

# Intra-Firm Trade, Multinational Production, and Welfare\*

Pamela Bombarda and Stefania Marcassa<sup>†</sup>

## Abstract

We propose a model where firms have access to competing market strategies: export and multinational production. Due to technological appropriability issues, foreign affiliates import an intermediate input from the home headquarters. The presence of export and multinational production alters the standard results obtained for welfare in heterogeneous firm models, through a double truncation of the productivity distribution. The model is then calibrated to analyze counterfactual scenarios. We find that welfare gains from intra-firm trade range from 6 to 12 percent depending on country characteristics.

*JEL classification:* F12, F23

*Keywords:* MNFs, multinational production, intra-firm trade, welfare.

---

\*Previously circulating with the title “Gains From Intra-Firm Trade and Multinational Production”.

<sup>†</sup>Bombarda, corresponding author: Université de Cergy-Pontoise THEMA (UMR CNRS 8184), 33 boulevard du Port, 95011 Cergy-Pontoise cedex, FR, [pamela.bombarda@u-cergy.fr](mailto:pamela.bombarda@u-cergy.fr). Marcassa: Université de Cergy-Pontoise THEMA, [stefania.marcassa@u-cergy.fr](mailto:stefania.marcassa@u-cergy.fr).

# 1 Introduction

Engaging in international trade is an exceedingly rare activity: In 2000, only 4 percent of all U.S. firms were exporting (Bernard et al. (2007), Eaton et al. (2011) among others). In this context, multinational firms play a key role: 90 percent of U.S. exports and imports occur through them with 50 percent of U.S. imports taking place within the same firm rather than through arms length (Bernard et al. (2009)). Moreover, several empirical studies convey the idea that intra-firm trade is mainly related to the transfers of capabilities within the corporation. For example, Ramondo et al. (2013) find that most U.S. foreign affiliates are not created for multistage production chains, but as outlets to produce and then supply in the local market. Similarly, Atalay et al. (2014) study domestic operations of U.S. multinational firms, and provide evidence of intra-firm transfers of intangible inputs.

We embed knowledge transfer in Helpman et al. (2004) to build a model of export and horizontal multinational production with intra-firm trade. In our general equilibrium framework with  $N$  asymmetric countries, each foreign affiliate imports an intermediate input from the home country due to technological appropriability issues. Therefore, an important activity of our multinational firm is to transfer capabilities or knowledge-intensive inputs from the home headquarter to the foreign affiliate. This mechanism renders the knowledge-intensive input used in multinational production mobile across regions. Moreover, this implies that geographical costs apply to both exports and multinational production because they involve transportation of a finished good and of an intermediate good, respectively. Intra-firm trade is left exogenous in line with recent findings: Ramondo et al. (2013) show that neither the presence nor the magnitude of an input-output link between the parent and the affiliate predicts the existence or the share of intra-firm trade in the affiliate's total sales, but only the existence and the size of the affiliate.

An increase in trade barriers affects the multinational production strategy in two different ways. First, sales of the existing foreign affiliates decrease, which generates a new margin of adjustment for multinational firms. This occurs because of the complementarity between export and multinational activities. In models where export and multinational activities are substitutes this margin disappears, since the sales of existing affiliate firms are not directly affected by a change in trade policy. Second, trade barriers increase the threshold productivity cutoff for multinationals: the need to import intermediate goods from the headquarter makes it more difficult to enter as a foreign affiliate when trade costs increase. Firms face an altered

proximity-concentration trade-off because their choice to maintain capacity in other markets crucially depends on trade costs and not only on the forgone economies of scale. By contrast, in [Helpman et al. \(2004\)](#), an increase in trade costs unambiguously makes the multinational production strategy more attractive. In their framework, the proximity-concentration trade-off arises from the fact that only exported goods are subjected to iceberg transport costs, while multinational activity is free of trade cost.

Our contribution is threefold. Firstly and most novelly, the presence of export and multinational production alters the standard results obtained for welfare in heterogeneous firm models, through a double truncation of the productivity distribution. Upon drawing its own efficiency parameter, each firm decides whether to exit or to produce. In the latter case, the firm must face additional fixed costs linked to the supply strategy chosen. When the firm decides to serve the foreign market, it chooses whether to export domestically produced goods or to produce abroad via affiliate production. The presence of two alternative ways of reaching the foreign location introduces a double truncation in the productivity distribution of exporters.<sup>1</sup> This affects average export sales which are now dependent on firm's productivity level. Furthermore, it implies that domestic trade share and trade elasticity are no longer sufficient statistics to evaluate welfare gains.<sup>2</sup>

Secondly, we exploit the absence of free entry to retrieve gravity equations and compare margins' sensitivity for exports and affiliate sales with respect to alternative models such as [Chaney \(2008\)](#) and [Helpman et al. \(2004\)](#). As regards the margin of exports, we find that, similarly to [Chaney \(2008\)](#), the intensive margin only depends on the elasticity of substitution. Differently from [Chaney \(2008\)](#), the extensive margin is not constant but a function of both export and affiliate sales. With respect to the margins of affiliate sales, we show that the intensive margin is unambiguously related to the elasticity of substitution and the share of the imported intermediate good; whereas, the sensitivity of the extensive margin depends on trade frictions.

Thirdly, we quantify the country level gains from multinational production with intra-firm trade. We calibrate three versions of the model: symmetric countries with export only (*à la* [Melitz and Redding \(2013\)](#)); symmetric and asymmetric countries with export, multinational production, and intra-firm trade. Our findings stress the role of intra-firm trade for welfare

---

<sup>1</sup>Intra-firm trade does not affect our findings on welfare, which depend on the double truncation of the productivity distribution.

<sup>2</sup>The results in [Arkolakis et al. \(2008\)](#) and [Arkolakis et al. \(2012\)](#) do not apply with a double truncation of the productivity distribution.

gains: they range from 6 to 12 percent depending on country characteristics. We also compare the total gains from a model of multinational production and intra-firm with a model of pure multinational production. The latter yields the largest rise in welfare due to trade liberalization. Finally, we compute the sensitivity of export and affiliate sales, confirming the important role of the elasticity of substitution for both modes of supply.

This paper relates to several strands of literature. As in [Horstmann and Markusen \(1992\)](#), [Brainard \(1997\)](#), [Helpman et al. \(2004\)](#), and [Grossman et al. \(2006\)](#) we capture the interaction between export, multinational production, and intra-firm trade. [Keller and Yeaple \(2012\)](#) measure the spatial barriers to transferring knowledge. They find that the knowledge intensity of production affects the level of affiliate sales around the world. Our theoretical setup is closely related to [Irrazabal et al. \(2012\)](#) and [Bombarda \(2007\)](#). [Irrazabal et al. \(2012\)](#) structurally estimate a model of trade and multinational production with firm heterogeneity. They reject the proximity versus concentration hypothesis which did not consider intra-firm trade. We add on to their findings and show that the welfare equation varies from the one obtained in models with no truncation. Moreover, we quantitatively compare welfare of alternative market access strategies. In concurrent research, [Bombarda \(2007\)](#) proposes a model of intra-firm trade with distant dependent fixed cost to highlight non-monotonic choices of modes of supply. Our model, which is isomorphic to the set up in [Bombarda \(2007\)](#), is used to evaluate theoretical and quantitative welfare implications arising from the existence of simultaneous modes of supply. [Corcos et al. \(2012\)](#), using French firm-level data, investigate the main determinants of the internalization choice. Their findings highlight the role of capital, skill and productivity in explaining the choice of intra-firm trade.

This paper also contributes to the growing literature that theoretically analyzes the welfare gains from openness. [Arkolakis et al. \(2012\)](#) show that there exists a group of models in which a country's domestic trade share and the elasticity of trade are sufficient statistics to measure aggregate welfare gains from trade. This result relies on the assumption of an unbounded productivity distribution. [Feenstra \(2013\)](#) uses a bounded Pareto distribution and non CES preferences to restore the role for product variety and pro-competitive gains from trade in heterogeneous firm models. [Melitz and Redding \(2013\)](#) show that the additional adjustment margin in heterogeneous firm models plays an important role for welfare gains.

Differently from [Arkolakis et al. \(2012\)](#) and similarly to [Feenstra \(2013\)](#) and [Melitz and Redding \(2013\)](#), our welfare measure is altered by the double truncation in the productivity distribution of exporters. This makes our welfare measure depending on trade barriers, and

not only on domestic trade share and trade elasticity.

Another related strand of literature quantifies the gains from international activities. [Edmond et al. \(2015\)](#) study gains from international trade in a quantitative model with endogenously variable markups. [Ramondo \(2014\)](#) uses a multi-country general equilibrium model with a continuum of goods produced under constant return to scale at the industry level to calculate the gains that a country would experience from liberalizing access to foreign firms. [Ramondo and Rodriguez-Clare \(2013\)](#) consider trade and multinational production into an Eaton-Kortum framework to measure the overall gains from openness. [Garetto \(2013\)](#) quantifies the gains from multinational activity, using an Eaton-Kortum type model, where multinational firms engage in vertical FDI.<sup>3</sup> [Irrarrazabal et al. \(2012\)](#) find that impeding multinational activity has a small effect on welfare. Similarly to most of these studies, we propose a mechanism through which intra-firm trade affects multinational production, and we rely on aggregate evidence to quantify its importance.

The rest of the paper is organized as follows. Section 2 describes the theoretical framework. Section 3 discusses gravity equations and derives intensive and extensive margins. In Section 4, we investigate the theoretical implications of the model on welfare. Section 5 contains the calibration. Finally, Section 6 concludes.

## 2 Theoretical Framework

In this section, we introduce the main ingredients of a model with export, multinational production, and intra-firm trade without free-entry. We define preferences and technologies, and characterize the optimal strategies of firms and consumers.

**Preferences.** There are  $N$  potentially asymmetric countries. Country  $n$  has a population  $L_n$  whose labor supply is inelastic. Consumers in each country share identical preferences over two final goods: a homogeneous good  $h$ , and a differentiated good  $c$ . We assume two-tier preferences with Cobb-Douglas in the upper tier and CES in the lower tier. If a consumer spends a fraction  $\beta$  of her income on  $c(v)$  units of each variety  $v$  of the differentiated good, and  $(1 - \beta)$  on the homogeneous good  $h$ , she gets a utility  $U$ ,

$$U = h^{1-\beta} \left[ \int_{v \in V} c(v)^{(\sigma-1)/\sigma} dv \right]^{\frac{\sigma}{\sigma-1}\beta}, \quad (1)$$

---

<sup>3</sup>An estimation of our model *à la* Eaton-Kortum is not possible due to the lack of data.

where  $\sigma > 1$  represents the elasticity of substitution between any two goods within the group.

**Supply.** The homogeneous good  $h$  is freely traded and is used as the numeraire. It is produced under constant returns to scale with one unit of labor in country  $n$  producing  $w_n$  units of good  $h$ . Its price is set equal to 1 so that if country  $n$  produces this good, the wage in country  $n$  is  $w_n$ . We will consider only equilibria where every country produces some of the numeraire. This assumption allows countries to differ both in size ( $L_n$ ) and factor return ( $w_n$ ), which in turns will be reflected in different productivity levels.

The differentiated sector produces a continuum of horizontally differentiated varieties,  $q(v)$ , from two intermediate goods,  $y_1$  and  $y_2$ . Both  $y_1$  and  $y_2$  are produced with one unit of labor, but  $y_1$  can only be made at home, due to technological appropriability issues. This assumption is crucial for multinational production strategy:  $y_1$  can be considered as transfer of capabilities between the headquarter and the foreign affiliate. Each variety is then supplied by a Dixit-Stiglitz monopolistically competitive firm which produces under increasing returns to scale originating from a fixed cost. We assume that the fixed cost is paid in units of labor of the country where the good is produced.

We consider three modes of supply in the differentiated sector: (i) firms which sell only domestically; (ii) firms which export; and, (iii) firms which supply the foreign market via multinational production. Hence, when a firm decides to serve a foreign market, it chooses whether to export domestically produced goods or to locate production via an affiliate in the foreign country. In making these decisions, firms compare the net profits from exports and multinational production.

In our model, the classical scale versus proximity trade-off is altered by the introduction of intra-firm trade, which makes the multinational production strategy sensitive to geographical frictions between countries. The fact that  $y_1$  can only be produced at home plays an important role. If a firm chooses to supply the foreign market via local sales of its affiliates, the affiliate must import the intermediate good  $y_1$  from the home country. This implies that the multinational production strategy does not entirely avoid geographical related costs. The trade link between the home parent and the affiliate captures the complementary relationship between export and multinational production.

Upon drawing its own parameter  $a$  from a cumulative density function  $G(a)$  that is common to every country, each firm decides whether to exit (if it has a low productivity draw), or to produce. In the latter case, the firm faces additional fixed costs linked to the mode of supply chosen: (i) if it chooses to produce only for the domestic market  $i$ , it pays the fixed market

entry cost,  $f_{ii}$ ; (ii) if it chooses to export, it bears the additional costs  $f_{ij}^X$  of meeting different market specific standards (e.g., the cost of creating a distribution network in a new country  $j$ ); finally, (iii) if the firm chooses to serve foreign markets through multinational production, it will bear a fixed costs  $f_{ij}^M$ , which is a combination of the fixed cost of creating a distribution network, and the fixed cost of building up new capacities in the foreign country.<sup>4</sup> We allow for the fixed costs to differ across countries.

**Demand for Differentiated Goods.** The CES utility function implies that the demand of a representative consumer from country  $i$  for a good of type  $a$  is given by

$$c_i(a) = A_i p_i(a)^{-\sigma} \quad \text{with } A_i \equiv \frac{\beta Y_i}{P_i^{1-\sigma}}, \quad (2)$$

where  $a$  denotes the unit labor coefficient,  $A_i$  is the demand shifter,  $p_i(a)$  is the final price of a variety produced by a firm with marginal cost  $a$ , and  $P_i$  is the CES price index of the final good.  $A_i$  is exogenous from the perspective of the firm: it is given by the ratio of the aggregate level of spending on the differentiated good  $\beta Y_i$ , and the CES price index  $P_i^{1-\sigma}$ .

**Organization and Product Variety.** We assume that the production of the final good combines two intermediate goods,  $y_1$  and  $y_2$ , in the following Cobb-Douglas function

$$q_i(a) = \frac{1}{a} \left( \frac{y_1}{\eta} \right)^\eta \left( \frac{y_2}{1-\eta} \right)^{1-\eta} \quad 0 < \eta < 1 \quad (3)$$

where  $1/a$  represents the firm specific productivity parameter, and  $\eta$  is the Cobb-Douglas cost share of  $y_1$ , common across all countries. When trade is possible, firms decide whether to sell to a particular market. The supply mode (export or multinational production) will depend on their own productivity, the trade costs between the origin and the destination country, and the fixed costs.

The marginal costs in the exporting sector will be higher than in the FDI sector. Since  $y_1$  and  $y_2$  are produced with labor,  $L$ , the marginal cost for domestic as well as export production is linear in  $\tau$

$$mc_{ij} = a w_i \tau_{ij}, \quad (4)$$

where  $\tau_{ij}$  is the trade cost with  $\tau_{ij} = 1$  when  $i = j$ . The marginal cost for supplying the

---

<sup>4</sup>In our model, if a firm chooses to serve foreign markets via multinational production, the local foreign affiliate will produce the intermediate good  $y_2$  only. Then  $y_2$  will be combined with the intermediate good imported from the headquarter,  $y_1$ .

foreign market  $j$  via local sales of foreign affiliates is concave in  $\tau$

$$mc_{ij}^M = aw_j^{1-\eta} (w_i \tau_{ij})^\eta. \quad (5)$$

$mc_{ij}^M$  combines inputs (i.e., labor) from home and host country. More precisely,  $w_j^{1-\eta}$  is the labor cost of an input produced in country  $j$ , while  $w_i^\eta$  is the labor cost of an the input imported by country  $j$  from the home country  $i$ .<sup>5</sup> Note that in this framework trade costs matter only in relation to the share of intermediate good  $y_1$  that is used in the production of final good  $\eta$ . Using the mark-up  $\sigma/(\sigma - 1)$ , we can easily derive the price for each particular mode of supply.

**Mode of Supply Decisions.** The choice of the mode of supply is made by comparing various profit levels. We can distinguish three relevant cases:

(i) If a firm decides not to supply a market and exits, its operating profits are zero.

(ii) If a firm in country  $i$  decides to supply market  $j$  via exports, the profits from exporting to market  $j$  are decreasing in  $\tau_{ij}$  in a linear fashion

$$\pi_{ij}^X = [p_{ij}(a) - aw_i \tau_{ij}] q(a)_{ij} - w_j f_{ij}^X, \quad (6)$$

where  $q(a)_{ij}$  denotes the quantity exported. Substituting the equilibrium price and quantity we have

$$\pi_{ij}^X = \frac{1}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{(1-\sigma)} Y_j (w_i a \tau_{ij})^{1-\sigma} / P_j^{1-\sigma} - w_j f_{ij}^X, \quad (7)$$

where the fixed cost of exporting  $f_{ij}^X$  is evaluated at the foreign wage rate  $w_j$ .<sup>6</sup>

(iii) If a firm in country  $i$  decides to supply market  $j$  via affiliate sales, the profits realized by a subsidiary located in the  $j$  country depend on  $\tau_{ij}$

$$\pi_{ij}^M = [p^M(a) - aw_j^{1-\eta} (w_i \tau_{ij})^\eta] q(a)_{ij}^M - w_j f_{ij}^M, \quad (8)$$

where  $q(a)_{ij}^M$  represents the quantity supplied by the foreign affiliate. Substituting the equilibrium price and quantity, we obtain

$$\pi_{ij}^M = \frac{1}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{(1-\sigma)} Y_j (aw_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} / P_j^{1-\sigma} - w_j f_{ij}^M, \quad (9)$$

<sup>5</sup>Further details for cases where  $\eta = 1$  (as in Chaney (2008)) and  $\eta = 0$  (as in Helpman et al. (2004)) are provided in Appendix E.

<sup>6</sup>Note that this mode of supply collapses to domestic production when  $i = j$ , since  $\tau_{ii} = 1$ .



where  $\tau_{ij}^\eta$  is the trade costs associated with the intermediate good  $y_1$ , that is imported from the home country. The foreign affiliate has to face both the fixed cost  $f_{ij}^M$ , evaluated at the foreign wage rate, and the trade costs (that hit the imported intermediate).

We set parameters to get the same ranking as in [Helpman et al. \(2004\)](#). Namely, only firms with sufficiently high productivity will supply the foreign market, with the most productive firms supplying it via multinational production rather than exports. Hence, the regularity condition is

$$f_{ij}^X < f_{ij}^M w_j^{(1-\eta)(\sigma-1)} (w_i \tau_{ij})^{(\eta-1)(\sigma-1)}. \quad (10)$$

Since the price index depends on the probability distribution, we have to assume a particular functional form for  $G(a)$  in order to obtain closed-form solutions. Following the empirical literature on firm size distribution, we assume that the unit labor requirements are drawn from a Pareto distribution. The cumulative distribution function of a Pareto random variable  $a$  is

$$G(a) = \left( \frac{a}{a_0} \right)^k, \quad (11)$$

where  $k$  and  $a_0$  are the shape and scale parameters, respectively.

Following [Chaney \(2008\)](#), we assume that the total mass of potential entrants in country  $i$  is proportional to its labour income,  $L_i$ . Hence, larger and wealthier countries have more entrants. The absence of free entry implies that firms generate net profits which are redistributed to workers (or shareholders), proportionally to each own share  $w_i$  of the global fund.

The solution of the model is relegated to the appendix. More specifically, [Appendix A](#) provides intermediate results for differentiated goods; [Appendices B and C](#) derive profits and price index; and [Appendix D](#) solves the equilibrium of the overall economy.

### 3 Gravity Equations

Using firms level exports and affiliate sales we can derive gravity equations.<sup>7</sup> In this model aggregate bilateral trade and overseas affiliate sales will behave differently from traditional models.

**Proposition 1** (*Aggregate Exports Sales*) Total export (f.o.b.)  $X_{ij}^X$  from country  $i$  to country

---

<sup>7</sup>Firms level exports and affiliate sales are derived in [Appendix D](#), see equations (50) and (51).

$j$  are

$$X_{ij}^X = \frac{Y_i Y_j}{Y} \theta_j^{b(\sigma-1)} (w_i \tau_{ij})^{1-\sigma} \left[ \left( \frac{w_j f_{ij}^X}{(w_i \tau_{ij})^{1-\sigma}} \right)^{1-b} - \left( \frac{w_j (f_{ij}^M - f_{ij}^X)}{(w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \right)^{1-b} \right]. \quad (12)$$

**Proof.** See Appendix E.1. ■

The gravity equation (12) suggests that exports are a function of country sizes  $Y_i$  and  $Y_j$ , wages, bilateral trade and fixed costs, and the measure of  $j$ 's remoteness from the rest of the world.

Differently from Chaney (2008), this expression for aggregate trade takes into consideration the interaction between export and multinational production. This interaction makes the gravity for export non linear in logarithm. We expect aggregate export sales to decrease with trade costs, and this decrease should be faster the larger is  $\sigma$ . This is reduced for large value of imported intermediate.

**Proposition 2** (*Aggregate Affiliate Sales*) Total affiliate sales  $X_{ij}^M$  in country  $j$  are

$$X_{ij}^M = \frac{Y_i Y_j}{Y} \theta_j^{b(\sigma-1)} (w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} \left[ \frac{w_j (f_{ij}^M - f_{ij}^X)}{(w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \right]^{1-b}. \quad (13)$$

**Proof.** See Appendix E.2. ■

The gravity equation (13) suggests that affiliate sales are a function of country sizes  $Y_i$  and  $Y_j$ , wages, bilateral trade and fixed costs, intra-firm trade intensity, and the measure of  $j$ 's remoteness from the rest of the world.

Depending on the intensity of imported headquarter intermediates,  $\eta$ , an increase in trade barriers might create an incentive to ship production to the foreign market to avoid a part of the trade costs. This increases the demand for labor in the destination country relative to the home country. When the difference between wages is not too big, an increase in trade barriers can lead to a raise in aggregate local sales. This effect is stronger the lower is the share of intra-firm trade.

### 3.1 Intensive and Extensive Margins

In what follows we derive margins for export and affiliate sale equations.

### 3.1.1 Affiliate Sales

In this section we examine the intensive and extensive margins of affiliate sales. We analyze how the elasticity of substitution as well as the share of intermediate inputs affects the sensitivity of these margins. Differentiating total affiliate sales in country  $j$ ,  $X_{ij}^M = w_i L_i \int_0^{\bar{a}_{ij}^M} x_{ij}^M dG(a)$ , with respect to  $\tau_{ij}$ , we derive the intensive and extensive margins of affiliate sales

$$\frac{\partial X_{ij}^M}{\partial \tau_{ij}} = \underbrace{w_i L_i \int_0^{\bar{a}_{ij}^M} \frac{\partial x_{ij}^M}{\partial \tau_{ij}} dG(a)}_{\text{Intensive Margin}} + \underbrace{w_i L_i x_{ij}^M G'(\bar{a}_{ij}^M) \frac{\partial \bar{a}_{ij}^M}{\partial \tau_{ij}}}_{\text{Extensive Margin}}, \quad (14)$$

where we applied the Leibniz rule to separate the margins.

**Proposition 3** *Defining  $\psi \equiv -\partial \ln X_{ij}^M / \partial \ln \tau_{ij}$ , a change in variable costs  $\tau_{ij}$  makes the margins of affiliate sales react in the following way:*

$$\psi = \underbrace{\eta(\sigma - 1)}_{\text{Intensive Margin Elasticity}} + (k - \sigma + 1) \underbrace{\frac{\left( \eta (w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma} \right)}{\left( w_j^{1-\eta} (w_i \tau_{ij})^\eta \right)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}}}_{\text{Extensive Margin Elasticity}}. \quad (15)$$

**Proof.** See Appendix F.1. ■

**Intensive Margin.** The intensive margin of affiliate sales depends on the constant elasticity of substitution and on the level of imported intermediate  $\eta$ . Therefore, when goods are very substitutable (high  $\sigma$ ), the sales of each individual affiliate is very sensitive to the trade barriers. Let us now focus the role of the parameter  $\eta$ .

When  $\eta$  is equal to one, no firm will supply via multinational production. In this case, the foreign affiliate is importing both intermediate inputs from the home country. This strategy is extremely costly, since it implies full trade costs as well as higher fixed costs. Therefore, when  $\eta = 1$  export is the only market access strategy.

Differently, when  $\eta$  is equal to zero, the foreign affiliate is producing using only foreign inputs (similarly to Helpman et al. (2004)). When all intermediates are realized in the foreign location, the volume of sales of already existing affiliates are not affected by changes in trade costs. Therefore, in this case the intensive margin elasticity is equal to zero.

For intermediate levels of  $\eta$ , both the extensive and the intensive margins of affiliate sales are affected by the intensity of imported headquarter intermediates. The behaviour of the intensive margin is unambiguous:  $\sigma$  magnifies the sensitivity of the intensive margin. When  $\sigma$

is high, the change in  $X_{ij}^M$  due to a change in  $\tau$  is mostly captured by the intensive margin: if  $\tau$  decreases, new affiliates enter the market, but a high  $\sigma$  leads to a high level of competition. In this environment, having a low productivity is an even bigger disadvantage as firms can only capture a small market share, and their impact on the overall affiliate sales is small.

**Extensive Margin.** The sensitivity of the extensive margin of affiliate sales to changes in trade costs is not constant and it is related to the elasticity of substitution  $\sigma$ . In general, we should expect that when the substitutability across varieties is low, an increase in  $\sigma$  makes entrance of new affiliates more sensitive to changes in  $\tau$ . On the one hand, trade liberalization makes easier to import the intermediate goods; on the other hand, the low degree of substitution keeps the level of competition down. This explains why more firms can survive as new affiliates after entry. Contrarily, a larger degree of substitutability among varieties makes entry of new affiliates less sensitive to changes in  $\tau$ . In fact, when the level of competition is high, new entrants will capture only a small fraction of market share despite the reduction in trade costs.

We leave to the calibration section the general equilibrium analysis of how trade policy affects the sensitivity of affiliate sales.

### 3.1.2 Exports Sales

In this section we examine the intensive and extensive margins of export sales. After differentiating the expression of total exports in country  $j$ ,  $X_{ij}^X = w_i L_i \int_{\bar{a}_{ij}^M}^{\bar{a}_{ij}} x_{ij}^X dG(a)$ , with respect to trade costs, we derive the intensive and extensive margins of export sales

$$\frac{\partial X_{ij}^X}{\partial \tau_{ij}} = \underbrace{w_i L_i \int_{\bar{a}_{ij}^M}^{\bar{a}_{ij}} \frac{\partial x_{ij}^X}{\partial \tau_{ij}} dG(a)}_{\text{Intensive Margin}} + \underbrace{w_i L_i \left[ x_{ij}^X G'(\bar{a}_{ij}) \frac{\partial \bar{a}_{ij}}{\partial \tau_{ij}} - x_{ij}^M G'(\bar{a}_{ij}^M) \frac{\partial \bar{a}_{ij}^M}{\partial \tau_{ij}} \right]}_{\text{Extensive Margin}}, \quad (16)$$

where we applied the Leibniz rule once again to separate the margins.

**Proposition 4** *Defining  $\Omega \equiv -\partial \ln X_{ij}^X / \partial \ln \tau_{ij}$ , a change in the variable costs  $\tau_{ij}$  makes the margins of export sales to react as follows:*

$$\Omega = \underbrace{(\sigma - 1)}_{\text{Intensive Margin Elasticity}} + (k - \sigma + 1) \underbrace{\left[ 1 - \frac{X_{ij}^M}{X_{ij}^X} (\Gamma - \omega) \right]}_{\text{Extensive Margin Elasticity}}, \quad (17)$$

where

$$\Gamma = \frac{\eta \frac{(w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma}}{\tau_{ij}} - \frac{(w_i \tau_{ij})^{1-\sigma}}{\tau_{ij}}}{(w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}}, \quad (18)$$

$$\omega = \left( \frac{w_i \tau_{ij}}{w_j} \right)^{(1-\eta)(1-\sigma)}. \quad (19)$$

**Proof.** See Appendix F.2. ■

**Intensive Margin.** Similarly to models with untruncated Pareto distribution, the volume of export sales depends on the constant elasticity of substitution. This implies that when goods are very substitutable (high  $\sigma$ ), the export of each exporter is very sensitive to the trade barriers.

**Extensive Margin.** Differently from models with an untruncated Pareto, the extensive margin in our model depends on variable trade costs and it is not constant at  $k - \sigma + 1$ . Equation (17) shows that the sensitivity of the extensive margin of exports to trade policy depends on the interaction between aggregate affiliate and export sales.<sup>8</sup> This is because the change in the number of varieties supplied via exports depends on the level of profits generated by the export and multinational production strategies, which in turns affect overall affiliate sales.

Let us focus on the second part of (17). If  $X_{ij}^M > X_{ij}^X$ , a decrease in trade cost reduces the extensive margin elasticity of export. Notice that the sign of the overall elasticity depends on the size of the intensive margin, which can compensate the negative extensive margin. When  $X_{ij}^M < X_{ij}^X$  the opposite is true, and a decrease in trade costs increases the extensive margin. To summarize, while the elasticity of the intensive margin is always positive (a decrease in trade costs increases the volume of trade), the behaviour of the extensive margin depends on how export and affiliate sales interact.

Differently from Chaney (2008), the elasticity of exports and affiliate sales with respect to variable costs depends on the elasticity of substitution between goods,  $\sigma$ , and on trade costs  $\tau_{ij}$ . This result, which is discussed more carefully in the next section, suggests that countries' asymmetries embedded in a multi-country supply framework are relevant to fully understand how variable costs affect bilateral flows. This bounded productivity framework reaffirms the importance of trade costs and  $\sigma$  in models of firm heterogeneity. To further stress this result,

---

<sup>8</sup>Note that  $\Gamma > \omega$  is true for certain parameter restrictions consistent with our calibration. For further details on  $\Gamma$  and  $\omega$ , see appendix F.

in the calibration section we propose an exercise to understand the effects of trade policy on both export and affiliate sales' margins.

## 4 Welfare

In this model the welfare of each representative consumer is given by  $C_j = w_j/P_j$ , which does not depend on the assumption of free entry. We follow the procedure suggested by [Arkolakis et al. \(2008\)](#) to obtain an expression for the domestic trade share  $\lambda_{jj}$ , and the wage  $w_j$ .

We start by deriving the average sales for export, affiliate sales, and domestic firms, which results in the following equations:

$$(\overline{p_{ij}q_{ij}})^X = \left( \frac{k\sigma}{k - \sigma + 1} \right) \left( \frac{a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{(k-\sigma+1)}}{a_{ij}^{1-\sigma} [a_{ij}^k - (a_{ij}^M)^k]} \right) w_j f_{ij}^X, \quad (20)$$

$$(\overline{p_{ij}q_{ij}})^M = \left( \frac{k\sigma}{k - \sigma + 1} \right) w_j f_{ij}^M, \quad (21)$$

$$(\overline{p_{jj}q_{jj}})^D = \left( \frac{k\sigma}{k - \sigma + 1} \right) w_j f_{jj}. \quad (22)$$

Equations (21) and (22) are standard with respect to the literature, and independent of the productivity levels.<sup>9</sup> On the contrary, average sales of exporting firms in equation (20) depend on cutoff productivities. This happens despite the assumptions of Pareto distribution and Dixit-Stiglitz preferences. Therefore, the result in (20) differs from [Arkolakis et al. \(2008\)](#) and [Melitz and Redding \(2013\)](#).

A few remarks can be made on equation (20). First, the second term, i.e., the ratio including the difference in the cutoffs, is lower than 1. This makes average export sales smaller than in models with only exporting firms. This effect is driven by the additional level of competition characterizing our set up. Second, average export sales decline with a reduction in average productivity of exporters, while it increases with a reduction in average productivity of multinationals.<sup>10</sup>

Equations (20), (21) and (22) are used to obtain total export and affiliate sales from country  $i$  to  $j$  as well as domestic sales in country  $j$ . Total export sales from country  $i$  to  $j$

<sup>9</sup>Appendix G provides the derivations for equations (20), (21) and (22).

<sup>10</sup>See Propositions 5 and 6 in Appendix G.

are

$$T_{ij}^X = \underbrace{w_i L_i [a_{ij}^k - (a_{ij}^M)^k]}_{\text{no. exporting firms}} \underbrace{\left( \frac{k\sigma}{k - \sigma + 1} \right) \left( \frac{a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{k-\sigma+1}}{a_{ij}^{1-\sigma} [a_{ij}^k - (a_{ij}^M)^k]} \right)}_{\text{avg. exporting sales}} w_j f_{ij}^X. \quad (23)$$

Total affiliate sales from country  $i$  to  $j$  are

$$T_{ij}^M = \underbrace{w_i L_i (a_{ij}^M)^k}_{\text{no. multinational production firms}} \underbrace{\left( \frac{k\sigma}{k - \sigma + 1} \right)}_{\text{avg. affiliate sales}} w_j f_{ij}^M. \quad (24)$$

From equations (23) and (24) we can obtain total sales in country  $j$  as

$$\sum_v T_{vj} = \sum_v T_{vj}^X + \sum_v T_{vj}^M, \quad (25)$$

where it is worth stressing that both  $T_{vj}^X$  and  $T_{vj}^M$  include domestic sales. We are now able to compute the domestic trade share, which is given by

$$\lambda_{jj} = \frac{w_j L_j a_{jj}^k \frac{k\sigma}{k-\sigma+1} w_j f_{jj}}{\sum_v T_{vj}} = \frac{T_{jj}}{\sum_v T_{vj}}. \quad (26)$$

From equation (26) we obtain an expression for  $w_j$  as a function of domestic trade share  $\lambda_{jj}$ ,

$$w_j^{k-1} = \frac{1}{\lambda_{jj}} \frac{L_j f_{jj}^{1-b}}{\sum_v w_v L_v \left[ \frac{f_{vj}^{1-b}}{(w_v \tau_{vj})^k} + (f_{vj}^M - f_{vj})^{1-b} [w_j^{1-\eta} (w_v \tau_{vj})^{1-\sigma} - (w_v \tau_{vj})^{1-\sigma}]^b \right]}. \quad (27)$$

**Welfare with Only Exporters.** For the sake of comparison, let us consider the measure of welfare derived in [Arkolakis et al. \(2008\)](#) and [Arkolakis et al. \(2012\)](#), that is

$$\frac{w_j}{P_j} = \lambda_{jj}^{-\frac{1}{k}} L_j^{1/(\sigma-1)} C, \quad (28)$$

where  $C$  is a mnemonic for constant terms. This result shows that within a class of trade models, welfare predictions only depend on two sufficient statistics: the share of expenditure on domestic goods,  $\lambda$ , and the trade elasticity  $k$ . Therefore, the change in real income,  $\hat{W} \equiv W'/W$ , can be computed as

$$\hat{W} = \hat{\lambda}_{jj}^{\frac{1}{k}}. \quad (29)$$

**Welfare with Truncation.** The presence of export and multinational production alters the standard results obtained for welfare in heterogeneous firm models, through a double truncation of the productivity distribution.<sup>11</sup> Since this truncation makes average export sales to depend on productivity, the expression for welfare becomes complex and highly non linear. More specifically, our welfare measure is<sup>12</sup>

$$\frac{w_j}{P_j} = \lambda_{jj}^{\frac{1}{1-k}} \left[ \frac{L_j f_{jj}^{1-b}}{\sum_v w_v L_v \left[ \frac{f_{vj}^{1-b}}{(w_v \tau_{vj})^k} + (f_{vj}^M - f_{vj})^{1-b} [w_j^{1-\eta} (w_v \tau_{vj})^{1-\sigma} - (w_v \tau_{vj})^{1-\sigma}]^b \right]} \right]^{\frac{1}{1-k}} \frac{1}{P_j}, \quad (30)$$

where  $w_j$  comes from equation (27), and  $P_j$  is

$$\begin{aligned} P_j &= [(\sigma/(\sigma-1))^{1-\sigma} (k/(k-\sigma+1)) \lambda_1^{1-b}]^{\frac{1}{b(1-\sigma)}} (Y_j)^{\frac{b-1}{b(1-\sigma)}} \\ &\times \left[ \sum_{k=1}^N \frac{Y_K}{Y} \frac{Y}{1+\pi} \left[ (w_j f_{kj}^M - w_j f_{kj}^X)^{1-b} \left[ (w_j^{1-\eta})^{1-\sigma} w_k^{\eta(1-\sigma)} \phi_{kj}^\eta - (w_k)^{1-\sigma} \phi_{kj} \right]^b \right. \right. \\ &\left. \left. + [w_j f_{kj}^X]^{1-b} ((w_k)^{1-\sigma} \phi_{kj})^b \right] \right]^{\frac{1}{b(1-\sigma)}}. \end{aligned} \quad (31)$$

The welfare expression in (30) cannot be further simplified and trade costs are left inside. This is different from models with only exporters, where welfare is a function of domestic trade share, trade elasticity and parameters, as in equation (28). We conclude that in models where alternative market strategies occur simultaneously, a country's domestic trade share and trade elasticity are no longer sufficient statistics to evaluate welfare gains.

To have a better understanding of what happens to welfare, we propose a calibration exercise to evaluate the effects of trade liberalization.

## 5 Quantitative Exercise

We examine the quantitative relevance of our model. In section 5.1, we show that there are relevant differences in welfare, probability of trading, and domestic trade share between a benchmark model *à la* Melitz and Redding (2013) ( $M1$ ); our model with exporting and intra-firm activity in symmetric countries ( $M2$ ); and, our model with exporting and intra-firm activity in asymmetric countries ( $M3$ ). We also examine welfare gains from intra-firm

<sup>11</sup>We care to clarify that Arkolakis et al. (2008)'s summary statistic for the welfare gains from trade is not invalidated by the presence of intra-firm trade, but more generally by the truncation.

<sup>12</sup>See Appendix B for more details on the price index.



trade and pure multinational production activity. In section 5.2, we exploit the absence of free entry to analyse the sensitivity of intensive and extensive margins of exports and affiliate sales to variable trade costs.

## 5.1 Comparative Static

We set the elasticity of substitution  $\sigma = 4$ , as in Broda and Weinstein (2006): over the 1999-2001 period they find average and median elasticities for SITC 5-digit goods of 13.1 and 2.7, respectively (see their Table IV).<sup>13</sup> Consistently with the literature, we choose the shape parameter of the Pareto distribution to be  $k = 4.25$ . Geographical and trade barriers are set to  $\tau = 1.83$ , as in Melitz and Redding (2013) and Irarrazabal et al. (2012). For the value of  $\eta$ , we follow the findings in the literature and assign a magnitude of 1/6 to intra-firm trade.<sup>14</sup>

We identify the Home country ( $H$ ) with the U.S. and the Rest of the World ( $RoW$ ) with an average of OECD countries excluding the U.S. In models  $M1$  and  $M2$ , wage is equal to one, and country size is equal to the U.S. labor force. In model  $M3$ , the wage in  $H$  is equal to one, while the wage in the  $RoW$  is set to match the wage ratio between the two countries of 0.85 (OECD (2000)).  $L_H$  and  $L_{RoW}$  equal the respective labor forces (OECD (2000)).

We calibrate exporting fixed costs  $f^X$  to match the average fraction of U.S. manufacturing firms that export (18 percent, as reported in Bernard et al. (2007)).<sup>15</sup> We choose  $H$  intra-firm fixed costs  $f_{H,RoW}^M$  to ensure that both symmetric and asymmetric models,  $M2$  and  $M3$ , are consistent with U.S. affiliate sales as a share of world export sales (32 percent, as in Ruhl (2016)). For model  $M3$ ,  $RoW$  intra-firm fixed costs  $f_{RoW,H}^M$  is calibrated to match world sales of foreign affiliate as a percentage of world GDP (25 percent, as in Ramondo (2014), Table 1). The calibrated parameters and targeted moments are listed in Table 1 and 2, respectively.

Table 1: Calibrated Parameters

Parameter		Model		
		$M1$	$M2$	$M3$
Export fixed cost	$f^X$	0.547	0.334	0.341
Home intra-firm export fixed cost	$f_{H,RoW}^M$		2.250	5.055
ROW intra-firm export fixed cost	$f_{RoW,H}^M$			1.030

<sup>13</sup>The value  $\sigma = 4$  implies a mark-up of 33 percent.

<sup>14</sup>See Garetto (2013), Irarrazabal et al. (2012) and Ramondo and Rodriguez-Clare (2013). Note that the data do not allow to distinguish between horizontal and vertical integration. We can interpret the value of  $\eta$  as an upper bound of intra-firm trade magnitude.

<sup>15</sup>For all models,  $M1$ ,  $M2$  and  $M3$ , we set  $f_{H,RoW}^X = f_{RoW,H}^X = f^X$ .

The baseline calibrated model ( $M1$ ) implies a fixed cost of export in line with the estimate of 0.545 in [Melitz and Redding \(2013\)](#). When multinational production is introduced in the model, the fixed cost decreases to 0.3 and the average productivity of exporting firms goes down.

Table 2: Moments targeted in the estimation

Calibration target	Data	Model			Source
		$M1$	$M2$	$M3$	
Share of exporting firms	0.180	0.180	0.180	0.168	<a href="#">Melitz and Redding (2013)</a>
IF trade as share of total export	0.320		0.320	0.320	<a href="#">Ruhl (2016)</a>
World multinational production as a share of GDP	0.250			0.250	<a href="#">Ramondo (2014)</a>

There is great heterogeneity in the fixed cost of engaging in multinational production activities that reflects the heterogeneity in multinational production flows: multinational production firms in  $H$  bear a fixed cost which is about 16 times higher than the exporting cost, and represents about 0.5 percent of the host country's GDP; multinational production firms in  $RoW$  are half productive than firms in  $H$  and bear a fixed cost which is about 0.08 percent of the host country's GDP.

In [Figure 1](#), we show the effects of adjusting variable trade costs from their calibrated value of  $\tau = 1.83$  (trade regime  $T_0$ ) to  $\tau \in [1.5, 1.83]$  (trade regime  $T_1$ ).<sup>16</sup>

In all panels, we compare the results of the three models,  $M1$ ,  $M2$  and  $M3$ . In general, the results for  $M1$  are consistent with the findings of [Melitz and Redding \(2013\)](#) ([Figure 1](#)). Panel A displays welfare gains, measured as welfare for each value of variable cost:  $W^{T_1}$ ; Panel B shows the probability of exporting  $[G(a^{X,T_1}) - G(a^{M,T_1})]/G(a^{D,T_1})$ ; Panel C shows the probability of multinational production activity  $G(a^{M,T_1})/G(a^{D,T_1})$ ; Panel D displays the domestic trade share  $\lambda^{T_1}$ .

As shown in Panel A, gains from trade are larger in economies with multinational production and intra-firm trade where the wage is equal to one ( $M2$  and  $H$  in  $M3$ ), but they are lower in the  $RoW$ , where the wage is 15 percentage points lower.<sup>17</sup> A reduction in  $\tau$  from 1.83 to 1.5 generates a welfare difference between the model with export ( $M1$ ) and the symmetric model with export and multinational production ( $M2$ ) of about 0.7 points (up to 0.8 points

<sup>16</sup>We only consider values of  $\tau$  for which there is trade in all models. We cannot consider values of  $\tau$  higher than 1.83 as for those values, there would not be trade in model  $M3$ .

<sup>17</sup>Welfare is computed using equation (30), where  $w_H = w_{RoW} = 1$  and  $L_{RoW} = L_H$  in the symmetric model  $M2$ .

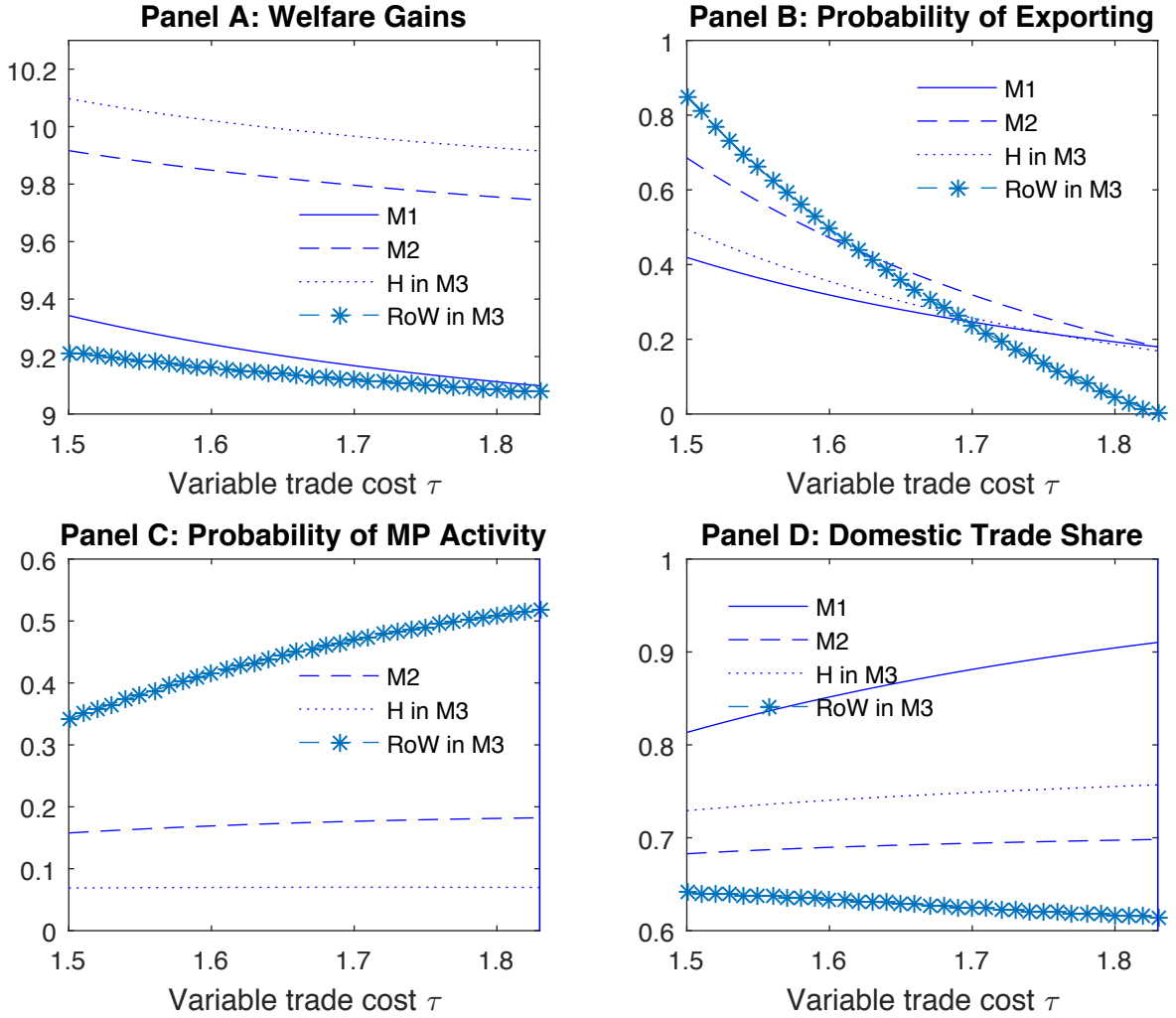


Figure 1: Reductions in variable trade costs

when the symmetric model with export is compared to  $H$ ).

As shown in Panel B and C, the sources of these welfare differences are endogenous selection into the domestic, export, and multinational production activity ( $M1$  and  $M2$ ) as well as wage differential ( $M3$ ). In Panel B, as variable trade costs fall from their calibrated value to  $\tau = 1.5$ , the probability of exporting rises from 0.42 in  $M1$  to 0.85 in the  $RoW$  (and about 0.68 in  $M2$ ). At the calibrated value of  $\tau$ , the probability of exporting is slightly higher than zero for  $RoW$ , and about 0.18 for  $M1$ ,  $M2$ , and  $H$ . In Panel C, decreasing variable trade costs, the probability of multinational production activity decreases considerably for the  $RoW$ . Multinational production activity in the  $RoW$  is discouraged by lower variable trade costs: in multinational production firms with lower average productivity (than exporting firms), a low  $\tau$  makes export more interesting than intra-firm activity. The probability of multinational production activity remains about constant and lower for  $M2$  and  $H$  in  $M3$ . In Panel D,  $M1$  exhibits the highest (and increasing) domestic trade share. In the  $RoW$ , the domestic trade

share varies between 61 and 64 percent.

In Table 3 we compute the welfare gains that the theory implies by comparing the calibrated economies ( $M2$  and  $M3$ ) and a counterfactual world where  $\eta \rightarrow 1$  in both countries. That is, we compute the gains as the ratio between the welfare in the calibrated economies with export and multinational production activities and the welfare in the economies with the calibrated parameters but shutting down all multinational production activities.

Table 3: Welfare Gains from Intra-Firm (GIF)

	Baseline Calibration $\sigma = 4, \eta = 1/6, k = 4.25$	$\sigma = 3.5$	$\sigma = 4.15$	$\eta = 1/8$	$\eta = 1/4$	$k = 3.75$	$k = 4.75$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$M2$	1.066	1.044	1.072	1.074	1.051	1.089	1.049
$M3$	$H$	1.069	1.055	1.072	1.076	1.081	1.059
	$RoW$	1.076	1.059	1.081	1.085	1.060	1.059

The first column shows the welfare gains in the calibrated economy. As the fixed costs of multinational production are higher for  $H$  than for the  $RoW$  firms, the gains from opening to intra-firm trade are lower for  $H$  than for the  $RoW$ . Intra-firm trade among  $H$  firms in the  $RoW$  allows final good producers to use more productive technologies and to pay lower wages. Moreover, the possibility of integration reduces the prices charged on traded intermediates. In the second and third columns, we report the same computation but changing  $\sigma$ . First, the elasticity of substitution is lowered to 3.5 which increases the market power to 40 percent. The welfare gains driven by intra-firm trade decrease for both countries because both price indices increase. Second, we show that a lower market power of 31 ( $\sigma = 4.15$ ) percent generates an increase in welfare when the economies open to intra-firm trade, especially for the symmetric case  $M2$ . In columns four and five, we show the impact of a lower and higher share of intra-firm trade on welfare. A higher  $\eta$  induces a welfare loss in all models, as the cost of intra-firm activity increases. Lastly in columns six and seven, we analyze the effect of changes in productivity dispersion,  $k$ . We consider a situation in which the economies are characterized by higher productivity dispersion (lower  $k$ ), which implies a larger number of high productivity firms. In this case, welfare gains driven by intra-firm trade are larger, especially in the  $RoW$ . On the contrary, a higher  $k$  implies an important decrease in welfare gains. Overall, the results in Table 3 suggest that the gains from intra-firm trade are larger

in asymmetric ( $M3$ ) than in symmetric ( $M2$ ) models.

In Table 4, we report the welfare gains from pure multinational production ( $GMP$ ), i.e. horizontal multinational production with no intra-firm trade ( $\eta = 0$ ). The absence of intra-firm flows makes this model to capture the proximity versus concentration hypothesis. In each column, the gains are computed as the ratio between the welfare from the economy with export and pure multinational production and the economy where only export activity is allowed. The parameters of the model are taken from the calibration in the first column and modified in the rest of the table.

Table 4: Welfare Gains from Pure Multinational Production (GMP)

		Baseline Calibration $\sigma = 4, k = 4.25$	$\sigma = 3.5$	$\sigma = 4.15$	$k = 3.75$	$k = 4.75$
		(1)	(2)	(3)	(4)	(5)
$M2$		1.105	1.074	1.113	1.133	1.084
$M3$	$H$	1.102	1.086	1.105	1.114	1.092
	$RoW$	1.120	1.088	1.130	1.159	1.094

Notice that asymmetries continue to play a role also in pure multinational production model. Welfare gains in the  $RoW$  are always higher than in  $H$  and in the symmetric case  $M2$ . More precisely, the  $RoW$  will always gain from multinational production, but relatively more in pure multinational production models. In fact, comparing Table 3 to Table 4, we observe that for both  $H$  and the  $RoW$  the gains are higher when intra-firm trade is not included in the model. We can conclude that the relative gains from trade is larger in pure multinational production models. This is not surprising since pure multinational production corresponds to a case in which the Pareto distribution is less constrained than in multinational production with intra-firm, where multinational firms face higher fixed and variable costs.

## 5.2 Margins of Trade

We exploit the presence of alternative market strategies to quantify the impact of trade liberalization on the sensitivity of intensive and extensive margins. In particular, in our model the sensitivity of the extensive margins will differ with respect to alternative models with untruncated Pareto, such as Chaney (2008) and Helpman et al. (2004).

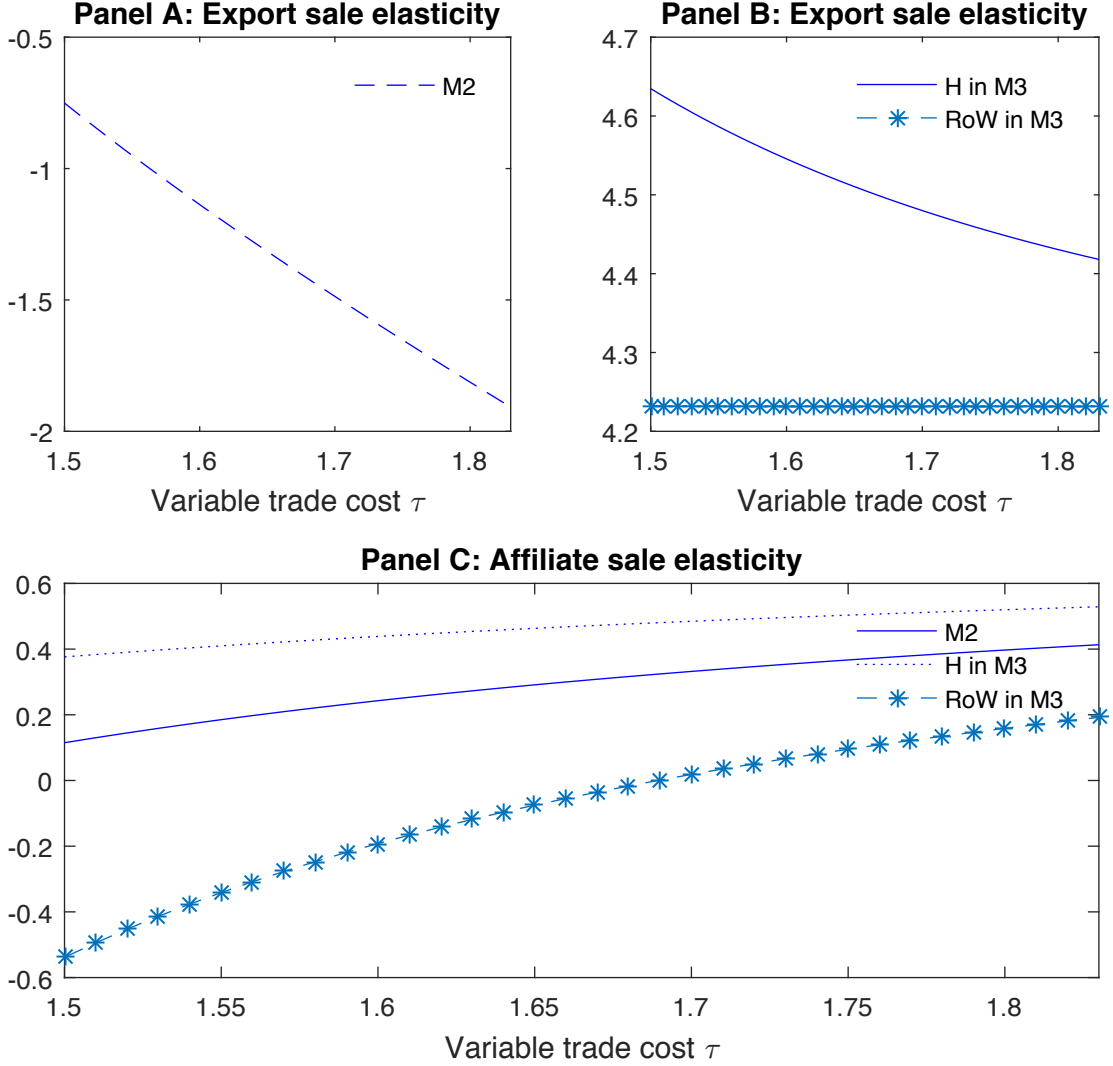


Figure 2: Reductions in variable trade costs

In Figure 2, we plot the extensive margin of trade for models  $M2$  and  $M3$ , considering a decrease in variable trade costs from 1.83 to 1.5.<sup>18</sup> Panel A and B show margins of export sales for the symmetric and asymmetric cases,  $M2$  and  $M3$  respectively. Panel C depicts margins of affiliate sales for  $M2$  and  $M3$ . An observation is in order. While the elasticity of affiliate sales, equation (15), depends only on parameters' values, equation, the sensitivity of export sales is a function of affiliate and export sales, equation (17). To simplify the analysis and focus on the trend (and not on the magnitude), we set  $X_{ij}^M = X_{ij}^X$  in equation (17).

The parameter values of our exercise are such that the extensive margins are negative in the symmetric model (as  $\Gamma > 1 + \omega$ ); while in the asymmetric model the extensive margins are positive both in  $H$  (as  $\Gamma < \omega$ ), and in the  $RoW$  (as  $\Gamma - \omega \in (0, 1)$ ). In the symmetric case  $M2$  and only for  $H$  in  $M3$ , the margins of export sales are decreasing with  $\tau$ : as variable

<sup>18</sup>Figure 2 uses equations (15) and (17).

trade costs increase, export sales go down. Margins of export sales are (almost) constant in the *RoW*, where the lower wage  $w_{RoW}$  mitigates the increase in  $\tau$ .

For the affiliate sales' margin, the result is the expected one: as variable export costs increase, affiliate sales increase in all the models, and especially for the *RoW*. For our parametrization, an increase in trade barriers makes the multinational production strategy less costly.

## 6 Conclusion

Our goal in this paper has been to evaluate welfare and gains from intra-firm trade in a general equilibrium model with export and multinational production. We have assumed that each foreign affiliate imports an intermediate input from the home country due to technological appropriability issues. This set up captures the interaction between alternative market access strategies, by allowing the knowledge-intensive input used in multinational production to move over geographical space. Therefore, geographical costs apply to both exports and multinational production because they involve transportation of a finished good and of an intermediate good, respectively. We have investigated the effects of an increase in trade barriers on multinational production. First, it increases the productivity cutoff: the need to import intermediate goods from the headquarter makes it more difficult to enter as a foreign affiliate when trade costs increase. Second, sales of the existing foreign affiliates decrease, which implies the existence of a new margin of adjustment for multinational firms.

An important theoretical result of the paper is that alternative market access strategies alter the standard results obtained for welfare in heterogeneous firm models, through a double truncated productivity distribution. Our model shows that with export and multinational production, the welfare gains from trade are also affected by trade costs and wage differential between countries.

To quantitatively assess the country level gains from multinational production with intra-firm trade, we calibrate the model to match aggregate U.S. and an average of OECD countries' data. Our findings stress the role of intra-firm trade for additional welfare gains: they range from 6 to 12 percent depending on country characteristics. Moreover, we exploit the delivered gravity equations to compare margins' sensitivity for exports and affiliate sales with respect to alternative models such as [Chaney \(2008\)](#) and [Helpman et al. \(2004\)](#). Our framework reaffirms the importance of trade costs and elasticity of substitution in models with firm heterogeneity.

Extensions of the model should be devoted to the introduction of a free entry condition. However, we believe the analysis conducted here is a useful starting point to understand the mechanism governing the firms' decisions about sourcing, and of the welfare consequences of having multiple market access strategies.



## Acknowledgments

We thank Costas Arkolakis, Richard Baldwin, Thierry Mayer, Gianmarco Ottaviano, Julien Prat and Peter Schott for constructive and helpful suggestions. Remarks from Stefanie Haller, Antonella Nocco, and participants at the 2013 workshop in Lecce also contributed to improve this paper. All errors are ours. This research has been conducted as part of the project Labex MME-DII (ANR11-LBX-0023-01).

## Appendix

Appendices **B-F** provide derivations for equilibrium variables and proofs of the propositions.

### A Demand for Differentiated Goods

Total income in country  $j$ ,  $Y_j$ , is computed as the sum of workers' labor income in country  $j$ ,  $w_j L_j$ , and the dividends from their portfolio,  $\pi w_j L_j$ , where  $\pi$  is the dividend per share. Given the optimal pricing of firms and the consumers' demand, the export value from country  $i$  to country  $j$  for a firm with unit labor requirement  $a$  is equal to

$$x_{ij}^X = p_{ij}^X q_{ij}^X = Y_j (p_{ij}^X)^{1-\sigma} / P_j^{1-\sigma}, \quad (32)$$

where  $p_{ij}^X = [\sigma / (\sigma - 1)] a w_i \tau_{ij}$  and  $q_{ij}^X = (p_{ij}^X)^{-\sigma} \beta Y_i / P_j^{1-\sigma}$ . Affiliate sales by a firm located in  $j$  are

$$x_{ij}^M = p_{ij}^M q_{ij}^M = Y_j (p_{ij}^M)^{1-\sigma} / P_j^{1-\sigma}, \quad (33)$$

where  $P_j$  represents the price index of good  $q$  in country  $j$ . We can observe that the values of export and total production in  $j$ 's foreign affiliates are similar to those derived in a setting with homogeneous firms. These equations are the basis for deriving gravity equations of export and affiliate sales.

## B Profits

In what follows we determine the dividend per share in the economy. In order to do this we use the total profits from exporting from  $i$  to  $j$  (including also trade within a country):

$$\begin{aligned}\Pi_{ij}^X &= w_i L_i \left[ \int \frac{1}{\sigma} x_{ij} dG(a) - \int w_j f_{ij}^X dG(a) \right] \\ &= \frac{X_{ij}}{\sigma} - w_j f_{ij}^X w_i L_i \int dG(a).\end{aligned}\quad (34)$$

Note that when  $i = j$ , this expression represents domestic profit.<sup>19</sup> Since  $n_{ij} = w_i L_i \int_{a_{ij}^M}^{a_{ij}^X} dG(a)$ , the expression above can be rewritten as

$$\Pi_{ij}^X = \frac{X_{ij}}{\sigma} - n_{ij} w_j f_{ij}^X. \quad (35)$$

The total profits for country  $j$ 's affiliates are:

$$\begin{aligned}\Pi_{ij}^M &= w_i L_i \int \frac{1}{\sigma} x_{ij}^M dG(a) - \int w_j f_{ji}^M dG(a) \\ &= \frac{X_{ij}^M}{\sigma} - n^M w_j f_{ji}^M,\end{aligned}\quad (36)$$

since  $n^M = w_i L_i \int_0^{a_{ij}^M} dG(a)$ .

Total profits in this economy are

$$\begin{aligned}\Pi &= \sum_i \sum_j (\Pi_{ij}^X + \Pi_{ij}^M) \\ &= \sum_i \sum_j \left[ \left( \frac{X_{ij}}{\sigma} + \frac{X_{ij}^M}{\sigma} \right) - (n_{ij} w_j f_{ij}^X + n^M w_j f_{ij}^M) \right].\end{aligned}\quad (37)$$

This expression is the sum of the overall profits produced by domestic, exporting and multi-national firms in every country. Remember that country  $j$  is receiving varieties from  $N - 1$ . More specifically, total sales in country  $j$  are determined by varieties sold by domestic firms, varieties exported to  $j$ , and varieties produced locally by foreign affiliates. Hence, total import in country  $j$  are  $\sum_i (X_{ij}^X + X_{ij}^M) = Y_j$ , where we used the fact that trade is balanced. Substituting the equilibrium number of exporters and affiliates we can rewrite the worldwide

---

<sup>19</sup>If we are interested in the domestic profits from serving market  $i$  we should compute:  $\Pi_{ii} = w_i L_i \int_0^{a_{ii}} \frac{1}{\sigma} x_{ii} dG(a) - \int_0^{a_{ii}} f_{ii} dG(a)$ . We should proceed in the same way for computing the number of firms entering a particular market  $i$ :  $N_{ii} = w_i L_i \int_0^{a_{ii}} dG(a)$ . This expression delivers the overall number of firms existing in  $i$ .

profits as:

$$\Pi = \sum_j \left[ \frac{Y_j}{\sigma} - \lambda_4^{-b} Y_j \right] = Y \frac{1 - \lambda_4^{-b}}{\sigma}. \quad (38)$$

Hence dividends per share are:

$$\begin{aligned} \pi &= \frac{\Pi}{\sum_i w_i L_i} = \frac{\Pi}{Y} (1 + \pi) = \frac{1 - \lambda_4^{-b}}{\sigma} (1 + \pi) \\ &= \frac{\frac{1 - \lambda_4^{-b}}{\sigma}}{\left(1 - \frac{1 - \lambda_4^{-b}}{\sigma}\right)}. \end{aligned} \quad (39)$$

## C Price Index

Only firms with  $a \leq a_{kj}$  will produce.<sup>20</sup> The price index adjusts depending on country characteristics, and the number of potential entrants,  $n_E$ , which is exogenously given,

$$P_j^{1-\sigma} = \sum_{k=1}^N w_k L_k \left[ \int_0^{a_{kj}^M} (w_j^{1-\eta} (w_k \tau_{kj})^\eta)^{1-\sigma} a^{1-\sigma} dG(a) + \int_{a_{kj}^M}^{a_{kj}} (w_k \tau_{kj})^{1-\sigma} a^{1-\sigma} dG(a) \right], \quad (40)$$

which becomes

$$\begin{aligned} P_j^{1-\sigma} &= (\sigma/(\sigma - 1))^{1-\sigma} [k/(k - \sigma + 1)] \\ &\quad \times \sum_{k=1}^N w_k L_k \left[ (a_{kj}^M)^{k-\sigma+1} \left[ (w_j^{1-\eta} (w_k \tau_{kj})^\eta)^{1-\sigma} - (w_k \tau_{kj})^{1-\sigma} \right] \right. \\ &\quad \left. + a_{kj}^{k-\sigma+1} (w_k \tau_{kj})^{1-\sigma} \right]. \end{aligned} \quad (41)$$

Plugging the productivity thresholds from (46) and (47) we can solve for the price index

---

<sup>20</sup>Since we are not conditioning by  $G(a/a_{ij})$ , the number of firms will be the number of entrants and not the number of active firms. Moreover, we consider  $a_{ij}$  to be the unit labor requirement for exporting. Note that when  $i = j$ ,  $\tau_{ii} = 1$ . Therefore  $a_{ij} = a_{ii}$  corresponds to the cutoff of domestic firms.

in the destination country  $j$ ,

$$\begin{aligned}
P_j^{1-\sigma} &= (\sigma/(\sigma-1))^{1-\sigma} [k/(k-\sigma+1)] \sum_{k=1}^N w_k L_k \\
&\times \left\{ \left[ \lambda_1 \frac{w_j f_{kj}^M - w_j f_{kj}^X}{Y_j} \frac{P_j^{1-\sigma}}{\left( w_j^{1-\eta} (w_k \tau_{kj})^\eta \right)^{1-\sigma} - (w_k \tau_{kj})^{1-\sigma}} \right]^{1-b} \right. \\
&\times \left[ \left( w_j^{1-\eta} (w_k \tau_{kj})^\eta \right)^{(1-\sigma)} - (w_k \tau_{kj})^{1-\sigma} \right] \\
&\left. + \left[ \lambda_1 \frac{w_j f_{kj}^X}{Y_j} \frac{P_j^{1-\sigma}}{(w_k \tau_{kj})^{1-\sigma}} \right]^{1-b} (w_k \tau_{kj})^{1-\sigma} \right\}, \tag{42}
\end{aligned}$$

where  $b = k/(\sigma-1)$ ,  $w_k$  is the wage paid to workers in country  $k$  for firms which are exporting the good, while  $w_j$  is the wage paid to the workers who are either producing the  $j$ -domestic varieties or the intermediate good  $y_2$  used by the foreign affiliate in country  $j$ . Then solving for  $P_j^{1-\sigma}$

$$\begin{aligned}
P_j^{b(1-\sigma)} &= (\sigma/(\sigma-1))^{1-\sigma} [k/(k-\sigma+1)] \lambda_1^{1-b} (Y_j)^{b-1} \\
&\times \sum_{k=1}^N w_k L_k \left[ (w_j f_{kj}^M - w_j f_{kj}^X)^{1-b} \left[ \left( w_j^{1-\eta} \right)^{1-\sigma} w_k^{\eta(1-\sigma)} \phi_{kj}^\eta - (w_k)^{1-\sigma} \phi_{kj} \right]^b \right. \\
&\left. + [w_j f_{kj}^X]^{1-b} \left( (w_k)^{1-\sigma} \phi_{kj} \right)^b \right],
\end{aligned}$$

where  $\phi_{kj} = \tau_{kj}^{1-\sigma}$ .

$$\begin{aligned}
P_j &= [(\sigma/(\sigma-1))^{1-\sigma} (k/(k-\sigma+1)) \lambda_1^{1-b}]^{\frac{1}{b(1-\sigma)}} (Y_j)^{\frac{b-1}{b(1-\sigma)}} \\
&\times \left[ \sum_{k=1}^N \frac{Y_K}{Y} \frac{Y}{1+\pi} \left[ (w_j f_{kj}^M - w_j f_{kj}^X)^{1-b} \left[ \left( w_j^{1-\eta} \right)^{1-\sigma} w_k^{\eta(1-\sigma)} \phi_{kj}^\eta - (w_k)^{1-\sigma} \phi_{kj} \right]^b \right. \right. \\
&\left. \left. + [w_j f_{kj}^X]^{1-b} \left( (w_k)^{1-\sigma} \phi_{kj} \right)^b \right] \right]^{\frac{1}{b(1-\sigma)}}, \tag{43}
\end{aligned}$$

which after rearrangements becomes:

$$P_j = \lambda_2 Y_j^{\frac{b-1}{b(1-\sigma)}} \theta_j \left( \frac{Y}{1+\pi} \right)^{\frac{1}{b(1-\sigma)}}. \tag{44}$$

where  $\lambda_2^{b(\sigma-1)} = (\sigma/(\sigma-1))^{\sigma-1} [(k-\sigma+1)/k] \lambda_1^{b-1}$ ,  $w_k$  is the wage paid to workers in country  $k$  by exporting firms, and  $w_j$  is the wage paid to the workers who are either producing the  $j$ -domestic varieties or the intermediate good  $y_2$  used by the foreign affiliate in country  $j$ .<sup>21</sup>

<sup>21</sup>Appendix C provides detailed derivations of the price index.

$\theta_j$  in equation (44) collects the following terms

$$\begin{aligned} \theta_j^{b(1-\sigma)} &= \sum_{k=1}^N \frac{Y_K}{Y} \left[ (w_j (f_{kj}^M - f_{kj}^X))^{1-b} \left[ (w_j^{1-\eta} (w_k \tau_{kj})^\eta)^{1-\sigma} - (w_k \tau_{kj})^{1-\sigma} \right]^b \right. \\ &\quad \left. + [w_j f_{kj}^X]^{1-b} ((w_k \tau_{kj})^{1-\sigma})^b \right], \end{aligned} \quad (45)$$

where  $Y$  is the world output.  $\theta_j$  is an aggregate index of  $j$ 's remoteness from the rest of the world, and it can be thought as the ‘‘multilateral trade resistance’’ introduced by [Anderson and van Wincoop \(2003\)](#). It takes into consideration the role of fixed and trade costs as well as the intermediate input traded.

Since total income  $Y$  will depend on the dividends received from the global fund, in equilibrium the amount of dividends per share is a constant.

## D Equilibrium with Heterogeneous Firms

To compute the equilibrium of the overall economy, we solve for the selection of firms into different modes of supply. We generate predictions for aggregate bilateral trade and affiliate sales.

### D.1 Productivity Threshold

The productivity threshold of the least productive firm in country  $i$  that exports to country  $j$  is

$$a_{ij}^{1-\sigma} = \lambda_1 \frac{w_j f_{ij}^X}{Y_j} \frac{P_j^{1-\sigma}}{(w_i \tau_{ij})^{1-\sigma}}, \quad (46)$$

where  $\lambda_1 = \sigma^\sigma (\sigma - 1)^{(1-\sigma)}$ .<sup>22</sup>

The productivity threshold of the least productive firm in country  $i$  which opens a foreign affiliate in country  $j$ , is obtained by equating the operating profits from doing multinational production in equation (8), to the operating profit from exporting in equation (7):

$$(a_{ij}^M)^{1-\sigma} = \lambda_1 \frac{w_j (f_{ij}^M - f_{ij}^X)}{Y_j} \frac{P_j^{1-\sigma}}{(w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}}. \quad (47)$$

Equations (46) and (47) are used in calibration.

---

<sup>22</sup>We interpret  $a^{1-\sigma}$  as a measure of productivity.

## D.2 Equilibrium Variables

The mode of supply choice depends on each firm's productivity, the trade costs, the aggregate demand, the amount of intermediates, and the set of competitors. Plugging the general equilibrium price index (44) into the productivity thresholds (46) and (47), we can solve for the equilibrium productivity thresholds:

$$\bar{a}_{ij}^{1-\sigma} = \lambda_4 \frac{w_j f_{ij}^X}{(w_i \tau_{ij})^{1-\sigma}} \theta_j^{1-\sigma} \left( \frac{Y}{Y_j} \right)^{\frac{1}{b}} (1 + \pi)^{-\frac{1}{b}}, \quad (48)$$

$$(\bar{a}_{ij}^M)^{1-\sigma} = \lambda_4 \frac{w_j (f_{kj}^M - f_{ij}^X)}{(w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \theta_j^{1-\sigma} \left( \frac{Y}{Y_j} \right)^{\frac{1}{b}} (1 + \pi)^{-\frac{1}{b}}, \quad (49)$$

where  $\lambda_4 = \lambda_1 / \lambda_2^{\sigma-1}$  is constant. The productivity threshold in (48) is unambiguously positively affected by the wage rate in the origin country, and distance trade costs. The productivity threshold in (49) is ambiguously affected by the wage rate in  $i$ , the intensity in headquarter services  $\eta$ , and trade costs.

The share of imported intermediates plays an important role in determining the substitutability or the complementarity between export and multinational production strategies. For low intensity in imported intermediate (low  $\eta$ ), the multinational production strategy becomes more attractive when trade costs increase, making multinational production and exports substitutes. On the contrary, higher level of  $\eta$  makes multinational production and export complements, so that both activities require a higher productivity when trade barriers increase.

Using the demand function, the equilibrium price, and the price index (44), we can find firm level exports, firm level affiliate sales, aggregate output, and dividends per share  $\pi$ :

$$x_{ij}^X = p_{ij}^X q_{ij}^X = \lambda_3 \theta_j^{\sigma-1} \left( \frac{Y_j}{Y} \right)^{\frac{1}{b}} (1 + \pi)^{\frac{1}{b}} (w_i \tau_{ij})^{1-\sigma} a^{1-\sigma}, \quad (50)$$

$$x_{ij}^M = p_{ij}^M q_{ij}^M = \lambda_3 \theta_j^{\sigma-1} \left( \frac{Y_j}{Y} \right)^{\frac{1}{b}} (1 + \pi)^{\frac{1}{b}} (w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{(1-\sigma)} a^{1-\sigma}, \quad (51)$$

$$\pi = \lambda_5, \quad (52)$$

$$Y_j = (1 + \pi) w_j L_j = (1 + \lambda_5) w_j L_j, \quad (53)$$

where  $\lambda_3 = \lambda_2^{\sigma-1} (\sigma / (\sigma - 1))^{1-\sigma}$  and  $\lambda_5 = ((1 - \lambda_4^{-b} \sigma) / \sigma) / (1 - (1 - \lambda_4^{-b} \sigma) / \sigma)$  are constants. Equations (50)-(53) are functions of: country size  $L_j$ , wages, trade barriers  $\tau_{ij}$ , fixed costs

$f_{ij}^M$  and  $f_{ij}^X$ , proportions of imported intermediate,  $\eta$ , and measures of the  $j$ 's location with respect to the rest of the world  $\theta_j$ .

Similar to [Chaney \(2008\)](#), exports by individual firms depend on the trade cost  $\tau_{ij}$  with an elasticity of  $(1 - \sigma)$ . Additionally, we characterize sales by a foreign affiliate: they depend on the share of intermediate  $y_2$  produced in the foreign location, and on the intermediate  $y_1$  imported from the home country. Intra-firm trade implies that firm level affiliate sales in [\(51\)](#) is unambiguously affected by trade costs: an increase in trade costs reduces firm level affiliate sales. The behaviour of a single firm is similar to what a traditional model of trade and multinational production with representative firms would predict for aggregate bilateral trade flows and affiliate sales.

## E Proofs

In what follows we provide proofs of the propositions and equilibrium variables.

### E.1 Proposition 1

**Proof.** Total exports from  $i$  to  $j$  are given by:

$$X_{ij}^X = w_i L_i \int_{\bar{a}_{ij}^M}^{\bar{a}_{ij}} x_{ij}^X dG(a). \quad (54)$$

A firm will be exporting if  $a(v) \leq \bar{a}_{ij}$ . Using [\(50\)](#), [\(51\)](#), [\(48\)](#) and [\(49\)](#) and the specific assumption about the distribution of the labor unit requirement,  $a$ , we obtain:

$$X_{ij}^X = w_i L_i \int_{\bar{a}_{ij}^M}^{\bar{a}_{ij}} \lambda_3 \theta_j^{\sigma-1} \left( \frac{Y_j}{Y} \right)^{\frac{1}{b}} (1 + \pi)^{\frac{1}{b}} (w_i \tau_{ij})^{1-\sigma} a^{1-\sigma} dG(a), \quad (55)$$

$$\text{with } \bar{a}_{ij}^{1-\sigma} = \lambda_4 \frac{w_j f_{ij}^X}{(w_i \tau_{ij})^{1-\sigma}} \theta_j^{1-\sigma} \left( \frac{Y}{Y_j} \right)^{\frac{1}{b}} (1 + \pi)^{-\frac{1}{b}}, \quad (56)$$

$$\text{and } (\bar{a}_{ij}^M)^{1-\sigma} = \lambda_4 \frac{w_j f_{ij}^M - w_j f_{ij}^X}{(w_j^{1-\eta} (w_i \tau_{kj})^\eta)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \theta_j^{1-\sigma} \left( \frac{Y}{Y_j} \right)^{\frac{1}{b}} (1 + \pi)^{-\frac{1}{b}}. \quad (57)$$

Using the assumption of the Pareto distribution and the productivity thresholds, we can then solve the integral and find [\(12\)](#). ■

## E.2 Proposition 2

**Proof.** Total affiliate sale in country  $j$  are given by:

$$X_{ij}^M = w_i L_i \int_0^{\bar{a}_{ij}^M} x_{ij}^M dG(a). \quad (58)$$

A firm will open a subsidiary in country  $j$  if  $a(v) \leq \bar{a}_{ij}^M$ . Using (51) and (47) and the specific assumption about the distribution of the labor unit requirement,  $a$ , we obtain:

$$X_{ij}^M = w_i L_i \int_0^{\bar{a}_{ij}^M} \lambda_3 \theta_j^{\sigma-1} \left( \frac{Y_j}{Y} \right)^{\frac{1}{b}} (1 + \pi)^{\frac{1}{b}} (w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{(1-\sigma)} a^{1-\sigma} dG(a), \quad (59)$$

$$\text{with } (\bar{a}_{ij}^M)^{1-\sigma} = \lambda_4 \frac{w_j f_{ij}^M - w_j f_{ij}^X}{(w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \theta_j^{1-\sigma} \left( \frac{Y}{Y_j} \right)^{\frac{1}{b}} (1 + \pi)^{-\frac{1}{b}}. \quad (60)$$

Then solving the integral we get (13). ■

Notice that if both the intermediates are produced at home,  $\eta = 1$ , there will be no firm supplying via multinational production because the cost will be prohibitive (trade costs plus greater fixed cost,  $f_{ij}^M > f_{ij}^X$ ). Thus every firm will end up being an exporter, since it is more profitable. In this case the only gravity equation will be for export sales, as in Chaney (2008):

$$X_{ij}^X = \beta \frac{Y_i Y_j}{Y} \theta_j^{b(\sigma-1)} (f_{ij}^X)^{1-b} (w_i \tau_{ij})^{-k}. \quad (61)$$

On the other side, when all the intermediates are produced in the foreign location,  $\eta = 0$ , we are back in the Helpman et al. (2004) framework. In this scenario, the gravity equations for export and affiliate sales are:

$$X_{ij}^X = \beta \frac{Y_i Y_j}{Y} \theta_j^{b(\sigma-1)} (w_i \tau_{ij})^{1-\sigma} \left[ \left( \frac{f_{ij}^X}{(w_i \tau_{ij})^{1-\sigma}} \right)^{1-b} - \left( \frac{f_{ij}^M - f_{ij}^X}{w_j^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \right)^{1-b} \right], \quad (62)$$

$$X_{ij}^M = \beta \frac{Y_i Y_j}{Y} \theta_j^{b(\sigma-1)} w_j^{1-\sigma} \left( \frac{f_{ij}^M - f_{ij}^X}{w_j^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \right)^{1-b}. \quad (63)$$

In this Helpman et al. (2004) set up there is no role for complementarity between trade and multinational production.



## F Intensive and Extensive Margin Elasticities

In what follows we derive in details the intensive and extensive margins for affiliate and export sales.

### F.1 Intensive and Extensive Margins of Affiliate Sales

1) Rearranging the definition of intensive and extensive margins of affiliate sales we get

$$-\frac{\partial X_{ij}^M}{\partial \tau_{ij}} \frac{\tau_{ij}}{X_{ij}^M} = \underbrace{-\frac{\tau_{ij}}{X_{ij}^M} \left( w_i L_i \int_0^{\bar{a}_{ij}^M} \frac{\partial x_{ij}^M}{\partial \tau_{ij}} dG(a) \right)}_{\text{Intensive Margin Elasticity}} \underbrace{-\frac{\tau_{ij}}{X_{ij}^M} \left( w_i L_i x_{ij}^M G'(\bar{a}_{ij}^M) \frac{\partial \bar{a}_{ij}^M}{\partial \tau_{ij}} \right)}_{\text{Extensive Margin Elasticity}}. \quad (64)$$

Using the definition of equilibrium individual affiliate sales, (51), and assuming that country  $i$  is small enough so that  $\partial \theta_j^{\sigma-1} / \partial \tau_{ij} \approx 0$ , we get:

$$\begin{aligned} \frac{\partial x_{ij}^M}{\partial \tau_{ij}} &= \eta(1-\sigma) \tau_{ij}^{\eta(1-\sigma)-1} \left( w_j^{1-\eta} (w_i d_{ij})^\eta \right)^{1-\sigma} \lambda_3 \theta_j^{\sigma-1} \left( \frac{Y_j}{Y} \right)^{\frac{1}{b}} (1+\pi)^{\frac{1}{b}} a^{1-\sigma} \\ &= \eta(1-\sigma) \frac{x_{ij}^M}{\tau_{ij}}. \end{aligned} \quad (65)$$

Therefore, the elasticity of the intensive margin of affiliate sales with respect to the variable costs is:

$$\begin{aligned} \varepsilon_{I, \tau_{ij}}^M &= -\frac{\tau_{ij}}{X_{ij}^M} \left( w_i L_i \int_0^{\bar{a}_{ij}^M} \frac{\partial x_{ij}^M}{\partial \tau_{ij}} dG(a) \right) \\ &= -\eta(1-\sigma) \frac{\tau_{ij}}{X_{ij}^M} \frac{w_i L_i \int_0^{\bar{a}_{ij}^M} x_{ij}^M dG(a)}{\tau_{ij}} \\ &= \eta(\sigma-1). \end{aligned} \quad (66)$$

2) Using the definition of the equilibrium productivity threshold from (49), we find:

$$\begin{aligned} \frac{\partial \bar{a}_{ij}^M}{\partial \tau_{ij}} &= -\bar{a}_{ij}^M \frac{\left( \eta \frac{(w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma}}{\tau_{ij}} - \frac{(w_i \tau_{ij})^{1-\sigma}}{\tau_{ij}} \right)}{\left( w_j^{1-\eta} (w_i \tau_{ij})^\eta \right)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \\ &= -\frac{\bar{a}_{ij}^M}{\tau_{ij}} \frac{\left( \eta \left( w_j^{1-\eta} (w_i \tau_{ij})^\eta \right)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma} \right)}{\underbrace{\left( w_j^{1-\eta} (w_i \tau_{ij})^\eta \right)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}}_{=\Gamma}}. \end{aligned} \quad (67)$$

The sign of this derivative is ambiguous. It is positive for low level of  $\tau$ , but than when  $\tau$  increases it becomes negative. If the elasticity of substitution is high, the ambiguity is preserved only if  $\tau$  or/and  $\eta$  are sufficiently low.

We now rewrite the equation for firm level affiliate sales in (51), as

$$x_{ij}^M = \lambda_{ij}^M a^{1-\sigma}. \quad (68)$$

Then, since the Pareto distribution assumption implies that  $G'(a) = ka^{k-1}$ , the aggregate affiliate sales equation becomes:

$$\begin{aligned} X_{ij}^M &= w_i L_i \int_0^{\bar{a}_{ij}^M} x_{ij}^M dG(a) \\ &= w_i L_i \int_0^{\bar{a}_{ij}^M} \lambda_{ij}^M a^{1-\sigma} k a^{k-1} da \\ &= w_i L_i \lambda_{ij}^M (\bar{a}_{ij}^M)^{1-\sigma} (\bar{a}_{ij}^M)^k (k/(k-\sigma+1)) \\ &= w_i L_i x_{ij}^M G'(\bar{a}_{ij}^M) \frac{\bar{a}_{ij}^M}{k-\sigma+1}, \end{aligned} \quad (69)$$

where we used the fact that  $\bar{a}_{ij}^M G'(\bar{a}_{ij}^M) = k(\bar{a}_{ij}^M)^k$ . Using equation (69), we can find a solution for the elasticity of the extensive margin:

$$\begin{aligned} \varepsilon_{E,\tau_{ij}}^M &= -\frac{\tau_{ij}}{X_{ij}^M} \left( w_i L_i x_{ij}^M G'(\bar{a}_{ij}^M) \frac{\partial \bar{a}_{ij}^M}{\partial \tau_{ij}} \right) \\ &= -\frac{\tau_{ij}}{X_{ij}^M} w_i L_i x_{ij}^M G'(\bar{a}_{ij}^M) \left( -\bar{a}_{ij}^M \frac{\left( \frac{(w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma}}{\tau_{ij}} - \frac{(w_i \tau_{ij})^{1-\sigma}}{\tau_{ij}} \right)}{(w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \right) \\ &= -\frac{\tau_{ij}}{X_{ij}^M} \frac{X_{ij}^M}{\tau_{ij}} (k-\sigma+1) \frac{-\left( \eta (w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma} \right)}{(w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \\ &= (k-\sigma+1) \frac{\left( \eta (w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma} \right)}{(w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}}. \end{aligned} \quad (70)$$

## F.2 Intensive and Extensive Margins of Export Sales

1) Rearranging the definition of intensive and extensive margins of exports we get

$$-\frac{\partial X_{ij}^X}{\partial \tau_{ij}} \frac{\tau_{ij}}{X_{ij}^X} = \underbrace{-\frac{\tau_{ij}}{X_{ij}^X} \left( w_i L_i \int_{\bar{a}_{ij}^M}^{\bar{a}_{ij}} \frac{\partial x_{ij}^X}{\partial \tau_{ij}} dG(a) \right)}_{\text{Intensive Margin Elasticity}} \underbrace{-\frac{\tau_{ij}}{X_{ij}^X} w_i L_i \left[ x_{ij}^X G'(\bar{a}_{ij}) \frac{\partial \bar{a}_{ij}}{\partial \tau_{ij}} - x_{ij}^M G'(\bar{a}_{ij}^M) \frac{\partial \bar{a}_{ij}^M}{\partial \tau_{ij}} \right]}_{\text{Extensive Margin Elasticity}}. \quad (71)$$

Using the definition of equilibrium individual affiliate sales, (50), and assuming that country  $i$  is small enough so that  $\partial \theta_j^{\sigma-1} / \partial \tau_{ij} \approx 0$ , we get:

$$\begin{aligned} \frac{\partial x_{ij}^X}{\partial \tau_{ij}} &= (1 - \sigma) \tau_{ij}^{-\sigma} (w_i)^{1-\sigma} \lambda_3 \theta_j^{\sigma-1} \left( \frac{Y_j}{Y} \right)^{\frac{1}{b}} (1 + \pi)^{\frac{1}{b}} a^{1-\sigma} \\ &= (1 - \sigma) \frac{x_{ij}^X}{\tau_{ij}}. \end{aligned} \quad (72)$$

Therefore, the elasticity of the intensive margin of export with respect to the variable costs is:

$$\begin{aligned} \varepsilon_{I, \tau_{ij}}^X &= -\frac{\tau_{ij}}{X_{ij}^X} \left( w_i L_i \int_{\bar{a}_{ij}^M}^{\bar{a}_{ij}} \frac{\partial x_{ij}^X}{\partial \tau_{ij}} dG(a) \right) \\ &= -(1 - \sigma) \frac{\tau_{ij}}{X_{ij}^M} \frac{w_i L_i \int_{\bar{a}_{ij}^M}^{\bar{a}_{ij}} x_{ij}^X dG(a)}{\tau_{ij}} \\ &= (\sigma - 1), \end{aligned} \quad (73)$$

which is identical to the elasticity in Chaney (2008).

2) In order to derive the extensive margin of trade we need to use the equilibrium productivity thresholds from (48) and (49). Deriving these thresholds with respect to  $\tau_{ij}$  we find:

$$\frac{\partial \bar{a}_{ij}^M}{\partial \tau_{ij}} = -\bar{a}_{ij}^M \Gamma / \tau_{ij} \quad (74)$$

$$\frac{\partial \bar{a}_{ij}}{\partial \tau_{ij}} = -\frac{\bar{a}_{ij}}{\tau_{ij}}. \quad (75)$$

Rewriting the equation for firm level exports in (50), as

$$x_{ij}^X = \lambda_{ij}^X a^{1-\sigma} \quad (76)$$

allows us to find a connection between the  $\lambda_{ij}^M$  in firm affiliate sales, (68), and  $\lambda_{ij}^X$  in

export sales, (76). This implies that firm level affiliate sales can be rewritten as,

$$x_{ij}^M = \lambda_{ij}^X \underbrace{\frac{(w_j^{1-\eta})^{1-\sigma}}{((w_i \tau_{ij})^{1-\eta})^{1-\sigma}}}_{=\lambda_{ij}^M} (\bar{a}_{ij}^M)^{1-\sigma}. \quad (77)$$

Then since the Pareto distribution assumption implies that  $G'(\bar{a}_{ij}) = k(\bar{a}_{ij})^{k-1}$ , we can rewrite the aggregate export sales in the following way:

$$\begin{aligned} X_{ij}^X &= w_i L_i \int_{\bar{a}_{ij}^M}^{\bar{a}_{ij}} x_{ij}^X dG(a) \\ &= w_i L_i \int_{\bar{a}_{ij}^M}^{\bar{a}_{ij}} \lambda_{ij}^X a^{1-\sigma} k a^{k-1} da \\ &= w_i L_i \lambda_{ij}^X (k/(k-\sigma+1)) \left[ \lambda_{ij}^X \bar{a}_{ij}^{1-\sigma} - \lambda_{ij}^X (\bar{a}_{ij}^M)^{1-\sigma} (\bar{a}_{ij}^M)^k \right]. \end{aligned} \quad (78)$$

Using the relationship between  $\lambda_{ij}^X$  and  $\lambda_{ij}^M$  highlighted in equation (77), we can modify part of the equation above as

$$\lambda_{ij}^X (\bar{a}_{ij}^M)^{1-\sigma} = x_{ij}^M \left[ ((w_i \tau_{ij})^{1-\eta})^{1-\sigma} / (w_j^{1-\eta})^{1-\sigma} \right]$$

which gives

$$\begin{aligned} X_{ij}^X &= w_i L_i (1/(k-\sigma+1)) \\ &\quad \times \left[ x_{ij}^X G'(\bar{a}_{ij}) \bar{a}_{ij} - x_{ij}^M \underbrace{\left[ ((w_i \tau_{ij})^{1-\eta})^{1-\sigma} / (w_j^{1-\eta})^{1-\sigma} \right]}_{=\omega} G'(\bar{a}_{ij}^M) \bar{a}_{ij}^M \right] \\ &= w_i L_i (1/(k-\sigma+1)) x_{ij}^X G'(\bar{a}_{ij}) \bar{a}_{ij} \\ &\quad - w_i L_i (\omega/(k-\sigma+1)) x_{ij}^M G'(\bar{a}_{ij}^M) \bar{a}_{ij}^M. \end{aligned} \quad (79)$$

From equation (79) we can find the solution for the elasticity of the extensive margin of export:

$$\begin{aligned} \varepsilon_{E, \tau_{ij}}^X &= -\frac{\tau_{ij}}{X_{ij}^X} w_i L_i \left[ x_{ij}^X G'(\bar{a}_{ij}) \frac{\partial \bar{a}_{ij}}{\partial \tau_{ij}} - x_{ij}^M G'(\bar{a}_{ij}^M) \frac{\partial \bar{a}_{ij}^M}{\partial \tau_{ij}} \right] \\ &= -\frac{\tau_{ij}}{X_{ij}^X} w_i L_i \left[ x_{ij}^X G'(\bar{a}_{ij}) \left( -\frac{\bar{a}_{ij}}{\tau_{ij}} \right) - x_{ij}^M G'(\bar{a}_{ij}^M) \frac{\Gamma}{\tau_{ij}} \right]. \end{aligned} \quad (80)$$

Rewriting equation (69) to get:

$$w_i L_i x_{ij}^M G'(\bar{a}_{ij}^M) \bar{a}_{ij}^M = (k - \sigma + 1) X_{ij}^M, \quad (81)$$

and equation (79) to obtain:

$$\begin{aligned} w_i L_i x_{ij}^X G'(\bar{a}_{ij}) \bar{a}_{ij} &= (k - \sigma + 1) [X_{ij}^X + w_i L_i (\omega / (k - \sigma + 1)) x_{ij}^M G'(\bar{a}_{ij}^M) \bar{a}_{ij}^M] \\ &= (k - \sigma + 1) [X_{ij}^X + \omega X_{ij}^M]. \end{aligned} \quad (82)$$

The expressions in (81) and (82) can now be plugged in equation (80), to find a more compact expression for  $\varepsilon_{E,\tau_{ij}}^X$ . This yields:

$$\begin{aligned} \varepsilon_{E,\tau_{ij}}^X &= -\frac{\tau_{ij}}{X_{ij}^X} \left[ (k - \sigma + 1) [X_{ij}^X + \omega X_{ij}^M] \left( -\frac{1}{\tau_{ij}} \right) - (k - \sigma + 1) X_{ij}^M \left( -\frac{\Gamma}{\tau_{ij}} \right) \right] \\ &= -\frac{\tau_{ij}}{X_{ij}^X} (k - \sigma + 1) \frac{1}{\tau_{ij}} [- (X_{ij}^X + \omega X_{ij}^M) + X_{ij}^M \Gamma] \\ &= -\frac{1}{X_{ij}^X} (k - \sigma + 1) [X_{ij}^M (\Gamma - \omega) - X_{ij}^X] \\ &= - (k - \sigma + 1) \left[ \frac{X_{ij}^M}{X_{ij}^X} (\Gamma - \omega) - 1 \right], \end{aligned} \quad (83)$$

where

$$\Gamma = \frac{\left( \eta \frac{(w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma}}{\tau_{ij}} - \frac{(w_i \tau_{ij})^{1-\sigma}}{\tau_{ij}} \right)}{(w_j^{1-\eta} (w_i \tau_{ij})^\eta)^{1-\sigma} - (w_i \tau_{ij})^{1-\sigma}} \quad (84)$$

$$\omega = \left[ ((w_i \tau_{ij})^{1-\eta})^{1-\sigma} / (w_j^{1-\eta})^{1-\sigma} \right], \quad (85)$$

Notice that  $\Gamma > \omega$  is true for certain parameter restrictions consistent with our calibration exercise.

We can conclude that

$$\text{if } X_{ij}^M > X_{ij}^X \longrightarrow \varepsilon_{E,\tau_{ij}}^X < 0, \quad (86)$$

$$\text{if } X_{ij}^M < X_{ij}^X \longrightarrow \varepsilon_{E,\tau_{ij}}^X > 0. \quad (87)$$

## G Welfare

In this section, we show how to derive equations (20)-(22). From the profit of the threshold exporting firm we retrieve the value of export

$$\frac{1}{\sigma} \left( \frac{\sigma}{\sigma-1} \right)^{(1-\sigma)} Y_j (w_i a_{ij} \tau_{ij})^{1-\sigma} / P_j^{1-\sigma} = w_j f_{ij}^X. \quad (88)$$

Rearranging (88) we get

$$\left( \frac{\sigma}{\sigma-1} w_i \tau_{ij} \right)^{(1-\sigma)} = \frac{w_j f_{ij}^X \sigma P_j^{1-\sigma}}{a_{ij}^{1-\sigma} Y_j}. \quad (89)$$

Total export sales are

$$\begin{aligned} p_{ij} q_{ij} &= \frac{Y_j p_{ij}}{P_j^{1-\sigma}} \\ &= Y_j a^{1-\sigma} \left( \frac{\sigma}{\sigma-1} w_i \tau_{ij} \right)^{(1-\sigma)} P_j^{\sigma-1} \\ &= \left( \frac{a}{a_{ij}} \right)^{1-\sigma} w_j \sigma f_{ij}^X. \end{aligned} \quad (90)$$

From this last equation, we can compute average export sales:

$$\begin{aligned} (\overline{p_{ij} q_{ij}})^X &= \int_{a_{ij}^M}^{a_{ij}} \left( \frac{a}{a_{ij}} \right)^{1-\sigma} w_j \sigma f_{ij}^X \frac{g(a)}{G(a_{ij}) - G(a_{ij}^M)} da \\ &= \int_{a_{ij}^M}^{a_{ij}} \frac{a^{k-\sigma}}{a_{ij}^{1-\sigma} [a_{ij}^k - (a_{ij}^M)^k]} w_j \sigma f_{ij}^X da \\ &= \left( \frac{k\sigma}{k-\sigma+1} \right) \left( \frac{a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{k-\sigma+1}}{a_{ij}^{1-\sigma} [a_{ij}^k - (a_{ij}^M)^k]} \right) w_j f_{ij}^X. \end{aligned} \quad (91)$$

Following a similar procedure we can obtain average affiliate

$$\begin{aligned} (\overline{p_{ij} q_{ij}})^M &= \int_0^{a_{ij}^M} \left( \frac{a}{a_{ij}^M} \right)^{1-\sigma} w_j \sigma f_{ij}^M \frac{g(a)}{G(a_{ij}^M)} da \\ &= \left( \frac{k\sigma}{k-\sigma+1} \right) w_j f_{ij}^M, \end{aligned} \quad (92)$$

and domestic sales

$$\begin{aligned} (\overline{p_{jj} q_{jj}})^D &= \int_0^{a_{jj}} \left( \frac{a}{a_{jj}} \right)^{1-\sigma} w_j \sigma f_{jj} \frac{g(a)}{G(a_{jj})} da \\ &= \left( \frac{k\sigma}{k-\sigma+1} \right) w_j f_{jj}. \end{aligned} \quad (93)$$

We analyze the behavior of the average export sales, and show that: (i) the ratio including the difference in the cutoffs is lower than 1; and, (ii) average sales decline with a reduction in average productivity of exporters, while it increases with a reduction in average productivity of multinationals.

**Proposition 5**

$$\left( \frac{a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{k-\sigma+1}}{a_{ij}^{1-\sigma} [a_{ij}^k - (a_{ij}^M)^k]} \right) < 1$$

*i.e., average export sales are smaller than in models with only exporting firms.*

**Proof.**

$$\begin{aligned} a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{k-\sigma+1} &< a_{ij}^{1-\sigma} [a_{ij}^k - (a_{ij}^M)^k] \\ a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{k-\sigma+1} &< a_{ij}^{k-\sigma+1} - a_{ij}^{1-\sigma} (a_{ij}^M)^k \\ (a_{ij}^M)^{k-\sigma+1} &> a_{ij}^{1-\sigma} (a_{ij}^M)^k \\ (a_{ij}^M)^{1-\sigma} &> a_{ij}^{1-\sigma} \end{aligned} \tag{94}$$

which is true since  $a^M < a_{ij}$  and  $\sigma > 1$ . ■

**Proposition 6** *Average export sales are decreasing (increasing) in average productivity of exporters (multinationals).*

**Proof.** We start by taking the derivative

$$\frac{\partial (\overline{p_{ij} q_{ij}})^X}{\partial a_{ij}} = \frac{(k-\sigma+1)(a_{ij}^M)^{k-\sigma} (a_{ij}^{1-\sigma} [a_{ij}^k - (a_{ij}^M)^k]) - (a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{k-\sigma+1}) [(k-\sigma+1)a_{ij}^{k-\sigma} - (1-\sigma)a_{ij}^{-\sigma} (a_{ij}^M)^k]}{(a_{ij}^{1-\sigma} [a_{ij}^k - (a_{ij}^M)^k])^2}.$$

Consider the sign of the numerator:

$$\begin{aligned} (k-\sigma+1)(a_{ij}^M)^{k-\sigma} (a_{ij}^{1-\sigma} [a_{ij}^k - (a_{ij}^M)^k]) &< (a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{k-\sigma+1}) \\ &\quad \times [(k-\sigma+1)a_{ij}^{k-\sigma} - (1-\sigma)a_{ij}^{-\sigma} (a_{ij}^M)^k] \\ \frac{(a_{ij}^M)^{k-\sigma+1} - a_{ij}^{1-\sigma} (a_{ij}^M)^k}{a_{ij}^{k-\sigma+1} - (a_{ij}^M)^{k-\sigma+1}} &< 1 + \frac{(\sigma-1)a_{ij}^\sigma (a_{ij}^M)^k}{a_{ij}^{(k-\sigma+1)(k-\sigma)}} \\ \frac{1 - (a_{ij}^M/a_{ij})^k}{1 - (a_{ij}^M/a_{ij})^{k-\sigma+1}} &< 1 + \frac{\sigma-1}{k-\sigma+1} \frac{(a_{ij}^M)^k}{a_{ij}^k} \\ \underbrace{\frac{1}{1 - (a_{ij}^M/a_{ij})^{k-\sigma+1}}}_{> 0 \text{ and } < 1} - \underbrace{\frac{(a_{ij}^M/a_{ij})^k}{1 - (a_{ij}^M/a_{ij})^{k-\sigma+1}}}_{> 0 \text{ and } < 1} &< 1 + \underbrace{\left(\frac{a_{ij}^M}{a_{ij}}\right)^k}_{< 1} \underbrace{\frac{\sigma-1}{k-\sigma+1}}_{< 1}. \end{aligned}$$

As

$$\frac{1}{1 - (a_{ij}^M/a_{ij})^{k-\sigma+1}} < \frac{(a_{ij}^M/a_{ij})^k}{1 - (a_{ij}^M/a_{ij})^{k-\sigma+1}},$$

we can conclude that  $\partial(\overline{p_{ij}q_{ij}})^X/\partial a_{ij} < 0$ . ■



## References

- Alfaro, L., Charlton, A., 2007. Intra-Industry Foreign Direct Investment. CEP Discussion Paper No 825.
- Anderson, J.E., van Wincoop, E., 2003. Gravity with Gravitas: A Solution to the Border Puzzle. *American Economic Review*, 93(1), March, 170–92.
- Arkolakis, C., 2010. Market Penetration Costs and the New Consumers Margin in International Trade. *Journal of Political Economy*, 118(6), 1151–1199.
- Arkolakis, C., Demidova, S., Klenow, P.J., Rodriguez-Clare, A., 2008. Endogenous Variety and the Gains from Trade. *American Economic Review*, 98(2), May, 444–50.
- Arkolakis, C., Costinot, A., Rodriguez-Clare, A., 2012. New Trade Models, Same Old Gains?. *American Economic Review*, 102(1), 94–130.
- Atalay, E., Hortaçsu, A., Syverson, C., 2014. Vertical Integration and Input Flows. Forthcoming: *American Economic Review*.
- Bernard, A., Jensen, B., Redding, S.J., Schott, P.K., 2007. Firms in International Trade. *Journal of Economic Perspectives*, 21(3), 105–130.
- Bernard, A., Jensen, B., Schott, P.K., 2009. Importers, Exporters, and Multinationals: A Portrait of Firms in the U.S. that Trade Goods, in: Dunne, T., Jensen, M.J., Roberts, M.J. (Eds.), *Producer Dynamics: New Evidence from Micro Data*. University of Chicago Press.
- Bombarda, 2007. The Spatial Pattern of FDI: Some Testable Hypotheses. IHEID Working Papers 24-2007, Economics Section, The Graduate Institute of International Studies.
- Brainard, S.L., 1997. An Empirical Assessment of the Proximity-Concentration Trade-off between Multinational Sales and Trade. *American Economic Review* 87(4), 520–44.
- Broda, C., Weinstein, D.E., 2006. Globalization and the Gains From Variety. *Quarterly Economic Journal*, 121(2), 541–85.
- Chaney, T., 2008. Distorted Gravity: The Intensive and Extensive Margins of International Trade. *American Economic Review*, 98(4), 1707–21.

- Corcos, G., Irac, D.M., Mion, G., Verdier, T., 2012. The Determinants of Intra-Firm Trade. *Review of Economics and Statistics*, forthcoming.
- Eaton, J., Kortum, S., 2002. Technology, Geography, and Trade. *Econometrica*, 70(5), 1741–79.
- Eaton, J., Kortum, S., Kramarz, F., 2011. An Anatomy of International Trade: Evidence from French Firms. *Econometrica*, 79(5), 1453–98.
- Edmond, C., Midrigan, V., Yi Xu, D., 2015. Competition, Markups, and the Gains from International Trade. *American Economic Review*, 105(10), 3183–3221.
- Ekholm, K., Markusen, J., Forslid, R., 2005. Export-Platform Foreign Direct Investment. *Journal of European Economic Association*, 5(4), 31–50.
- Feenstra, R.C., 2013. Restoring the Product Variety and Pro-competitive Gains from Trade with Heterogeneous Firms and Bounded Productivity. University of California, Davis, mimeograph.
- Garetto, S., 2013. Input Sourcing and Multinational Production. *The American Economic Journal: Macroeconomics*, 5(2), 118–151.
- Grossman, G.H., Helpman, E., Szeidi, A., 2006. Optimal Integration Strategies for the Multinational Firm. *Journal of International Economics*, 70(1), 216–238, September.
- Hanson, G.H., Mataloni, R.J., Slaughter, M.J., 2001. Expansion Strategies of U.S. Multinational Firms. .
- Hanson, G.H., Mataloni, R.J., Slaughter, M.J., 2004. Vertical Production Networks in Multinational Firms. .
- Head, K., Ries, J., 2004. Exporting and FDI as Alternative Strategies. *Oxford Review of Economic Policy*, 20(3), Autumn, 409–23.
- Helpman, E., Melitz, M.J., Yeaple, S.R., 2004. Exporting and FDI with Heterogeneous Firms. *American Economic Review*, 94(1), 300–16.
- Horstmann, I.J., Markusen, J.R., 1992. Endogenous market structures in international trade (natura facit saltum). *Journal of International Economics*, 32(1-2), 109–29.

- Irrarrazabal, A., Moxnes, A., Opmomolla, L.D., 2012. The Margins of Multinational Production and the Role of Intra-Firm Trade. *Journal of Political Economy*, forthcoming.
- Keller, W., Yeaple, S.R., 2012. Gravity in the Knowledge Economy. *American Economic Review*, forthcoming.
- Markusen, J.R., 1984. Multinationals, multi-plant economies, and the gains from trade. *Journal of International Economics*, 16(3-4), 206–26.
- Melitz, M.J., 2003. The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. *Econometrica*, 71(6), 1695–1725.
- Melitz, M.J., Redding, S.J., 2013. Firm Heterogeneity and Aggregate Welfare. NBER Working Paper No. 18919.
- OECD, 2000. Summary tables. [/content/data/data-00286-en](#).
- OECD, 2000. Summary tables. [/content/data/data-00571-en](#).
- Ramondo, N., 2014. A Quantitative Approach to Multinational Production. *Journal of International Economics*, forthcoming.
- Ramondo, N., Rappoport, V., Ruhl, K.J., 2014. Horizontal versus Vertical Foreign Direct Investment: Evidence from U.S. Multinationals. Working Paper.
- Ramondo, N., Rodriguez-Clare, A., 2013. Trade, Multinational Production, and the Gains from Openness. *Journal of Political Economy*, 121(2), 273–322.
- Ramondo, N., Rappoport, V., , 2016. Intrafirm Trade and Vertical Fragmentation in U.S. Multinational Corporations. *Journal of International Economics*, 98, 51–59.
- Yeaple, S.R., 2003. The Role of Skill Endowments in the Structure of U.S. Outward Foreign Direct Investment. *The Review of Economics and Statistics*, 85(3), 726–34.
- Yeaple, S.R., 2009. Firm Heterogeneity and the Structure of U.S. Multinational Activity: An Empirical Analysis. *Journal of International Economics*, 78(2), 206–15.