Solving Trigonometric Equations

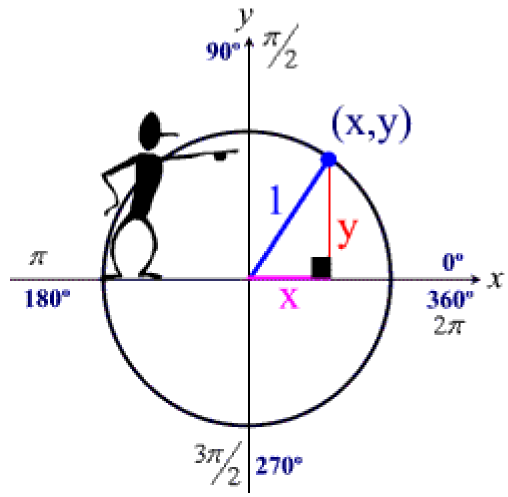
We have been doing Trig identities, reference angles and graphing lately. Today we will take a look at solving trigonometric equation.

Before starting anything, we need to review our reference angles

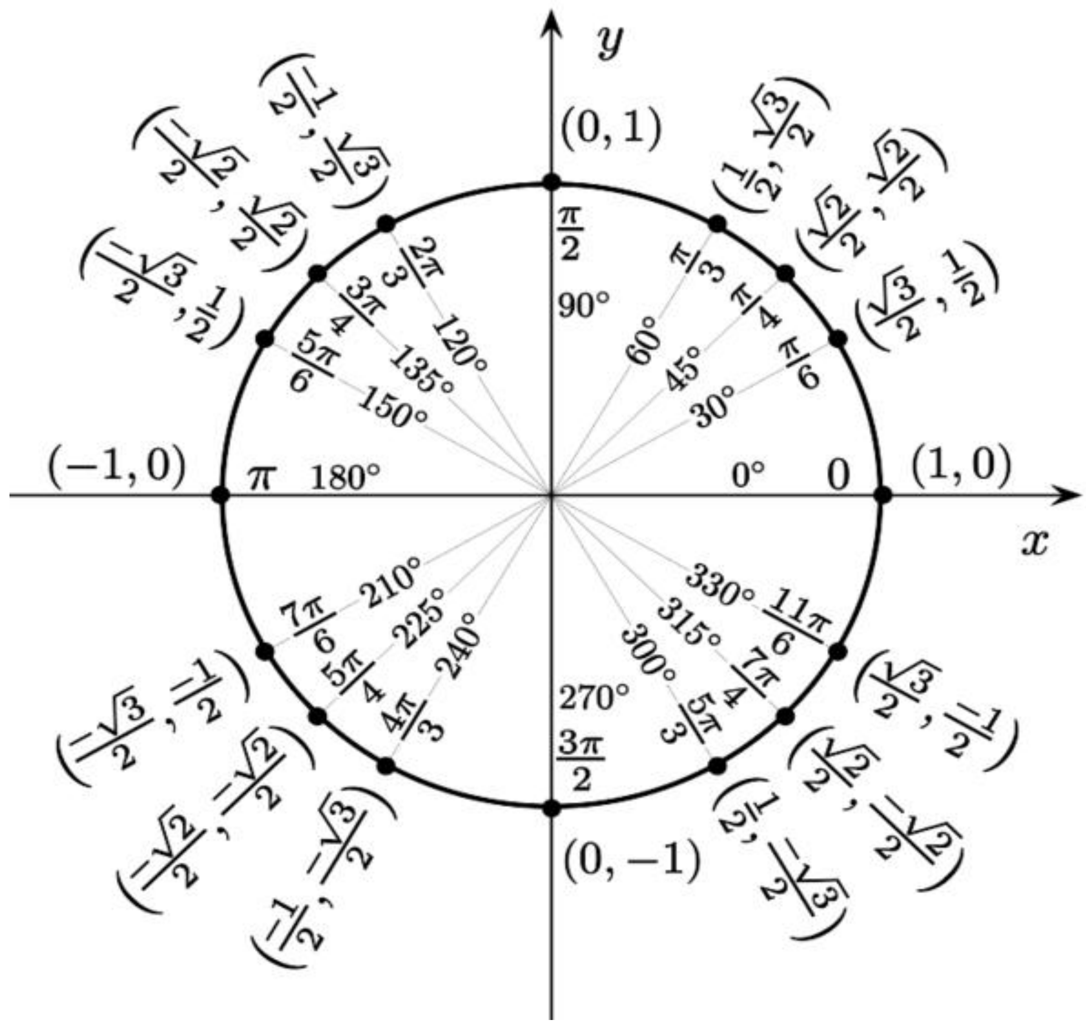
$$\sin\theta = \frac{opposite}{hypothenuse} = SOH$$

$$\cos\theta = \frac{adjacent}{hypothenuse} = CAH$$

$$\tan\theta = \frac{opposite}{adjacent} = TOA$$



This is the trigonometry unit circle, studying this chart will help a lot in solving trig.



The difficulties can be varied from problems to problems. Let's start with the easy one.

Example 1:

$2\cos x - 1 = 0$

$Step 1$: add 1 on both side

- $2\cos x - 1 = 0$

  $+1 = +1$

Step 2: divide 2 on both side

- $2\cos x = 1$
- $\cos x = \frac{1}{2}$

Step 3: take the arccosx (the inverse function of cosine) on both side

- $\arccos(\cos x) = \arccos\frac{1}{2}$

Step 4: cross out arccos(cos), leaving the x

- $x = \arccos\frac{1}{2}$

Step 5: What is the $\arccos\frac{1}{2}$? it's $60°$ based on the unit circle above

- $x = 60°$

$Example\ 2: 3\sec^2 x - 4 = 0$

$Step\ 1:$ Add 4 on both sides [always try to isolate X since that's what we're looking for]

$3\sec^2 x - 4 = 0$

$\qquad +4 = +4$

$3\sec^2 x = 4$

$Step\ 2:$ divide 3 on both sides

$\left(\frac{1}{3}\right)3\sec^2 x = 4\left(\frac{1}{3}\right)$

$\sec^2 x = \frac{4}{3}$

$Step\ 3:$ take the square root of 2 on both sides

$\sqrt{\sec^2 x} = \sqrt{\frac{4}{3}}$

$\sec x = \pm\frac{2}{\sqrt{3}}$

$Step\ 4:$ take the inverse of sec which is arcsec

$\text{arcsec}(\sec x) = \text{arcsec} \pm \frac{2}{\sqrt{3}}$

$$\left(\begin{array}{l}\text{bubble thought}: \sec = \frac{1}{\text{cosx}} = \pm\frac{2}{\sqrt{3}}, \cos x = \pm\frac{\sqrt{3}}{2} \text{ since we flip both side upside down.} \\ \arccos \pm\frac{\sqrt{3}}{2} \text{ is } 30° \textit{and} 150° \text{ based on the unit circle, this is why memorizing it is helpful}\end{array}\right)$$

FINAL ANSWER: $x = 30°, 150°$

$Example\,3: \tan x \sin^2 x = 2\tan x$

$Step\,1:$ *subtract* $2\tan x$ on both sides to set the equation equal to zero

$$\left(\begin{array}{l}\text{some of you might be asking the question } \textit{why} \text{ dont we divide tanx} \\ \text{, we will answer it later on}\end{array}\right)$$

$\tan x \sin^2 x = 2\tan x$

$-2\tan x \qquad -2\tan x$

$\tan x \sin^2 x - 2\tan x = 0$

$Step\,2:$ *Factor* out $\tan x$

$\tan x(\sin^2 x - 2) = 0$

$Step\,3:$ Set $\tan x = 0$ and $(\sin^2 x - 2) = 0$

$\tan x = 0 \qquad , \qquad (\sin^2 x - 2) = 0$

$Step\,4:$ Solve for x

$\arctan(\tan x) = \arctan\ 0$

$$\left(\begin{array}{l}\text{Bubble thought : tanx} = \dfrac{\text{sinx}}{\text{cosx}} \to 0 = \dfrac{\text{sinx}}{\text{cosx}} \to 0 = \dfrac{0}{1} \\ \sin \textit{ and } \cos \text{ might be at } 0^\circ \text{ or } 180^\circ \\ \text{since we go with the first answer, } 0^\circ \text{ is the correct answer} \\ \end{array}\right)$$

$x = 0^\circ$

$Step\,5:$ solve for $(\sin^2 x - 2) = 0$

$\sin^2 x - 2 = 0$

$\sin^2 x = 2$

$\sin x = \pm\sqrt{2}$

$\text{acrsin}(\sin x) = \arcsin \pm\sqrt{2}$

$x = \arcsin \pm\sqrt{2}$

since this is not our reference angles , we can' t solve unless we use calculator

you can write the final solution as

$x = \arcsin \pm\sqrt{2}$

so we have 3 answers for x : 0° and $\arcsin \pm\sqrt{2}$

- IMPORTANT: you <u>do not shorten equations by eliminating variables</u> when it comes to SOLVING. When you ELIMINATE the VARIABLE, you are ELIMINATING ANSWERS.

- When it's solving, you are trying to **ISOLATE X**

- When solve for x, the **solutions might be various** due to its reference angle. For example

  $$\sin 30° = \frac{1}{2}$$

  AND

  $\sin 150° = \frac{1}{2}$ Even though the angles are different but since they are reference angles, their solution is one-half.

- We normally **account for the first solution** if they do not ask **to list all the possible values for x within a certain interval**

- **General solution is more like a formula for infinite solution**

- **Arc** means the **inverse** of whatsoever comes after it.

- If you have double angle, you have to account for 2 angles, for example

  $$\sin 2\theta = 0$$

  $$2\theta = \arcsin 0$$

  $$2\theta = 0°, 180°$$

  $$divide \text{ by } 2$$

  $$\theta = 0°, 90°$$