‘BELL’S INEQUALITY’

AN HYPOTHESIS INTRODUCING TIME DILATION INTO THE BELL THEOREM

BY

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ABSTRACT

The publication of the Einstein, Podolsky, Rosen (EPR) thought experiment in 1935 appeared to show that either there is a faster than light exchange of information between quantum particles, or that alternatively, Quantum Theory is incomplete

In 1966, a remarkable theorem known as Bell's Inequality was published and and which proved conclusively that the non-local hidden variable model of Quantum Mechanics is indeed the the true description of nature at least as far as the behaviour of particles at the quantum level is concerned. These results were finally proven following the publication of the results of an Experiment by Alain Aspect and his colleagues in 1986.

Bell's theorem and the Aspect experiment showed conlusively that there is seemingly a fundamental conflict between the quantum world and the classical world which is formally undecidable.

The purpose of this paper is to show that in fact there may be a simple solution to this conundrum through the application of a well known relativistic relationship applied in a slightly unconventional manner.

Probably one of the most profound and contraversial question in physics is that of the EPR-Einstein, Podolsky, Rosen experiment.

It will be recalled that in1935, the EPR 'thought experiment' was published and the 'experiment' appeared to show that a measurement on either one of a pair of quantum particles instantaneously fixed the state of the other particle, no matter the distance between the particles at the time of the measurement. The results of the experiment appeared to imply two possible alternatives; either there was an exchange of information between the particles which would mean that a signal could travel between the particles at a velocity faster than light (a state which is commonly described as the 'non local hidden variable' model, which is a result not permitted by relativity theory) or there is some other , unknown influence (known as the 'local hidden variable' model) which intervenes to resolve the outcome of the entangled quantum state of a pair of particles.

In 1966, the physicist John Bell published his Inequality Theorem which has since been described by some as the 'most profound discovery of science'. In a nutshell, Bell's theorem demonstrated conclusively that non-locality was indeed the correct interpretation of quantum theory. This interpretation was given massive support with the publication of an experiment carried out by Alain Aspect and his colleagues in 1986 which established that the quantum world does not function in the same way as the world of common sense with which we are all familiar.

In the past a great deal has already been said on the question of the local hidden variable model compared with the non-local hidden variable interpretation of quantum theory, but despite all the discussion which has taken place, the fact remains that neither interpretation is satisfactory and in summary, we can conclude that either causality is violated in th quantum world or the world is fundamentally indeterminate and is subject to the influence of the observer.

Clearly the two descriptions are both contradictory and mutually exclusive and whichever interpretation one prefers, both contain difficulties which have not yet been explained.

The purpose of this paper is to offer an explanation as to how the two interpretations can be united and we will begin by summarising the results of Bell's inequality in a version of the theorem published by David Bohm and later expanded upon by Messrs. Clauser, Holt, Home and Shimony as follows.

If a spin *'0'* particle decays into two particles each of spin ½, then the spin component of one of the particles has two eigenstates, β_{+1} and β_{-1} and the spin component of the other particle is β_{+2} and β_{-2} . The total spin of this entangled system is zero and the total state vector of the system is :-

$$\psi = \left(1/\sqrt{2}\right)\left[\beta_{+1}\ \beta_{-2} - \beta_{-1}\ \beta_{+2}\right]$$

When the measurement of the spin component of one particle is made the state vector of the total system will collapse to either $\psi = \beta_{+1}\beta_{-2}$ or $\psi = \beta_{-1}\beta_{+2}$ from which it follows that if the spin of one particle is measured as $+\hbar/2$, then the spin of the other particle must be $-\hbar/2$ and this proposition holds, no matter the distance between the two particles at the time when the measurement has been made. (Ref. 1)

This being the case, we must conclude that the spin measurement made on one particle instantaneously fixes the spin state of the other particle and we must further conclude that information has been passed from one particle to the other at a velocity faster than light, or we must conclude that quantum theory is incorrect.

Accepting that quantum theory and relativity theory have both been so very successful and yet that both seem to contradict each other, we must seek to solve this contradiction. In 1981 the foregoing results were conclusively proved in an experiment carried out by Alain Aspect and his colleagues which showed that entangled quantum particles do indeed behave exactly as predicted by quantum theory. That is to say, the spin state of one component of the entangled states *instantaneously* fixes the spin state of the other component regardless of the distance between the two events.

The Aspect experiment was designed in part at least, to verify that there are absolutely no pre-set conditions which can produce quantum mechanical probabilities in the classical world. This remarkable fact totally rules out so-called local realistic models. In other words no 'message' can travel from the first measured particle to the second particle indicating the intended direction of measurement before the particles separate.

In the Aspect experiment, spin is measured in one wing of an apparatus as being in direction *'a'* and in the other wing as being in direction *'b'*, thus spin can be either up or down in each wing.

The difference between the probability of both results being the same i.e. both up or both down and both being different we define as *E(a,b)* and we can describe this difference as:-

$$E(a,b) \equiv P(up, up; a,b) + P(down, down; a,b) - P(up, down;a,b) - P(down, up; a,b)$$

If now we take four experiments with directions a and a' in one wing of the apparatus and b and b' in the other wing, we obtain the result:-

$$X(a,b,a'b') \equiv \left| \; E(a,b) \; E(a,b') + E(a',b) - E(a',b') \; \right| \leq 2 \qquad \text{Equ. (1)}$$

which is of course, the inequality attributed to John Bell. (Ref. 1)

This result assumes that the 'local hidden variable' interpretation of nature is correct and was developed by Clauser et al. from John Bell's original theorem. Bell's inequality which appears to show that both quantities *'a'* and *'b'* have 'exchanged information' at the outset of the experiment as to what the results (i.e. direction of spin) will be on examination of the individual spin states.

However, the inequality is violated by quantum theory because quantum theory gives the result:-

E(a,b) = -cos (a-b)

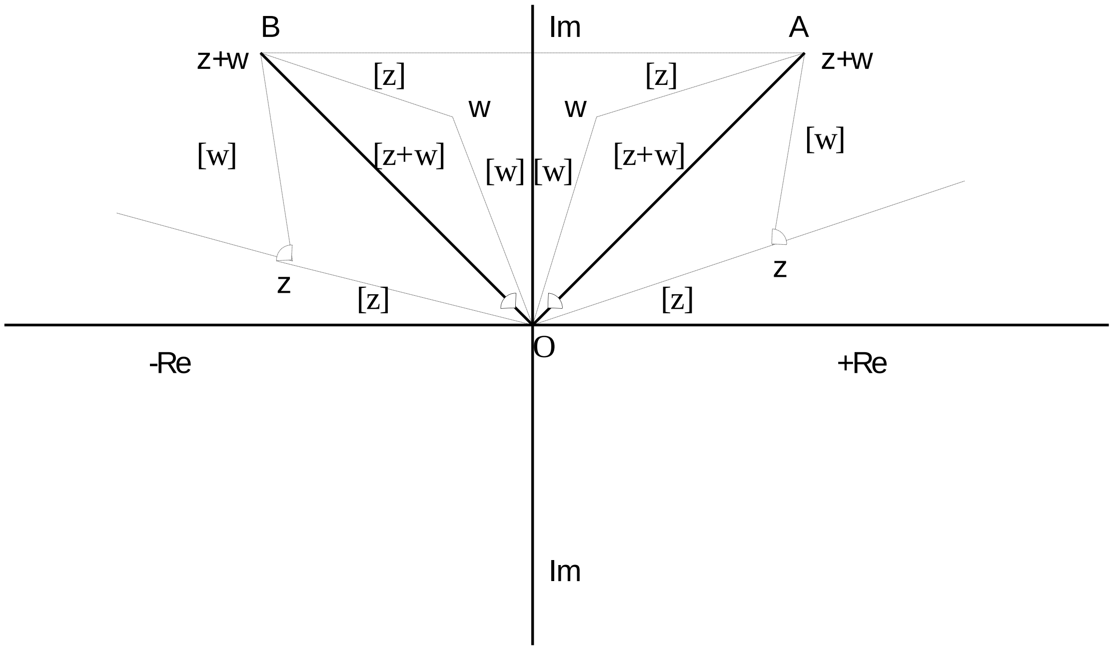
because if $a = 0^0, a^{'} = 90^0, b = 45^0, b^{'} = -45^0$ then

$X(a, b, a^{'} b^{'}) = 2\sqrt{2}$ Equ.(2) (Ref. 2)

and thus Bell's inequality is violated and it follows that the 'local hidden variable' theory is incorrect and that quantum theory is fundamentally indeterminate in character.

So how do we attempt to solve the problem of quantum entanglement?

This paper is a proposition that the answer is contained in the geometry of space-time. To illustrate this point more clearly, the spatial probability of an entangled quantum state is more clearly defined by the Argand diagram below. That is to say the *+Re* axis of the apparatus resolves the alternative state in the *-Re* axis of the experiment and vice versa. Moreover, it follows that the squared modulus in the negative *(-Re)* of the experiment is equal but opposite to the squared modulus in the positive arm *(+Re).*



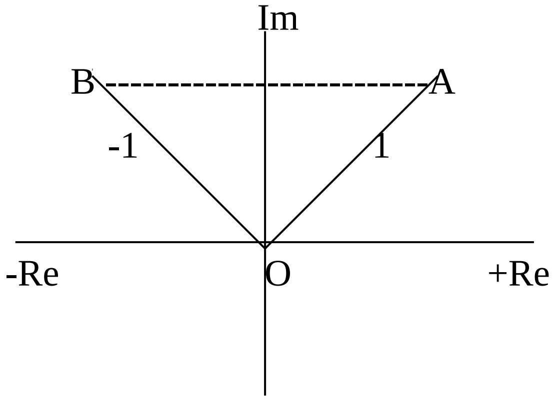
(Ref. 3)

In a two-armed quantum experiment carried out in an apparatus configured as in FIG.1 above, the quantum state defined at the points *A* and *B* is described as the squared modulus of the sum of two complex numbers i.e $|z+w|^2$ or $-|z+w|^2$ and that the quantum state at *A* and *B* is defined by a vector distance in space-time *(OA)* or *(OB)* and that both states are real numbers but of opposite sign, but because quantum probabilities are given by the squared modulus of the complex numbers 'w' and 'z' we obtain the sum of their squared moduli simutaneously as required by quantum theory and not consecutively as required by clasical theory.

If we now examine the fundamental difference between the probabilities manifested in quantum theory and the probabilities manifested in the macro world as compared in Equations (1) and (2) above we find that the difference is a factor of *2*, that is to say $X(a,b,a',b')$ equals either $2\sqrt{2}$ or ≤ 2 depending on whether the probability is being observed in either the quantum reference frame or the classical reference frame respectively.

It being the case that the fundamental difference between the classical world and the quantum world is that the states of a system in the quantum world are revealed simultaneously and in the classical world the states are revealed sequentially, then the difference between the two states is simply a difference in the perception of time between the two states and the manifestation of that difference is somehow reflected in the factor *2* or $\sqrt{2}$.

In order to explain this phenomenon, let us re-examine the Argand diagram in Fig.(1) in a simplified form:-



We describe the line *OA* as unit *1* and the line *OB* as unit *–1* and the angle *AOB* as being 90^0. This being the case we describe the triangle *AOB* as :-

$$(AB)^2 = (OA)^2 + (OB)^2$$
$$\therefore (AB)^2 = (-1)^2 + 1^2$$
$$\therefore (AB) = \sqrt{2}$$

and from Equ.1 we can infer that $X(a, b, a', b') = (AB)^2 \;\therefore\; (AB) = \sqrt{2}$ which is, of course less than *2* as required by Equ.(1), and we can further infer that events at points *A* and *B* do not occur simultaneously.

Bearing in mind that the distance *(AB)* is the resultant of two vectors *(OA)* and *(OB)* let us consider the relativistic implications of the experiment.

Ordinarily, the combined velocity of two particles *A* and *B* receding from each other is $V_A + V_B$, or if both velocities are equal then the combined velocity can be written as *2V*, however Special Relativity describes the combined velocity of two particles receding from each other as being:-

$$V_{AB} = \frac{V_A + V_B}{1 + \frac{V_A V_B}{c^2}} \qquad \text{Equ. (3)}$$

and this being the case we can proceed as follows.

Noting that the experiment is performed with particles which are travelling at velocity *'c'*, the velocity of particles moving in each arm of the experiment can be designated V_A and V_B respectively.

Now, since V_A and V_B are both equal to *'c',* we can infer from Equ.(3) that the expression V_{AB} is also equal to *'c'* and thus we can deduce that the distance between points *A* and *B* in Fig.(2) is not given by $(AB) = \sqrt{(OA)^2 + (OB)^2}$ but by $(AB) = \frac{\sqrt{(OA)^2 + (OB)^2}}{2}$ since Equ.(3) describes the distance *(AB)* as being *½(AB)* and not *(AB)* as in classical theory.

Thus, in interpreting the results of the Aspect experiment, the familiar cosine rule should be re-written as:-

$$(AB) = \frac{\sqrt{(OA)^2 + (OB)^2 - 2(OA)(OB)\cos\theta}}{2}$$

or :-

$$(AB) = \frac{\sqrt{(OA)^2 + (OB)^2 - 2(OA)(OB)\cos\theta}}{\left(1 + \frac{V_A V_B}{c^2}\right)}$$

so if *(OA)=1* and *(OB)=1* and $\theta = 90^0$ we can infer that $(AB) = \frac{\sqrt{2}}{2}$ and generally we can write $X\left(a,b,a^{'},b^{'}\right) \le 2$ in accordance with Bell's inequality.

Having introduced Special Relativity into the Argand diagram, we can consider the effects of Time Dilation on the results of the experiment. In quantum theory, the distance between points *A* and *B* is always half that predicted by conventional theory and we can infer that the time taken for a signal to travel between points *A* and *B* is half of that predicted by conventional theory and it is this phenomenon which explains the so called 'faster than light' signalling between *A* and *B*. Alternatively, one can imagine that signals appear to travel at a velocity of *2c* due to the spatial contraction between the two points and therefore the outcome of a quantity at *B* measured by an observer at *A* will appear to have been decided before the result of the measurement at *A* is known, thus re-defining the result of the Aspect experiment and demonstrating that it is the local hidden variable model which prevails and not the non-local model.

It being the case that information is exchanged between points *A* and *B* after commencement of the experiment and that local hidden variable model is therefore the preferred interpretation of the result of the Aspect experiment, the information in question must be propagated in a field by a carrier particle.

I believe that it is in the area of a gravitational field and an appropriate carrier particle to which research should be directed and accordingly work has commenced in that direction.

END

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