COMPLEXITY

A study of perturbations in self organizing systems which are in statistical equilibrium.

With particular reference to evolutionary theory.

BY

David J.M. Short

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The conventions of modern scientific theory describe the Universe as a collection of particles randomly organised with blind forces acting on those particles. Our experience of the world tells us that it is anything but random, on the contrary we see the world as a complex organisation moving towards even greater complexity and organisation with the passage of time. Scientific theory does not attempt to explain how it is that a totally directionless collection of particles and forces can become the immensly complex organisations such as the helical structure of DNA and similar structures which are fundamental to life and to our ability to observe nature.

In my previous works, in particular 'The Large Scale Structure of the Universe', I described the intimate relationships between the very largest and very smallest structures in nature. However that description offers no indication as to how those relationships arise from a seemingly random and directionless pool of component parts.

The purpose of this paper is to describe how it is that large numbers of components of a system can conspire together to arrange themselves into organised structures --in other words to describe how it is that order is produced from chaotic systems and to create a general mathematical model which can be used to describe the beaviour of any macroscopic system to include anything from changing weather patterns to evolutionary processes. Essentially, my approach is to describe such systems as self organising systems.

Before describing the precise mechanism of self organizing systems, let us take a brief look at the philosophical and historical background of the subject particularly with regard to the physiological processes of life and evolutionary theory.

The idea of self organising systems is not new. c.546 B.C, Anaximander expressed notions about the development of human life forms which seem to indicate that he was aware that the life forms of his time must have been different in the past and had arrived at their present state of development as a result of undergoing changes in their structure and organisation. About 200 years later, Aristotle had defined the notion that nature was a holistic self organising system and, as we know, Aristotle's theories on this and virtually every other philosophical and scientific subject were the major formative ideas to influence the intellectual world for the next 500 years. Following the disastrous period of the Dark Ages which dominated the Western world for the next 1000 years or so, there was a revival of Classical ideas in the West, dominated again by the influence of Aristotle. However, early in the period of this enlightenment, modern scientific thinking and methods took hold. Descartes and others considered that nature was described by linear sequences of events, strictly causal in nature. The greatest influence on the enlightenment, at least on scientific thought, was Sir Isaac Newton and it was Newton's ideas which did more than anything else to shape the world we live in today by his invention, but more particularly by the application of, the calculus. These ideas held sway for some 200 years until the beginning of the 20th. century when yet another change in philosophical outlook took place and the uncertainties of quantum theory began to hold sway and holistic scientific thinking finally became unfashionable and fell out of favour.

It is a fact that all the principal equations of physics, which are mainly products of the 20th. century, are reversible in time and that time has no specific direction. Obviously, as observers of the universe, this is not our experience, and I hope that my previous works have gone some way to dispelling this notion and to unify the largest and smallest strucures in the universe and by this method to unify the laws of physics. However, now is the time to move on and to attempt to create a mathematical description and model of the behaviour of the macro-world in which we find ourselves. I will show by means of a mathematical model which, in principle, can be applied to any large number of physical entities that it is possible to model the behaviour of complex physical systems and how it is that order is produced from seemingly complex and random systems.

Let me attempt to describe a self organizing system. A self organizing system is exactly that, it does not require an input of energy nor does it lose energy. It is not a thermodynamic system in the accepted sense of the word and therefore entropy is not involved. A self organizing system can be described as a collection of a large number of units which can be anything from microscopic particles to macroscopic components such as pebbles on a beach or shoals of fish or indeed any collection of 'things' which can be described collectively. Clearly, all the components of a system are different, but most are similar. Individual components possessing similar characteristics are the most likely to associate together and to occupy the the most favoured or most probable state according to some similar characteristic or intrinsic quality possessed by those components.Thus we are led to conclude that only so many components can occupy any one state at any one time

It is the collective mathematical behaviour of these individual components which I wish to describe in this paper and which I intend to approach in the following way.

Let us consider a system which is composed of components which are similar in nature but are different enough to be organised in groups or collective bodies. For example one could consider a shingle beach whose pebbles are graded according to size and where each occupies a given position on the beach which is the result of it's size or some other intrinsic quality common to that group. The whole system is composed of a large number N of individual components. Each component has available to it several possible states of occupancy with a probability of occupancy given by $E_{1,}, E_2, E_{3.}$. At a particular time the components are distributed so that n_1 components occupy probability level $E_{1,}$ n_2 components occupy E_2 probability level, n_3 components occupy E_3 probability level and so on.

The total number of components of the system is:-

$$N = n_1 + n_2 + n_3 + \ldots . n_i = \sum_i n_i \qquad \text{Equ. (1)}$$

The total probability state of the entire system is:-

$$U = n_1 E_1 + n_2 E_2 + n_3 E_3 + \ldots = \sum_i n_i E_i \qquad \text{Equ. (2)}$$

The set of numbers $(n_{.}, n_2, n_3 \ldots .)$ we describe as a distribution. A distribution defines the micro state of the system consistent with the macro state or physical condition of the system which is determined by the total number of components, the total probability state, the structure of each component (e.g. mass, size etc.) and some external conditions which may affect the system (such as tides, gravity etc.)

Equations (1) and (2) do not determine the distribution $(n_1, n_2, n_3 \ldots .)$ in a unique way and the distribution of the components among the available

probability levels may be changing continuously. However, given the physical conditions of an isolated system, there is a most probable distribution compatible with those conditions. When the most probable distribution is achieved, the system is said to be in statistical equilibrium.
A system which is in statistical equilibrium will not depart from the most probable distribution unless it is disturbed by some external action (environmental change for example).

In order to find the most probable distribution law we can proceed as follows.
Consider a system composed of a large number of components. Suppose that the components can occupy the probability levels E_1, E_2, E_3 and that there are n_1, n_2, n_3 components in each state. It follows that the total number *N* of components and the total probability for those components *U* are given by Equations (1) and (2).
We need to calculate the probability *P* of a distribution $(n_1, n_2, n_3....)$. We shall assume that the probability of a particular distribution is proportional to the number of different ways that the components can be distributed among the available probability levels compatible with the values of *N* and *U*. Next we must find the distribution for which the probability *P* has the maximum value for the given values of *N* and *U*.
There are no restrictions on the number of components which can occupy a given probability level, but it may happen that there are probability levels that are more likey to be occupied that others. Thus the probability levels themselves have different intrinsic probabilities, designated g_i of being occupied. Thus the larger the g_i the greater the probability that the level will be occupied. We can assume that when statistical equilibrium is reached and the probabilty *P* is a maximum, the occupation numbers n_i

should be proportional to the intrinsic probabilities g_i ,since the larger g_i the more probable that it is in state E_i.

At equilibrium, components tend to favour lower prbability levels and components in higher probability levels may tend to pass to states of lower probability and we can say that the larger the probabilitylevel E_i the less probable that a component will be in that state when statistical equilibrium is reached.

A negative exponential of the form $e^{-\beta E_i}$ where β is a positive parameter satisfies this requirement therefore we can assume that the occupation numbers of the most probable or equilibrium distribution should be of the form;-

$$n_i = \alpha g_i e^{-\beta E_i} \qquad \text{Equ. (3)}$$

where α is some constant that depends on the structure of the system.

We now require to relate the parameters α and β with the physical properties of the system.

The quantity α may be expressed in terms of the number of components in the system i.e:-

$$\begin{aligned} N &= n_1 + n_2 + n_3 + \ldots. \\ &= \alpha g_1 e^{-\beta E_1} + \alpha g_2 e^{-\beta E_2} + \alpha g_3 e^{-\beta E_3} + \ldots. \\ &= \alpha (g_1 e^{-\beta E_1} + g_2 e^{-\beta E_2} + g_3 e^{-\beta E_3} + \ldots.) \\ &= \alpha \left(\sum_i g_i e^{-\beta E_i} \right) \\ &= \alpha Z \end{aligned}$$

where

$$Z = \sum_i g_i e^{-\beta E_i} \qquad \text{Equ.(4)}$$

and we define the quantity Z as the distribution function. Here Z is a function of the parameter β and depends on the values of g_i and E_i. Thus we can write $\alpha = N/Z$ and equation (4) becomes:-

$$n_i = \frac{N}{Z} g_i e^{-\beta E_i} \qquad \text{Equ. (5)}$$

and Equation (5) constitutes a distribution law for a system in statistical equilibrium and we can define the total probability of a system of components in total equilibrium as;-

$$\begin{aligned} U &= n_1 E_1 + n_2 E_2 + n_3 E_3 + \ldots. \\ &= \frac{N}{Z}\left(g_1 E_1{}^{-\beta E_1} + g_2 E_2{}^{-\beta E_2} + g_3 E_3{}^{-\beta E_3} + \ldots.\right) \qquad \text{Equ. (6)} \\ &= \frac{N}{Z}\left(\sum_i g_i E_i e^{-\beta E_i}\right) \end{aligned}$$

However, our concern is not with systems in equilibrium as such, but is with fluctuations and perturbations in those systems which cause the probability distribution to move in one direction or another. Having described the physical conditions relating to states which are in equilibrium, we must now examine the probability of a particular distribution of components and show how even a minor perturbation of a system in equilibrium, can produce a major and significant change in the probability distribution and we will proceed as follows.

We wish to estimate the probability of a distribution $(n_1, n_2, n_3, ...n_i ....)$ for a system of identical components. First we note that the intrinsic probability g_i gives the probability that the probability level E_i is occupied by one component. If there are no restrictions on the occupation of the state E_i by more than one component, then the probability of occupation by two components is $g_1 x g_2$ or g_i^2 or for three components is $g_i x g_i x g_i$ or g_i^3. Thus the intrinsic probability of finding n_i components in the state whose probability level is E_i is $g_i^{n_i}$ and we can assume that the probability P of the distribution $(n_1, n_2, n_3 ....n_i ....)$ is proportional to $g_1^{n_1}, g_2^{n_2}, g_3^{n_3} ....$.

Any permutation of components in the same probability level does not give rise to a new distribution. The number of permutations of n_i components is $n_i!$. Therefore we must expect that the probability P is inversely proportional to $n_1!, n_2!, n_3!, ...$ and we can write :-

$$P = \frac{g_1^{n_1} g_2^{n_2} g_3^{n_3} ....}{n_1!, n_2!, n_3! ....} \qquad \text{Equ. (7)}$$

To illustrate the meaning of the expression for *P*, let us consider two different systems each composed of 4000 components in three probability levels, each with the same intrinsic probability as in Figs. (1) and (2) below.

E_3 2ε________n_3 577
E_2 ε ________n_2 1146 Fig. (1)
E_1 0 ________n_1 2277

The most probable or equilibrium distribution for this system is $n_1 = 2277$, $n_2 = 1146$, $n_3 = 577$.

The corresponding distribution probability is:-

$$P_1 \approx \frac{g^{4000}}{2277!\,1146!\,577!}.$$

The change in P when two components are removed from the middle level and transferred to the upper and lower levels is given by a new distribution probability i.e:-

$$P_2 \approx \frac{g^{4000}}{2278!\,1144!\,578!}.$$

The ratio of these two probabilities is;-

$$\frac{P_2}{P_1} = \frac{1146x1145}{2278x578} = \frac{1312170}{1316684} = 0.9966.$$

Therefore the two probabilities are essentially the same since if P_1 is a maximum, the change in P_1 must be very small for a change in the distribution numbers. Thus the equilibrium microstate may fluctuate among partitions close to the most probable equilibrium distribution without too much change. This confirms that the Distribution Law corresponds to the maximum of the probabilty P_1 for a system satisfying Equations (1) and (2).

Let us now examine the same 4000 components distributed differently as per Fig. (2) below:-

$E_3 = 2\varepsilon$ _______ $n_3 = 300$

$E_2 = \varepsilon$ _______ $n_2 = 1700$ FIG. (2)

$E_1 = 0$ _______ $n_1 = 2000$

We first compare the relative probabilities of two non-equilibrium distributions. The distribution in Fig. (2) has 2000 components in the lower level, 1700 components in the middle level and 300 components in the upper level.

The second distribution results from the transfer of one component from the middle level to the lower level and one component from the middle level to the upper level.

According to the above expression for p in Equ. (7) above the probabilities for the first and second distributions are :-

$$P_1 = \frac{g^{4000}}{2000!\,1700!\,300!} \quad \text{and} \quad P_2 \approx \frac{g^{4000}}{2001!\,1698!\,301!} \ .$$

The ratio of the two probabilities is :-

$$\frac{P_2}{P_1} = \frac{1700x1699}{2001x301} = \frac{2888300}{602301} = 4.8 \, .$$

Thus the mere transfer of two components out of 4000 to other levels increases the probability by a factor of 4.8. This means that the distributions P_1 and P_2 are far from equilibrium distribution. This is due to the excessive population of the middle level, Thus the system will try to evolve to a state where the middle level is less populated. That is to say it is a self organizing system in that it has a natural propensity to attain a state of equilibrium and this propensity is entirely due to the inherent physical properties of the individual components of the system.

We can use the relationships in Fig. (2) to calculate the most probable or equilibrium distribution.

Each component can be in one of three probability levels whose probabilities are *0,* ε and 2ε and we assume that each probability level has the same intrinsic probability *g.*

The system is composed of 4000 particles and it's total probability is 2300ε. i.e.:-

$$
\begin{aligned}
U &= \sum_i n_i E_i \\
&= 2000x0 + 1700x\,\varepsilon + 300x(2\varepsilon) \qquad \text{Equ. (8)} \\
&= 2300\varepsilon
\end{aligned}
$$

Using Equation (5) for the most probable distribution and setting $g_1 = g_2 = g_3 = g$ we have $n_1 = \alpha g, \quad n_2 = \alpha g e^{-\beta\varepsilon}, \quad n_3 = \alpha g e^{-\beta\varepsilon}$. If we designate $e^{-\beta\varepsilon}$ by x we can write $n_2 = n_1 x$ and $n_3 = n_1 x^2$. Thus Equation (1) and (2) become $n_1 + n_1 x + n_1 x^2 = 4000$ and $(n_1 x)\varepsilon + (n_1 x^2)(2\varepsilon) = 2300\varepsilon$. Cancelling the common factor ε in the second relation above we have :-

$$n_1(1 = x = x^2) = 4000 \text{ and } n_1(x + 2x^2) = 2300 \qquad \text{Equ. (9)}$$

Dividing one equation by the other to eliminate n_1 we obtain a quadratic equation for x i.e: $57x^2 + 17x - 23 = 0$ or *x=0.50377.*

Returning to Equ. (9) above and using this value for x we find $n_1 = 2277, n_2 = 1146$ and $n_3 = 577$ which gives the equilibrium distribution for the system. This distribution is different from that given in Equ. (8)above and therefore Equ. (8) was not in statistical equilibrium.

The foregoing is all very well but we must establish exactly what it is that causes these perturbations in statistical equilibrium and we discussed earlier the physical parameter β.

Using the distribution function, Equation (5) we note that n_i depends on the structure of the system through the $g_i's$ and the $E_i's$ and on it's physical state through β.

The total probability of an entire system in statistical equilibrium is;-

$$
\begin{aligned}
U &= n_1 E_1 + n_2 E_2 + n_3 E_3 + ... \\
&= \frac{N}{Z}(g_1 E_1 e^{-\beta E_1} + g_2 E_2 e^{-\beta E_2} = g_3 E_3 e^{-\beta E_3} + ...)
\end{aligned}
$$

Using Equ. (5) and noticing that $\frac{d}{d\beta}(e^{-\beta E_i}) = -E_i e^{-\beta E_i}$ we can write U in the following alternative form:-

$$U = -\frac{N}{Z}\frac{d}{d\beta}\left(\sum_i g_i e^{-\beta E_i}\right) = -\frac{N}{Z}\frac{dZ}{d\beta}.$$

This expression can be written as:-

$$U = -N\frac{d}{d\beta}(\ln Z) \qquad \text{Equ. (9)}$$

The average probability state of the component is then;-

$$E_{AVE} = \frac{U}{N} = -\frac{d}{d\beta}(\ln Z) \qquad \text{Equ. (10)}$$

Thus we can calculate the properties of a system in statistical equilibrium in terms of it's internal structure which is determined by the values of g_i, E_i and the parameter β, and therefore Equ.(8) relates to E_{AVE}. The value of the exponential $e^{-\beta E_i}$ decreases as βE_i increases and vice versa. Likewise as βE_i becomes larger, the occupation of the state with probability level E_i becomes smaller and vice versa and we can conclude that a large β favours the smaller values of E_i and reduces the average probability level of the particles. Likewise a small β favours larger values of of E_i and increases the average probability level of the component parts of the system.

From this we can see that the effect of a change in β is precisely the opposite to that of a change in exterior environment λ of the the system. This suggests a general statistical definition of the exterior environment λ of the system by relating it to the parameter β. Because the effects of β and λ are opposite we can obtain a statistical definition of the exterior environment by the inverse relation;-

$$\beta = \frac{1}{k\lambda} \quad \text{or} \quad k\lambda = \frac{1}{\beta} \qquad \text{Equ. (11)}$$

where k is some constant.

Using Equ. (11) we can now write Equ. (6) in the form:-

$$n_i = \frac{N}{Z} g_i e^{-E_i/k\lambda} \qquad \text{Equ. (12)}$$

which gives the average occupation numbers when the system is in statistical equilibrium. Similarly the distribution function, Equ.(4) becomes:-

$$Z = \sum_i g_i e^{-E_i/k\lambda}$$

and this a function of the external environment of the system.

From Equ. (11) we can write $d\beta = -d\lambda / k\lambda^2$ and therefore Equations (9) and (10) can be written in terms of the external environment of the system as:-

$U = kN\lambda^2 \frac{d}{d\lambda}(\ln Z)$ and $E_{AVE} = k\lambda^2 \frac{d}{d\lambda}(\ln Z)$ and thus we can conclude that the total probability U and the average probability E_{AVE} of the components are both determined by the external environment of the system.

Since the exponential $e^{-E_i/k\lambda}$ in Equ. (12) is a decreasing function of $E_i / k\lambda$, the larger the ratio $E_i / k\lambda$, the smaller the value of the occupation number n_i, therefore the larger the probability level E_i, the smaller the value of n_i. In other words, the occupation of probability levels available to the components decreases as their probability level increases. It is for this reason we can say that as the value of λ in a system increases, the system becomes more disordered. At higher levels of λ components transfer from more probable to less probable states. Therefore at lower levels of λ components transfer from less probable to more probable states. Conversely, if a system becomes more disordered, the value of λ will increase and if a system becomes more ordered the value of λ will decrease. It follows that the probability level of a system, that is to say its state of equilibrium, and the state of the environment which the system inhabits are inversely proportional to each other, thus constituting an inverse ratio. More simply, we can say that the more stable a system is, the less likely it is that the environment will change and the more chaotic the system then the more likely it is that the environment will change .

Recalling that stabilising parameter β of any statistical system is the inverse of the environmental input λ we can write a general rule:-

‘The internal statistical equilibrium of a system is inversely proportional to the influence of its external environment’.

From the foregoing we can say that a disordered state has an intrinsic tendency to stabilise itself towards a state of statistical equilibrium. Moreover we note that a system which is out of statistical equilibrium is stable until it is acted upon by an outside force and that it cannot achieve equilibrium spontaneously. The macro world can only change as a result of environmental change and change is induced by the external parameter λ. Thus the forces which produce change in any system are non-linear and logarithmic in nature. But we should note that these changes are induced in systems which are themselves linear in nature.
This then is a mathematical model which is essentially holistic in character in that it is simultaeously both linear and non-linear.

The model can be applied to any statistical system comprised of any large numbers of components and which can be applied to evolutionary theory.
Clearly evolutionary theory can be mathematized in the manner described above in that all life forms have an intrinsic tendency towards a form statistical equilibrium
Conventional evolutionary theory states that random mutations of certain individuals within species are more likely to survive than others but, if we think about this scenario, the chances of a particular mutation coinciding with a subtle, or for that matter radical change in the environment, are slim if not unlikely. Thus, it is my contention that evolutionary processes are statistical in their nature and if, for any reason, the population of a specific environment should find itself in a state of dis-equilibrium, then that population has an intrinsic tendency to regain its statistical equilibrium.

That equilibrium state is a product of the intrinsic qualities of the population which is in turn solely a product of the numerical structure of the system and my model shows that the slightest perturbation from equlibrium can produce masive distortions in the probability distributions of genetic change.

To conclude, we can state that the genetic content of any species is inherently stable unless strongly influenced by non linear processes in the environment. Conversely these non linear environmental processes can be stongly influenced by the equilibrium state of the genetic content of a particular species.

This being the case,we must conclude that if the environment and the gene pool together costitute a self regulating system, then evolutionary processes can only change over time and time is not, in itself, an environmental parameter. Since it is the repulsive gravitational field which is the creator of time (see 'The Large Scale Structure of the Universe') then gravity is the creator and engine of evolutionary processes and indeed, of life itself.

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