best way to to describe this situation is to imagine a body placed at the centre of a field which is acting on all parts of the body at the same time and in all directions thus there is no directional tidal effect producing a distortion in the shape of the body. There is however, an increasing rate of acceleration experienced by successive 'layers' of the body as one moves from the centre of the body towards the outer 'shell' of the body. This situation can best be described by resorting to the equations of Gauss's law and by treating the universe not exactly as a solid, but as a body whose mass is uniformly distributed throughout its structure. To this end we can state that;-

'The gravitational flux for an arbitrarily closed surface is equal to the net mass enclosed divided by 'g' i.e:-

$$1/ \qquad \Phi = \oint g.d\bar{R} = \int gdR = g\oint dR = g(4\pi r^2)$$

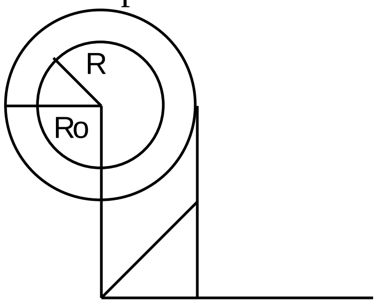
that is to say, treating the universe as a sphere, the flux is a product of the gravitational force and the area of the sphere.

Now equation (1) is derived from $\Phi = \frac{\Sigma m}{G} \quad or \quad \oint g.dR = \frac{\Sigma m}{G}$

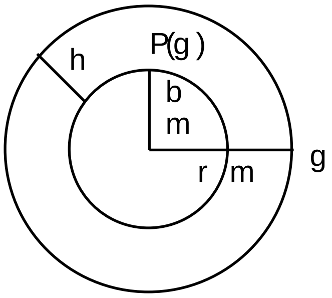
where G = gravitational constan*t* and m = mass of any galaxy.

The integral $\oint dr$ is the surface area $4\pi r^2$ of the sphere. By using symmetry arguments we can determine 'g' at any point inside a uniform spherical distribution of mass m and radius R_o by using spherical Gaussian surfaces with the same centre as the mass distribution. To find the field 'g' inside the mass distribution we use a Gaussian surface whose radius is less than the radius of the mass distribution i.e. $R < R_o$. The expression for the flux of the surface is again $\Phi = g\left(4\pi r^2\right)$. The mass contained within the the Gaussian sphere depends on the radius 'R' of the sphere. If we let 'ρ' represent the mass density and the volume of the Gaussian sphere where 'M' is the total mass of the sphere and is given by

$\frac{4}{3}\pi R_o^3$ then we can say that $\Sigma m = \rho \frac{4\pi r^3}{3} = \frac{M}{4/3\pi R_o^3}\frac{4\pi R^3}{3} = M\frac{R^3}{R_o^3}$. Gauss's law $\Phi = \frac{\Sigma m}{G}$ for this case is $g(4\pi R^2) = \frac{M\,R^3/R_o^3}{G}$ and solving for *'g'* gives $g = \frac{MR}{4\pi G R_o^3}$ $\quad (R \le R_o)$ i.e.the gravitational field increases linearly with *'g'* at points inside the sphere in the manner shown in FIG.1. below.



Referring now to FIG. 2. below :-



We can see that since the gravitational field strength increases linearly with distance it follows that the gravitational acceleration *'g'* at *'P'* is due entirely to the shell of thickness *'h'* where h = r-b . So we can write $g_1 = \frac{r}{b}/g$ since b = r-h $\therefore$ $g_{1=}\left(\frac{r}{r-h}\right)/g$ $\therefore$ $h \to 0,\ g_1 \to g$

Now the mass of the total sphere can be expressed as :- $M = G\rho\frac{4\pi r^3}{3} - \frac{4\pi b^3}{3} = r^3 - b^3 G\rho$. Since $h = r - b$ and $b = r - h$ we can write $M = 4\pi\left(\frac{r-b}{h}\right)^2 G\rho$ and we can state that the acceleration due to gravity is due entirely to the area $4\pi r^2$ and we can write $g = \Lambda = 4\pi G\rho$ because $r = b$ where r-$b/h = 1$ (here Λ is defined as the cosmological constant to be discussed in more detail later).

Furthermore, utilising Gauss's law again we can state that the mass M can be said to be distributed uniformly over the surface of the sphere i.e;-

$g=\dfrac{M}{4\pi GR_0^2}$. This produces the remarkable condition whereby it can be said that the gravitational force acting on any particle within the sphere is proportional to the entire mass of the universe and that the particle can be said to be lying in a symmetrical force field about the point 'O'. There is no tidal (i.e. directional) effect here and therefore the tensor Weyl = 0.

Keen students of cosmology will recognise the expression $\Lambda = 4\pi G\rho$ as the result obtained by Einstein as the condition for a static universe where Λ is described as the cosmological constant or the Λ factor and, for the sake of good order I describe below the derivation of the Λ term because of its importance in the development of the arguments outlined herein paper.

Now the fundamental equations of General Relativity as described by Einstein are written in terms of the Ricci tensor $R_{\mu\nu}$ and are given by ;-

2/ $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \left(\frac{8\pi G}{c^4}\right) T_{\mu\nu}$$ where $T_{\mu\nu}$ is the energy momentum tensor of the source producing the gravitational field. The Ricci scalar is defined by :- $R = g^{\mu\nu} R\mu\nu$ where $g^{\mu\nu}$ is the contravariant tensor corresponding to $g_{\mu\nu}$. All the information about the gravitational field is contained in the covariant tensor $g_{\mu\nu}$ and the number of indices gives the rank of the tensor. The 00 and 11 components of Equ. 2 can be written as

$$3\left(\dot{R}^2 + c^2 k\right) = 8\pi G\varepsilon R^2 / c^2$$ and $$2\ddot{R}R = \dot{R}^2 = kc^2 = -8\pi G p R^2 / c^2$$ here

p = pressure and ε = mass energy density. The critical density at any time is :-

$\varepsilon_c = 3H^2 / 8\pi G$ where $H = \dot{R}/R$ and here H = Hubbles constant (to be evaluated later). (Ref. 15 p.15)

Fortunately these equations can be translated into Newtonian expressions by referring to FIG. 5 in the following manner.

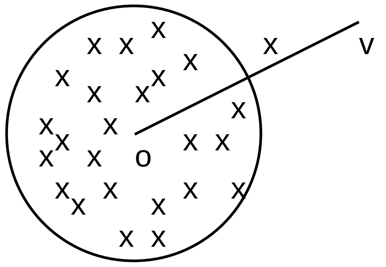


FIG. 3.

The force F on x is due to all the mass within the sphere i.e :- $M = 4\pi \frac{\rho(t)}{3} x^3(t)$ so Newtons second law for the galaxy of *m(x)* is :- $F = ma = \frac{m_x d^2 x(t)}{dt^2} = \frac{4\pi G m_x}{3} \frac{x(t)\rho(t)}{x^2(t)}$. We substitute for the proper distance *x(t) = R(t)r* (where *R = scaling factor*) and for the density $\rho(t) = \frac{\rho_{m0}}{R^3(t)}$ where $\rho_{m0} = \rho(T_0)$, the current mean density. With these substitutions Equation 2. becomes a differential equation for *R(t)* i.e;-

(3) $\frac{d^2 R}{dt^2} = -\frac{4\pi G\rho}{3}\frac{1}{R^2}$. Integrating once yields a constant of integration $-kc^2$ and we get:-

(4) $\dot{R}^2 = \frac{8\pi G\rho}{3}\frac{1}{r} - kc^2$ which contains information about space time geometry. The present values of the scaling factor and its rate of change are $R = 1$ and $\dot{R} = H_0$ respectively thus :-

(5) $H_0^2 = \frac{8\pi G\rho}{3}\frac{1}{1} - kc^2$. From Equ.5 we can write $H_0^2 = \frac{8\pi G}{3}\frac{3H_0^2}{8\pi G}\Omega - kc^2$. Here we note that $1 - \Omega = -H_0^2 c^2$ and $\Omega = \frac{\rho_{m0}}{\rho_c}$ and $\rho_c = \frac{3H_0^2}{8\pi G}$. Lastly using Equ.5. and inserting it into Equ.4 we can write :- $\dot{R}^2 = \frac{8\pi G\rho}{3}\frac{1}{R} - H_0^2(\Omega - 1)$

(Ref. 33 p.33)

Moving on now we have already written that $\Lambda = 4\pi G\rho$ which we derived from the notion that universal expansion results from the fact that the Weyl tensor is equal to zero and that universal expansion is not the result of an explosive expansion but rather is the result of the repulsive gravitational effect of the lambda term already described. Of course it is well known that the Λ term fulfifls the condition for a static universe. The

Λ term modifies the laws of general relativity at large distances into into a repulsion proportional to distance i.e. $r = \Lambda r$ with Λ constant . The Λ term is consistent with Einsteins field equations and we will therefore adopt this term in order to modify Equtions (3) and (4) to produce the required repulsive effect so (3) becomes :-

(8) $\ddot{R} = -4\pi \frac{G}{3} \rho R_0^3 + \frac{\Lambda}{3} R$ and (4) becomes :-

$\dot{R}^2 = 8\pi \frac{G}{3} \rho R_0^3 \frac{1}{R} - kc^2 + \frac{\Lambda}{3} R^3$. That is to say the Λ term has negligible effect near $R = 0$ its effect is at a maximum and the universe is in a steady state. If $k = 1$ there is a critical value of Λ i.e. Λ_c such that $R = 0$ and $\dot{R} = 0$ can be satisfied simultaneously. From Equ. 8 $\ddot{R} = 0$ implies that $R = R_0 \left(\frac{4\pi G \rho_0}{\Lambda} \right) = R_c$ and Equ.9 implies that $0 = \left(4\pi G \rho_{m0}\right)^{\frac{2}{3}} \Lambda^{\frac{1}{2}} - kc^2$ so that

$\Lambda_c = \frac{\left(kc^2\right)^3}{R_0^6 \left(4\pi G \rho_{m0}\right)^2}$. This means that there is a possibility of static model of the universe with $R = R_c$, $\Lambda = \Lambda_c$ for all time *'t'* provided that :-

(10) $\Lambda = 4\pi G \rho_c = kc^2 / R^2 c$ and since $\rho_c \geq 0$ it follows that *'k'* must be positive for this to happen. (Here we note that $R_o = R\left(T_0\right) = 1$ i.e. the scale factor is chosen to be unity at the present epoch). (Ref.29 p.132)

Thus we are now in accord with our conclusions on page 3. From this we conclude that not only is the gravitational attraction between the galaxies cancelled by the repulsive effect of the Λ factor but that the expansion of the universe is the result of a 'negative' gravitational force, the geometry of which we have already described in the paragraphs leading to the conclusions on page 3. Furthermore we can say that each galaxy is, in effect, falling in a gravitational field and can therefore be described in terms of its gravitational potential in this field. As will be seen this will enable us to establish the radius and consequently the size and mass density of this static universe with some degree of accuracy and we will approach this question in the following manner.

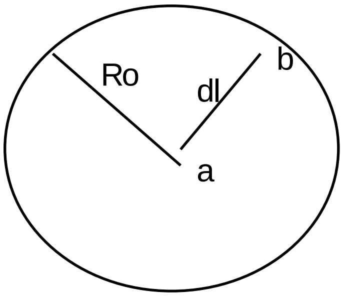


FIG.4

From FIG.4. we can imagine that a test galaxy is falling in a gravitational field and that the gravitational field inside the mass $'M'$, which represents the entire mass of the universe, is given by $\bar{g} = \frac{Mr}{4\pi G R_0^3}$ .Fig. 4. illustrates schematically a spherically symmetrical uniform volume mass and we can find the potential difference inside the mass by integrating along a radial path i.e.

$$V_b - V_a = -\int_{r_a}^{r_b} \left(\frac{Mr}{4\pi G R_0^3} \hat{r} \right) . \left(dr\, \hat{r} \right)$$

$$= -\frac{M}{4\pi G R_0^3} \int_{r_a}^{r_b} r \; dr$$

(11) $$= \frac{M}{8\pi G R_0^3} \left(r_a^2 - r_b^2 \right)$$

To find the potential at any point inside the mass distribution we can say that the potential V_0 evaluated at any point on the surface of the sphere is given by :-

(12) $$V_0 = \frac{M}{4\pi G R_0}$$ and from Equ.11. we can let r_a correspond to a point inside the sphere i.e. $\left(r_a = R, V_a = V(R) \right)$ and let r_b correspond to a point on the surface of the sphere $\left(r_b = R_0, V_b = V_0 \right)$. This gives ;- $V(R) - V(O) = \frac{M}{4\pi G R_0^3} \left(R_0^2 - R^2 \right) \quad \left(R \le R_0 \right)$. Thus the potential varies within the sphere and each galaxy can be said to be lying on an equipotential surface within the sphere. The gravitational field lines are perpendicular to the equipotentials and point from a higher to a lower

potential. From Equ. 12. we note that potential is highest when *R* is smallest and decreases with increasing *R*. This means that a galaxy is moving from a higher to a lower potential which means that the galaxy gains kinetic energy expressed by the familiar $K.E.=\frac{1}{2}mv^2$ which is in fact the energy of any galaxy at any point within the sphere.

From the foregoing we now have a mechanism for treating the motion of any individual galaxy of mass 'm' within the larger mass *'M'* of the universe. Since we are treating the galaxy as a particle in a gravitational field we can say that the potential energy due to gravity is $E_p = mgr$. A particle in a gravitational field has potential energy because the field does work moving the particle from one place to another . The gravitational potential at a point in a gravitational field is is defined as the potential energy per unit mass ata the point. Designating the gravitational potential by *'V'* and the potential energy of a mass *'m'* by E_p we have :- $V=\frac{E_p}{m}$ or $E_p = mv$. If a mass moves from one point *'r'* to another point *'R'* then the work done by the gravitational field is ;-

(13) $$W = E_{p(r)} - E_{p(R)} = m\left(V_r - V_R\right)$$

so the difference in potential between points *'r'* and *'R'* is $V_{(r)} - V_{(R)} = \frac{W}{m}$ or $\Delta V = -\frac{W}{m}$. Now since *W = Fr* and *W = mgr* and sine the galaxy (or particle) has been effectively accelerated from rest to some velocity *'v'* then, in this case, $v^2 = 0^2 + 2gR$ $i.e.$ $gR = \frac{v^2}{2}$ and since *F = ma =mg* it follows that *W = mgr*, therefore $W=\frac{1}{2}mv^2$. Noting that $W = -\Delta E_p$ and using Equ. 13. we can state that $\frac{1}{2}mv^2 = m\left(V_{(r)} - V_{(R)}\right)$ and cancelling 'm' on both sides we have $\frac{1}{2}v^2 = V_{(r)} - V_{(R)}$. It follows that, since the maximum velocity which can be achieved by any moving particle (or galaxy) is $\approx c$, then the maximum radius of the universe is

$\frac{1}{2}c^2$ light years and therefore $R_0 = \frac{1}{2}c^2$ light years where R_0 is the maximum possible radius of the universe.

A static model of the universe allows us to develop several interesting ideas. Firstly, since the volume of the universe is static and the radius is known we can utilise the Schwarzchild solution to write $R_0 = \frac{2GM}{c^2}$. Furthermore we can state that $G = \frac{R_0 c^2}{2M}$ which implies that *'G'* varies with distance and of course with time. Referring again to Fig.4. , any observer at point *'O'* with a view radially outwards into the universe is looking backwards into time. This implies that time *'T'* increases from the point T_0 to the point *'O'*. This implies that $G(t) \propto \frac{1}{T}$ and from this we deduce that the gravitational constant *G(t)* is proportional to Hubbles constant *H(t)* which is again proportional to $\frac{1}{T}$. However since $T \propto R$ we can state that

(13) $$G \propto \frac{1}{R} \; and \; G \propto H(t) \; .$$ (Ref. 29 p.142)

I end this section of the discussion by pointing out that, of course, the galactic redshift is a gravitational effect caused by a body (in this case any galaxy) falling in a gravitational field and will continue to develop the model of a static universe in the following manner.

It will be recalled that Sir Arthur Eddington produced many dimensionless ratios of the order of 10^{40} and various derivatives of this number and I intend to use Eddingtons work to develop the steady state model further. Eddingtons work is particularly useful in this respect, particularly because we have been able to define precisely the the radius and volume of the universe.

Eddingtons 'magic number' is, in fact, $\sqrt{10^{80}}$ which we describe as an expression for the total number of baryons in the universe (and therefore by implication, the bulk of the mass of th universe *'M'* and I will now take time to describe Eddington's derivation of this value.

$'N'$ is defined as the total nuber of elementary particles in the universe and was defined by Eddington as an effetive number of degrees of freedom of the universe. Wave mechanics defines $'N'$ as the number of independent wave systems in the universe and is therefore equal to the nuber of separate constituents of energy of the universe. In classical theory each of $'N'$ particles would have several degrees of freedom, but the exclusion principle limits the freedom of a particle by forbidding it to enter an orbit already occupied by another particle.

Present observations suggest that for matter the density is $\rho_m \approx 10^{-11} Jm^{-3}$ from which we can deduce $'N'$. The actual form Eddington gives the argument arises by considering not one number $'N'$ but a sequence i.e:-

$N_r = n_r(n_r+1)2^{n_r^2}$, $r = 1, 2, 3 \ldots .$ $where\, n_1 = 2, n_r = n_{r-1}^2$. The first few values of n_r are 2, 4, 16, 256.... from which can be obtained ;-

$$N_1 = 2x3x2^4 = 96$$

$$N_2 = 4x5x2^{16} = 2x10x2^{16} = 131072($$

$$N_3 = 16x17x2^{256} = 2x136x256$$

$$\approx 10^{18}$$

And thus for future reference we can define the number $'p'$ as:-

$$p = \sqrt{N} = 10^{40} \qquad \text{(Ref. 16 p.200)}$$

Furthermore we note that present observations of matter density indicate that the amount of cosmic material inside the Hubble radius is of a very similar order i.e:- $\rho \approx 10^{-11}\, J\, m^{-3} \approx 10^{80}$ protons. (Ref. 4 pp58 and 59)

It will be recalled that in our cosmological model, every particle is surrounded by a symmetrical gravitational field and we can describe this field as a force acting over the whole surface area of each individual particle contained within the sphere. By nominating the Planck length $1x10^{-35} m.$ as our lowest boundary condition and by introducing a gravitational force equal to the universal mass $'M'$, we can define the ratio of the mass of the proton to the mass of the universe and can thus define the origin of the mass of the proton and the inter-relationship of the other physical constants in the following manner.

We have already defined the mass of the universe to be $M = \frac{R_0 c^2}{2G}$ and we can calculate the actual value of M by inserting the following values i.:-

$G = 6.67x10^{-11} m^3.kg^{-1}.s^{-2}$, $c = 3.0x10m.s^{-1}$, $R_0 = \frac{1}{2}c^2$ *lightyears* $= 4.26x10^{32}$ *m.*

from which it follows that $M = \frac{4.26x10^{32}\ x\ 9x10^{16}}{1.33x10^{-10}} = 2.88x10^{59}\,kgs$.

having established the values of M and p we can move on.

From Equ. 13 we have shown that Hubbles constant varies with time and this bears out Eddingtons original hypothesis that if :-

(14) $$\frac{G / H_0}{\hbar\varepsilon_0 / m_p^3 e^2} \approx 1$$

then one of the fundamental constants G, $\hbar$, m_p, and e must change with time (here m_p and *'e'* are the mass of the proton and the charge on the electron recpectively).

Now the average density of matter in space expressed on protons per unit atomic volume $\left(e^2 / mc^2\right)$ turns out to be a number of the order of p^{-1} . Using R(t) as the scale factor of expansion and assuming mass to be conserved $\left(\rho R^3 = const.\right)$ the identification of the density and Hubbles constant $\left(\dot{R} / R\right)$ leads to the relation $R(t)^3 \approx p^{-1} \approx \dot{R} / R$ and we get

$$\frac{G(t)}{H(t)} = \frac{G(t)\,R(t)}{\dot{R}} = const.\ and\ \frac{\rho(t)\,G(t)}{H^2(t)} \propto \frac{1}{R(t)^3}\,G(t)\,\frac{G(t)}{R(t)\,\dot{R}(t)^2} = const..$$

Now *G(t)* can be eliminated to obtain $R^2(t)\,\dot{R}(t) = const.\ so\ that\ R(t) \propto t^{1/3}$ and we can write:-

(15) $$H(t) = \frac{\dot{R}}{R} = \frac{1}{3}t$$ (Ref. 2 p. 161)

In the atomic units 'e' and 'm' are constant but e^2 / Gm^2 ia a number of the order p and is proportional to T_0 . Therefore G must be proportional to t^{-1}. Furthermore as R increases so H decreases. Clearly the value of H is of vital importance to establishing the value of T_0 .

To establish the value of Hubbles constant H_0 for a universe of radius $\frac{1}{2}c^2$ light years we can proceed as follows:-

$$T_0 = \frac{1}{H_0} = \frac{10^6\ pars}{H_0} = 1.4x10^{18}\ s.$$

$$= \frac{3.09x10^{19}\ kms.}{H_0.} = 1.4x10^{18}\ s.$$

$$H_0 = \frac{3.08x10^{19}\ k.}{4.74x10^{17}\ s.} = 65.18 \quad \left(i.e. 1.5x10^{10}\ years\ x\ 3.16x10^7\ s. = 4.74x10^{17}\ s.\right)$$

$$\therefore H_0 = \frac{3.09x10^{19}\ km.}{1.4x10^{18}\ s.} = 22.07\ km./s.$$

$$i.e: \frac{1}{2}c^2\ light\ years = \frac{\left(3x10^5\right)^2}{2} = 4.5x10^{10}\ years = 1.4x10^{18}\ s.$$

but from Equ.15 we have shown that $H_0 = \frac{1}{3}t$ so that $T_0 = \frac{1}{3}H_0^{-1}$ $\quad \therefore \quad T_0 = \frac{4,5x10^{10}}{3}$.

$$= 1.5x10^{10}\ years.$$

This confirms a new value of H_0 as stated above i.e:-

$$H_0 = \frac{3.08x10^{19}\ km.}{4.74x10^{17}\ s.} = 65.18 \quad \left(i.e.\ 1.5x10^{10}\ yrs.\ x\ 3.16x10^7 s, = 4.74x10^{17} s.\right) .$$

A well known cosmological model is that produced by Paul Dirac in the 1930's. The drawback with this model was it's unfortunate implication that the universe was younger than some of its component parts. The foregoing derivation of of H_0 solves this problem and and enables us to study his work with greater confidence.

Dirac considered the ratio of the reciprocal of Hubbles constant $(1.5x10^{10} years)$ to an atomic unit of time $e^2/4\pi\varepsilon_0 m_e c^3$. Now $\frac{1}{H_0} = T_0$ so we can write $\frac{4.74x10^{17}}{2.57x10^{-38}/4\pi x 8.5x10^{-12}\ x 9.11x10^{-31}\ x 2.7x10^{25}} \approx 10^{40} = p$.

(Ref. 2 p. 161)

A further proposition by Dirac was that the value of $'G'$ would change with time and in Equ. 13. we have shown this to be the case. Dirac's theorem went on to compare the ratio of the electrostatic force to the gravitational force in the following manner:-

$\frac{F_E}{F_G} = 10^{40}$ *where* $F_E = \frac{e^2}{r^2}$ *and* $F_G = G\frac{m_e\, m_p}{r^2}$ where the radius of the electron is expressed as $r = \frac{e^2}{m_e c^2}$. Dirac went on to compare the radius of the universe with the radius of the electron $'r'$ and found the relationship $\frac{R}{r} = 10^{40}$ where the radius of the universe is expressed as $cT_0 = \frac{c}{H} = R_0$ and we will now proceed to explore these relationships further and we begin by writing:-

$\frac{R_0 c^2 / 2G}{GMr_p^2} = 10^{40} = p$ and in a similar manner we can write $\frac{1/H_0}{e^2 / 4\pi\varepsilon_0 m_e c^3} = 10^{40} = p$ where $e^2 / 4\pi\varepsilon_0 m_e$ is the atomic unit of time. This enables us to write a further three equalities as follows:-

(17) $\frac{cT_0}{e^2 m_e c^2} = p$ where cT_0 is the radius of the universe and $e^2 / m_e^2 c^2$ is the classical electron radius and :- (Ref. 29 p. 141)

(18) $\frac{e^2}{4\pi\varepsilon_0 G m_e m_p} = p$ where e^2 is the electromagnetic force between electron and proton and $4\pi\varepsilon_0 m_e m_p$ is the gravitational force between electron and proton. And:-

(19) $\frac{\rho (cT_0)^3}{m_p} = p$ where $\rho = 6.67x10^{-26}\, kg.m^{-3}$.

In a similar manner we can establish the link between electromagnetic force, the weak force and the gravitational force. The link between electromagnetism and the weak force is manifested by the weak coupling constant α_w by writing ;-

$\alpha_w = \alpha (m_e / m_w)^2$ where α is the electromagnetic fine structure constant i.e:- $\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = 7.3x10^{-3} = 137^{-1}$ *and* m_w is the mass of the $'w'$ particle.

Now gravitation and the weak coupling constant are linked by:- $\alpha_w = g_w (m_e^2 c / h^3) = \sqrt[4]{p}$ (Ref. 4 p. 80)

$\alpha_w^4 = \alpha_g$ where α_g is the gravitational fine structure constant i.e:- $\alpha_g = \frac{Gm_p^2}{\hbar c} = 5.9x10^{-39}$ and the relation between electromagnetism and gravitation is given by :- $\frac{\pi}{p\alpha_g^{-1}} = \alpha^{-1}$ *where* $p = 10^{40}$. Thus we can write the following :-

$\sqrt[4]{\frac{1}{Gr_p^2}} = \sqrt{\alpha_g} = \alpha_w = \sqrt[4]{p} = \alpha\left(m_e / m_w\right)^2 = \left(\frac{\alpha_g^{-1}\alpha^{-1}}{\pi}\right)^4$ and here we note that :-

$\alpha_w = \left(m_e^2 c / h^3\right) = \sqrt[4]{p}$ and $g_w m_e^2 c / \hbar^3 = \left(Gm_e^2 / \hbar c\right)^{\frac{1}{4}}$ where g_w is the weak force constant i.e. $g_w = 1.43x10^{-62}$. This leads to the further relationships :-

$\sqrt[4]{\frac{t_H}{t_N}} = \sqrt[4]{p} = \sqrt[4]{\alpha_g^{-1}} = \sqrt[4]{\left(\frac{t_N}{t_P}\right)^2}$ where :- (Ref. 5 p ???)

$t_N = nuclear\ time\ scale = \frac{h}{m_p c^3}\ \left(Compton\ time\right)$

$t_P = Planck\ time \left(\frac{G\hbar}{c^5}\right)^{\frac{1}{2}}$

$t_H = Hubble time\ H^{-1}$

The constants used in these expressions are as follows :-

$G = 6.72x10^{-11}\ m^3 kg^{-1} s^{-2}$
$H_0 = 2.109x10^{-18}\ s.$
$\varepsilon_0 = 8.85419x10^{-12}\ F.m^{-1}$
$m_p = 1.67265x10^{-27}\ kg.$
$e = 1.60219x10^{-19}\ C.$
$r_p = 10^{-15}\ m.$
$T_0 = 4.74x10^{17}\ s.$
$c = 3x10^8\ m.s^{-1}$
$m_e = 9.1095x10^{-31}\ kg.$

Lastly, to solve the problem of whether or not the universe is bound or unbound we can proceed as follows.

If ρ_{m0} is the present value of the average matter density of the universe, the total mass inside the sphere can be written as :-

$M = \frac{4\pi\rho_{m0}R_0^3}{3}$ where $M = 2.88x10^{59}\,kg.$ and $R_0 = 4.26x10^{32}\,m.$ so we can write -

$$\rho_{m0} = \frac{3M}{4\pi R_0^3}$$

$$= \frac{8.64x10^{59}}{1.1328x10^{99}} \quad .$$

$$= 7.63x10^{-40}\ kg.\,m^{-3}$$

The critical mass density is given by:-

$$E = \frac{1}{2}mH_0^2R_0^2 - \frac{Gm}{R}\frac{4\pi\rho_c R^3}{3} = 0$$

$$\therefore \rho_c = \frac{3H_0}{8\pi G}$$

which in our case gives the result :-

$$\frac{1.335x10^{-35}}{1.7x10^{-9}}$$

$$= 7.85x10^{-27}\,kg.\,m^{-3}$$

If $\rho_{m0} \geq \rho_c$ the universe is bound and if $\rho_{mo} \leq \rho_c$ the universe is unbound.

Therefore we conclude that our universe is unbound but finite.

END

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