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Magnetohydrodynamics Effects on Radiative and Dissipative Heat Transfer Near a Stagnation Point with Variable Viscosity and Thermal Conductivity

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ABSTRACT

The study of thermal radiative heat transfer over a continuously stretching sheet in the presence of a uniform magnetic field with dissipation near stagnation point flow on non-conducting permeable stretching sheet for power-law variation in the sheet temperature are investigated. The fluid flow viscosity and thermal conductivity are assumed to vary as a function of temperature. The governing partial differential equations of the model are reduced to a system of coupled non-linear ordinary differential equations by applying similarity variables and solved numerically using shooting with fourth-order Runge-Kutta method. The local similarity solutions for different values of the physical parameters are presented for velocity and temperature. The results for Skin friction and Nusselt numbers are presented and discussed.

Keywords: Stagnation point; Variable Viscosity; Thermal Conductivity; Radiation; Dissipation

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1. INTRODUCTION

For the past few years, the study of the magnetic field effects on the flow of a viscous, incompressible, laminar and electrically conducting fluid flow is important due to its industrial applications such as nuclear technology, polymer production, exchangers, coolers and many more. The variation in variable viscosity and thermal conductivity of an ambient fluid has been studied by large numbers of researchers due to their applications over a broad spectrum of science and engineering process. As a result of its numerous applications, Heat transfer on MHD viscous flow over a stretching sheet with prescribed heat flux was examined by Adhikari & Sanyal [1]. Laminar mixed convection boundary layers induced by a linearly stretching permeable surface was analyzed by Mohamed & Fahd [2]. It was found that the heat transfer coefficient increases as mixed convection parameter assisting the flow for all injection parameter for uniformly or linearly heated surface and as Prandtl number increases it becomes almost independent of mixed convection parameter. Srinivasacharya & Reddy [3] investigated chemical reaction and radiation effects on mixed convection heat and mass transfer over a vertical plate in power-law fluid saturated porous medium. The ordinary differential equations were obtained using similarity transformations and solved numerically with shooting method. Ahammad *et al.* [4] presented analysis of MHD free convection flow along a vertical porous plate embedded in a porous medium with magnetic field and heat generation while Lakshmi *et al.* [5] considered MHD boundary layer flow of heat and mass transfer over a moving vertical plate in a porous medium with suction and viscous dissipation. The above studies were carried out for fluid with constant viscosity and thermal conductivity without stagnation point.

It is known that fluid physical properties may change significantly with temperature. However, Modather *et al.* [6], Choudhury & Hazarika [7] studied the influences of temperature dependent viscosity and thermal conductivity on the unsteady flow and heat transfer fluid over a stretching sheet. Hazarika & Jadav [8] examined the effects of variable viscosity and thermal conductivity on magnetohydrodynamic forced convective boundary layer flow past a stretching/shrinking sheet prescribed with variable heat flux in the presence of heat source and constant suction. The effects of variable viscosity and thermal conductivity on MHD flow and heat transfer of a dusty fluid was investigated by Manjunatha & Gireesha [9]. Nasrin & Alim [10], Hazarika *et al.* [11] and Khaled [12] analyzed the effects of variable viscosity and thermal conductivity on MHD flow past a vertically moving plate with viscous and joule dissipation.

Santana & Hazarika [13] and Rahmana & Salahuddin [14] presented the effects of variable viscosity and thermal conductivity on MHD free convection and mass transfer flow over an inclined vertical surface. It was found that there are was significant difference in the results between variable Prandtl and constant Prandtl numbers when viscosity dependents on the temperature. Also, the results show that skin friction and Nusselt number are lower for the constant electric conductivity than those of the variable electric conductivity. Moreover, Ferdows *et al.* [15] considered double diffusion, slips and variable diffusivity effects on combined heat mass transfer with variable viscosity via a point transformation while mixed convection flow by a porous sheet with variable thermal conductivity and convective boundary condition was investigated by Hayat *et al.* [16]. But all the cited authors above did not consider variable viscosity and thermal conductivity effects near a stagnation point.

A point in a fluid flow where the flow has come to rest (i.e. speed is equal to zero adjacent to some solid body immersed in the fluid flow) is of special significance. It is of such importance that it is given a special name a stagnation point. Alireza *et al.* [17] verified the analytical solution for magnetohydrodynamic stagnation point flow and heat transfer over a permeable stretching sheet with chemical reaction while Ahmed & Nasser [18] presented lie group analysis for the effects of chemical reaction on MHD stagnationpoint flow of heat and mass transfer towards a heated porous stretching sheet with suction or injection. Salem & Fathy [19] examined the effects of variable properties on MHD heat and mass transfer flow near a stagnation point towards a stretching sheet in a porous medium with thermal radiation. Hunegnaw & Kishan [20] studied scaling group analysis on MHD effects on heat transfer near a stagnation point on a linearly stretching sheet with variable viscosity and thermal conductivity, viscous dissipation and heat source/sink. Also, Bazid *et al.* [21] reported on the soret and dufour effect on heat and mass transfer in stagnation point flow towards a stretching surface in the presence of buoyancy force and variable thermal conductivity.

Following from all the above cited authors, analysis were carried out without considering the effect of combine viscous dissipation, radiation and buoyancy force near a stagnation point. However, present study consider the effect of variable viscosity and thermal conductivity throughout the flow regime in the presence of buoyancy force, viscous dissipation and radiation near a stagnation point flow over a permeable stretching sheet with power-law variation in the sheet temperature..

2. MATHEMATICAL FORMULATION

A free convective heat transfer of two dimensional flow of an electrically conducting, incompressible fluid of variable viscosity and thermal conductivity in the vicinity of a stagnation point above a heated stretching sheet with radiation and dissipation under the influence of uniform magnetic field, buoyancy force and heat source/sink are considered. The flow is assumed to be in the *x*-direction with *y*- axis normal to it. The magnetic field of uniform strength B_0 is introduced in the direction of the flow. The stretching velocity and the free stream velocity are assumed to vary proportional to the distance *x* from the stagnation point, i.e. $u_w(x) = ax$ and $U(x) = bx$. The plate is maintained at the temperature T_w and free stream temperature T_∞ respectively. The governing equations of motion for the steady flow under the influence of externally imposed transverse magnetic field with variable thermal conductivity and variable viscosity in the boundary layer are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right) - \frac{1}{\rho}\sigma B_0^2(u-U) + g\beta_T(T-T_\infty) \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho C_p}\frac{\partial}{\partial y}\left(k(T)\frac{\partial T}{\partial y} - q_r\right) + \frac{\mu(T)}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{\rho C_p}Q_0(T-T_\infty) \qquad (3)$$

subject to the following boundary conditions:

$$u = u_w(= ax), v = -v_w, T = T_w(= T_\infty + Ax^m) \text{ at } y = 0 \qquad (4)$$

$$u = U(x)(= bx), T = T_\infty \text{ as } y \to \infty$$

Where
u, *v* and *T* are the velocity component in the *x* direction, velocity component in the *y* direction and temperature of the fluid respectively. B_0 is the magnetic field strength and v_w is the suction/injection.

The physical quantities μ, ρ, σ, C_p, k and Q_0 are the coefficient of viscosity, density, electric conductivity of the fluid, specific heat at constant pressure, thermal conductivity and rate of specific internal heat generation or absorption respectively. g is the gravitational acceleration, β_T is the thermal expansion coefficient, A, m, a and b are prescribed constants and q_r is the radiative heat flux respectively.

The viscosity is assumed to vary as a reciprocal of a linear function of temperature [22].

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \gamma(T - T_\infty)]$$
$$or \quad \frac{1}{\mu} = \alpha(T - T_r) \qquad (5)$$

where $\alpha = \frac{\gamma}{\mu_\infty}$ and $T_r = T_\infty - \frac{1}{\gamma}$

Both α and T_r are constants which depend on the reference state and the thermal property of the fluid, where $\alpha \succ 0$ for liquids and $\alpha \prec 0$ for gases.

The fluid thermal conductivity, k, is assumed to vary as a linear function of temperature in the form
[23] ,

$$k = k_\infty(1 + \epsilon\theta) \quad (6)$$

where $\epsilon = \frac{(k_w - k_\infty)}{k_\infty}$, is the thermal conductivity parameter.

Using Rosseland diffusion approximation for radiation [24] and [25]

$$q_r = -\frac{4\sigma_0}{3\delta}\frac{\partial T^4}{\partial y} \quad (7)$$

where σ_0 and δ are the Stefan-Boltzmann and the mean absorption coefficient respectively, assume the temperature difference within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature, using Taylor series to expand T^4 about the free stream T_∞ and neglecting higher order terms, this gives the approximation

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \qquad (8)$$

Applying equation (8), equation (7) can be express as follow.

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma_0 T_\infty^3}{3\delta}\frac{\partial^2 T}{\partial y^2} \qquad (9)$$

Using the stream function $\psi(x,y)$ with similarity transforms

$$\psi = (a\nu)^{\frac{1}{2}} x f(\eta), \eta = \left(\frac{a}{\nu}\right)^{\frac{1}{2}} y \qquad (10)$$

where the velocity components and temperature respectively become

$$u = \frac{\partial \psi}{\partial y} = axf'(\eta), v = -\frac{\partial \psi}{\partial x} = -(a\nu)^{\frac{1}{2}} f(\eta) \quad and \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \qquad (11)$$

Substitute equations (9)-(11) along with the variable viscosity and thermal conductivity, the continuity equation is satisfied and equations (2) and (3) becomes.

$$f''' - \frac{\theta-\phi}{\phi} f f'' - \frac{1}{\theta-\phi}\theta' f'' + \frac{\theta-\phi}{\phi} f'^2 - \frac{\theta-\phi}{\phi}\lambda^2 - \frac{\theta-\phi}{\phi} M(\lambda - f') - \frac{\theta-\phi}{\phi} G_r\theta = 0 \qquad (12)$$

$$(1+\frac{4}{3}R)(1+\epsilon\theta)\theta'' - P_r E_c \frac{\phi}{\theta-\phi}(f'')^2 + \epsilon(\theta')^2 + P_r f\theta' - P_r(mf' - Q)\theta$$

$= 0$ (13)

The corresponding boundary conditions are.

0
$f(0) = 1, f(0) = f_w, \theta(0) = 1$
0 (14)

$f(\infty) = \lambda, \theta(\infty) = 0$

where $\phi = \frac{T_r - T_\infty}{T_w - T_\infty a} = -\frac{1}{\gamma(T_w - T_\infty)}$ and $\frac{1}{\mu} = s(T - T_r)$ takes the form $\mu = \frac{\mu_\infty}{(1-\theta\phi^{-1})}$, $\lambda = \frac{b}{a}$ is the velocity ratio parameter, $Q = \frac{Q}{\rho C_p a}$ is the heat source/sink parameter, $M = \frac{\sigma B_0^2}{\rho a}$ is the magnetic parameter, $G_r = \frac{g\ \tau(T_w - T_\infty)}{a^2 x}$ is the Grashof number, $E_c = \frac{u_w^2}{C_p(T_w - T_\infty)}$ is the Eckert number, $R = \frac{4\sigma_0 T_\infty}{\delta k_\infty}$ is the radiation parameter, $P_r = \frac{\mu_\infty C_p}{k_\infty}$ is the Prandtl number and $f_w = -\frac{v_w}{\sqrt{a\nu}}$ is the suction parameter.

The biggest variation in the fluid viscosity from its free stream value μ_∞, happen at the surface of the plate when $\mu = \frac{\mu_\infty}{(1-\phi^{-1})}$ where φ is negative for liquids and positive for gases. From the expansion, as $-\varphi \to \infty$, $\mu \to \mu_\infty$, that is the viscosity difference in the boundary layer is negligible while as $-\varphi \to 0$ the viscosity variation accelerates significantly.

The physical parameters of interest for this flow are the local skin friction C_f and the Nusselt number N_u given respectively as:

$$C_f = \frac{\tau_w}{\rho_\infty u_w^2/2}, \quad Nu = \frac{x q_w}{k_\infty(T_w - T_\infty)} \qquad (15)$$

τ_w and q_w are respectively given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = k\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad (16)$$

Hence,

$$Re_x^{\frac{1}{2}} C_f = \frac{2\phi}{\phi - 1} f''(0) \qquad (17)$$

$$Nu_x Re_x^{-\frac{1}{2}} = (1+n)\theta'(0) \qquad (18)$$

Where
$Re_x = \frac{u_w x}{\nu}$ is the Reynolds number.

Table 1: Comparison of $\theta'(0)$ for $K_1 = 0,\ M = 0,\ \epsilon = 0,\ \phi \longrightarrow -\infty,\ E_c = 0,\ R = 0,\ G_r = 0,\ \lambda = 0$ for various values of m, Q and P_r

λ	m	P_r	Grubka and Bobba [26]	Ahmed [27]	Present results
0	0	0.7	-	-0.456052	-0.456049
0	0	1.0	-0.5820	-0.582229	-0.582225
0	0	10.0	-2.3080	-2.308000	-2.307950
-1.0	2	5.0	-	-4.028226	-4.028228

Table 2: Effect of $M,\ \lambda,\ Q,\ \phi,\ m$ and f_w on $f''(0)$ and $\theta'(0)$ (P-Parameters)

P	values	$f''(0)$	$\theta'(0)$	P	values	$f''(0)$	$\theta'(0)$
M	0.5	-0.28571	-1.04982	φ	1.2	-0.28571	-1.04982
	1.5	-0.37870	-1.01293		1.5	-0.51621	-1.02337
	3.0	-0.49870	-0.96825		2.0	-0.70968	-1.00142
	5.0	-0.63621	-0.92109		3.5	-0.92152	-0.97814
λ	15^0	-1.97298	-1.18584	m	0.5	-0.28522	-0.89303
	30^0	-2.99824	-0.73662		3.0	-0.28771	-1.54020
	45^0	-4.02276	-0.19831		5.0	-0.28918	-1.91415
	60^0	-4.84173	0.28576		7.0	-0.29020	-2.22658
Q	0.0	-0.28844	-1.24359	f_w	0.0	-0.19995	-0.64697
	0.5	-0.28571	-1.04982		0.5	-0.23918	-0.83287
	1.0	-0.28046	-0.78015		1.0	-0.28571	-1.04982
	1.5	-0.27255	-0.31202		2.0	-0.39638	-1.55110

3. RESULTS AND DISCUSSION

The set of non-linear differential equations along with the boundary conditions are solved numerically using shooting technique together with fourth-order Runge-Kutta integration algorithm. Confirmation is made to check that the smoothness conditions at the edge of the boundary layer are satisfied. Calculations are carried out for different values of the following default parameters: $M = Q = 0.5$, $R = 0.01$, $\lambda = 0.1$, $m = 1,\ \phi = 1.2,\ \epsilon = 0.1,\ E_c = 0.03,\ G_r = 1,\ P_r = 0.71$ and $f_w = 1$.

Table 1 represents the numerical results, which show the behavior of some physical parameters on the heat transfer aspects of the present results compared with the existing results. The comparison are found to be in an excellent agreement as shown in the tables.

Table 2 shows the numerical results, which depict the effect of physical parameters on the flow with heat transfer aspect of the study. The table shows that an increase in the values of parameters M, λ, φ, f_w and m decrease the skin friction but cause increase in the temperature gradient at the wall except m and f_w which cause decrease in the thermal boundary later thickness while Q causes increase in the skin friction but decreases the temperature temperature gradient at the wall.

Figure 1 shows the effect of magnetic field parameter M on the fluid flow. It is found that the rate of fluid transport is considerably retarded with increase in the values of M and make it warmer as it moves along the plate by causing decrease in the velocity profiles. Therefore, the transverse magnetic field opposes the flow phenomena due to the fact that Lorentz force produces more resistance to the transport phenomena.

The influence of thermal Grashof numbers G_r for heat transfer on the velocity profiles is showed in Figure 2. It is noticed that an increase in the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer increases the velocity of the flow field. Thus, heat transfer has a strong effect on the flow profiles.

Figure 3 exhibits the impact of suction/injection on the boundary layers. It is observed that the velocity profiles decrease monotonically, with an increase in the values of suction parameter, indicating the fact that suction stabilizes the boundary layer growth and hence, prevents boundary layer separation. Thus, sucking the decelerated fluid particles reduces the growth of the fluid boundary layer.

Figures 4 and 5 display the velocity and temperature profiles for different values of the heat generation parameter Q. It is noticed from figure 4 that the velocity profile is influenced considerably and increases as the values of heat generation parameter increases. It is evident from the figure 5 that as the values of heat generation parameter increase, the temperature profile increases due to an increase in the boundary layer thickness.

Figures 6 and 7 illustrate the Velocity and temperature distribution for variation in the values of velocity ratio parameter λ. The phenomenon of thinning boundary layer implies increased shear stress at the sheet. It is important to note for $\lambda = 1$, that there is no formation of boundary layer since the sheet velocity is equal to free stream velocity. It is found from figure 7 that the fluid temperature decreases due to increase in velocity ratio parameter λ.
The influence of viscosity parameter on the velocity and temperature profiles are depicted in figures 8 and 9. Increase in the value of φ decreases the velocity profile due to increase in the skin friction parameter while temperature profile increases as viscosity parameter increases because of thickness in the thermal boundary layer.

Figure 10 demonstrates the effect of thermal conductivity parameter on the temperature profiles. It is found that temperature increase with increasing values of which results to reduction in the rate of heat transfer from the flow to the surface and cause the boundary layer to generate energy which make the temperature to increase.

Figure 11 represents the temperature profiles with different values of Prandtl number. The prandtl number which is the ratio of momentum diffusivity to thermal diffusivity is justified by the fact that an increase in P_r causes decrease in temperature distributions as shown in the figure. This is due to the fact that the boundary layer gets thinner and causes fall in the average temperature within the boundary layer.

The influence of viscous dissipation parameter is illustrated in figure 12. Eckert number is the ratio of the kinetic energy of the flow to the boundary layer enthalpy difference. Kinetic energy is converted into internal energy by work done against the viscous fluid stresses. It is evident that an increase in Eckert number enhances the temperature at any point whereby causes the temperature profiles to increase. Figure 13 reports on the behavior of temperature profiles with variation in the values of m. It is observed that an increase in values m causes the temperature profiles to retard. Heat flows from the stretching sheet into the ambient environment when $m \succ 0$, otherwise it flows from the ambient environment to the stretching sheet. Overshoot in the temperature is exhibited because fluid particle moves from the heated wall temperature to where the wall temperature is otherwise.

The impact of radiation on the temperature profiles is illustrated in figure 14. It is found that as the value of R is increase, there is corresponding increase in the temperature distributions which results in an increase in the thermal boundary layer thickness.

4. CONCLUSION

Magnetohydrodynamic effects on radiative and dissipative heat transfer near a stagnation point with variable viscosity and thermal conductivity are studied. From the computation results, it is found that, an increase in the values of M, f_w and φ, reduced the motion of the fluid by causing decrease in the velocity distributions while increase in the values of G_r, λ and Q increase the velocity profiles. Increase in the values of Q, ϕ, ϵ, R and E_c increase the thermal boundary layer thickness which resulted in increase in the temperature distributions while variational increase in the values of λ, m and P_r decrease the thermal boundary layer and cause heat to diffuse away from the system and thereby decreasing the temperature distributions.

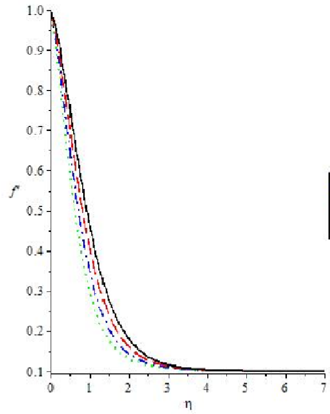

Figure 1 : Velocity Profiles for different values of *M*

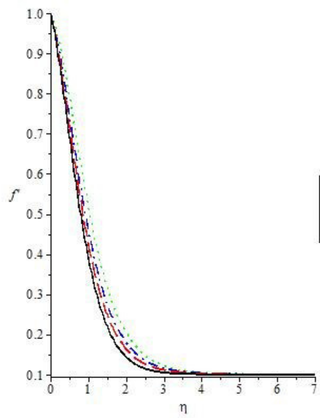

Figure 2 : Velocity Profiles for different values of G_r

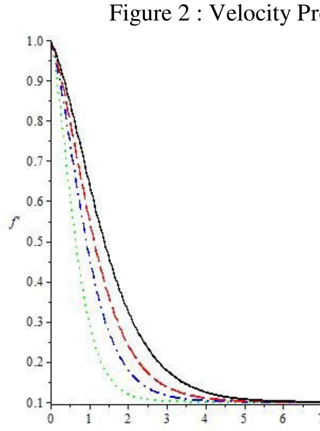

Figure 3 : Velocity Profiles for different values of f_w

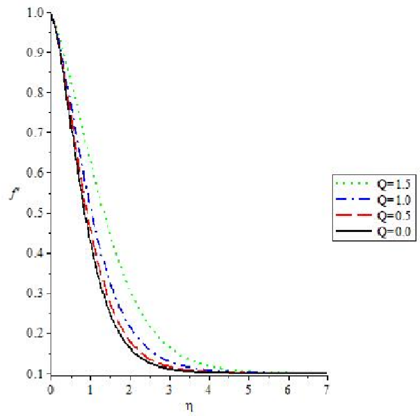

Figure 4 : Velocity Profiles for different values of *Q*

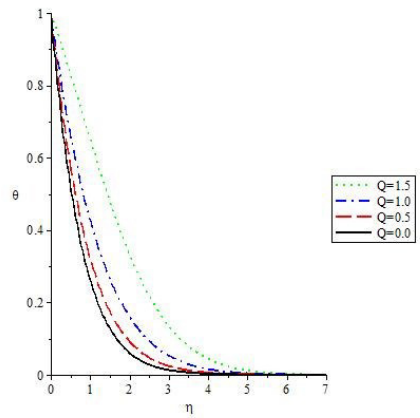

Figure 5 : Temperature Profiles for different values of *Q*

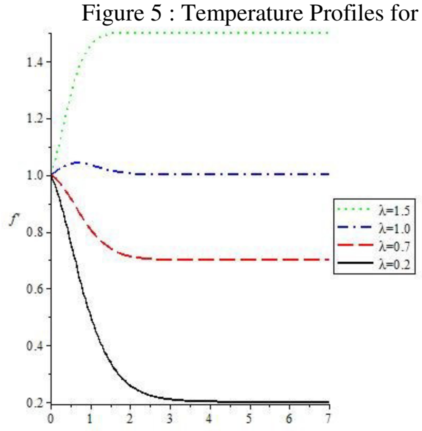

Figure 6 : Velocity Profiles for different values of *λ*

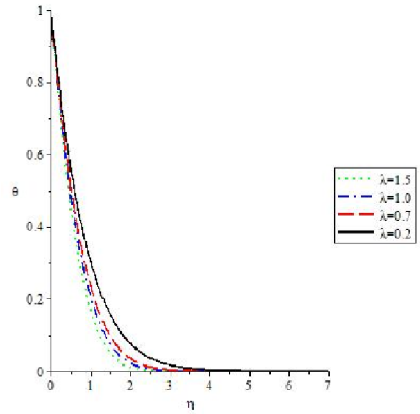

Figure 7 : Temperature Profiles for different values of λ

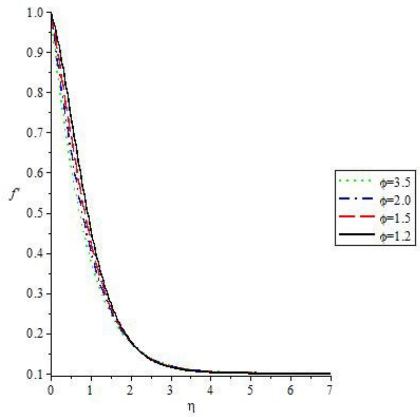

Figure 8 : Velocity Profiles for different values of φ

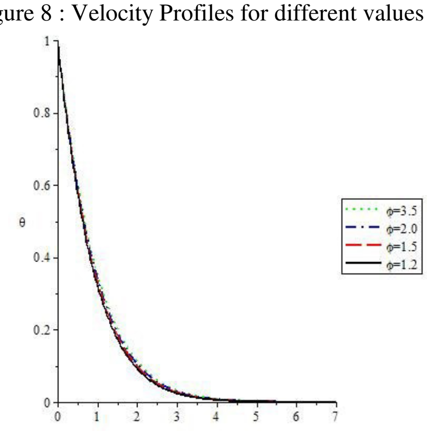

Figure 9 : Temperature Profiles for different values of φ

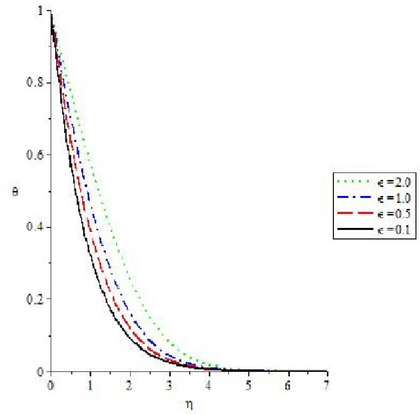

Figure 10 : Temperature Profiles for different values of

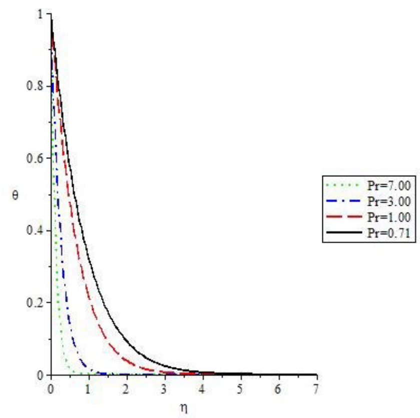

Figure 11 : Temperature Profiles for different values of P_r

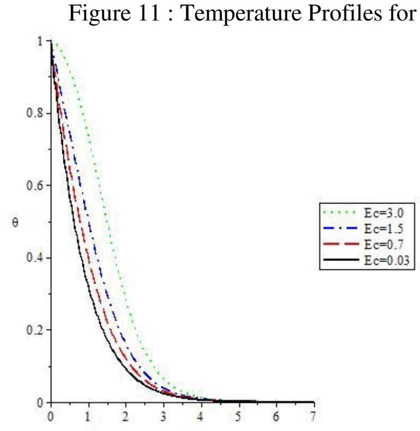

Figure 12 : Temperature Profiles for different values of E_c

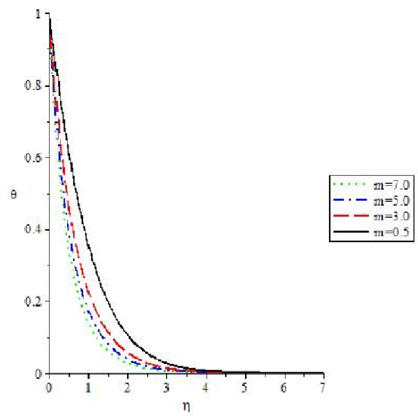

Figure 13 : Temperature Profiles for different values of m

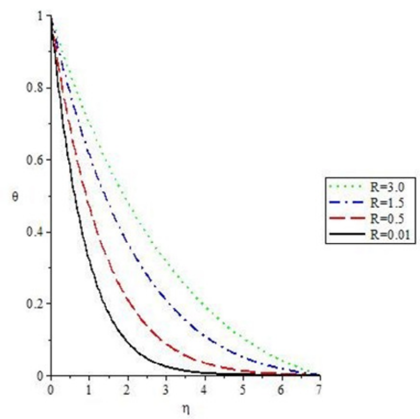

Figure 14 : Temperature Profiles for different values of R

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