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The Response of a Pre-Stressed Bernoulli- Euler Beam Carrying an Attached Mass to a Number of Concentrated Moving Loads

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ABSTRACT

The dynamic behaviour of a pre-stressed Bernoulli-Euler beam carrying an attached mass at one end while the other end is arbitrarily supported and which is under a moving concentrated masses is studied. The elastics properties of the beam and speeds of the masses are assumed constants. It is observed that for the moving mass problem, irrespective of whether the simply supported pre-stressed Bernoulli-Euler beam has an attached mass or not, the values of deflection increase as x increases. On the other hand, the value of the deflection decrease as x increases for moving force problems.

Keywords: Pre-stressed Beam, Lumped Mass, moving Loads

Nomenclature
G - Acceleration due to gravity,
μ - Constant mass per unit length of the beam,
E - Modulus of elasticity
I - moment of inertia,
EI -constant flexural stiffness,
η - Convective acceleration,
V(x,t) -lateral displacement,
$K_{nt}(x,t)$ -continuous moving force,
N- Pre-stressed constants
x - Position co-ordinate in the axial direction.

1. INTRODUCTION

For beams loaded with concentrated loads, the point of zero shears usually occurs under a concentrated load and so the maximum moment. Beams and girders such as in a bridge or an overhead crane are subject to moving concentrated loads, which are at fixed distance with each other. The problem here is to determine the moment under each load when each load is in a position to cause a maximum moment. The largest value of these moments governs the design of the beam. The real problem of explaining the vibrational behaviour of Railway and highway bridges traversed by train and vehicles is of technological importance and being studied in several fields of applied Mathematics, Engineering and applied Physics [1-12]. Such problems may be tackled by modelling the system (the bridge and the vehicles moving on it) as an elastic beam subjected to a moving load. The moving load may be considered as either a moving force (without inertial effect) or a moving mass (having inertial effect).

The early work on the topic has been described by Timoshenko et al in [1], where the governing equation for a uniform Bernoulli beam subjected to moving harmonic force with constant velocity was solved by the mode superposition method. In [2], Fryba presented a solution for vibration of simply supported beam under moving loads and axial forces [3]. Beams, on the other hand, are classified into several groups, depending primarily on the kind of support used. It is also known that the dynamic behaviour of beams differs from one end support to another. Frequently occurring support condition are simply supported end, fixed (or clamped) end, sliding end and free end. However, different end conditions arise when, for example, a lumped mass or a spring is attached to the end of the beam. As a matter of fact the problem involving the frequently occurring support conditions has been studied by many authors [6-8]. Gadeyan and oni [9] and Gbadeyan and Idowu[10] are example of the few exceptions.

Here, we study the effect of an attached mass on the dynamic response of pre-stressed Bernoulli-Euler beam which is traversed by a number of concentrated masses. The attached mass is assumed to be at x=L of the beam and the masses move at constant velocities. Illustrative example involving pre-stressed Bernoulli-Euler beam which is simply supported at the other end (x=0) is given. Numerical analysis of the example is also carried out.

2. MATERIAL AND METHODS

The equation governing the response of pre-stressed Bernoulli-Euler beam carrying an attached mass at one of its end traversed by an arbitrary number of concentrated masses is given by [11]

$$EI\ \frac{\partial^4 V(x,t)}{\partial x^4} + \mu\frac{\partial^2 V(x,t)}{\partial t^2} - \frac{\partial^2 V(x,t)}{\partial x^2} = P(x.t) \qquad (1)$$

Where $P(x,t) = K_{nt}(x,t)[1 - \frac{\eta}{g}(V(x,t))]$,

g is the acceleration due to gravity, μ is constant mass per unit length of the beam, E is the modulus of elasticity. I is the moment of inertia, EI is the constant flexural stiffness, η is the convective acceleration, V(x,t) is the lateral displacement, $K_{nt}(x,t)$ is the continuous moving force, N is the pre-stressed constants and x is the position co-ordinate in the axial direction.

The operator η is defined as

$$\eta = \frac{\partial^2}{\partial t^2} + 2v_i\frac{\partial^2}{\partial x\partial t} + v_i^2\frac{\partial^2}{\partial x^2} \qquad (2)$$

and $K_{nt}(x,t)$ is defined as

$$K_{nt}(x,t) = \sum_{i=0}^{s} M_{ig}\delta(x - v_{it}) \qquad (3)$$

$\delta(\ )$ is the Dirac delta function, L is the length of the beam, v_i is the constant velocity of the i[th] mass and the time t is such that 0$0 \le v_i t \le L$. Hence, from equation (1) to (3)we have

$$EI\ \frac{\partial^4 V(x,t)}{\partial x^4} + \mu\frac{\partial^2 V(x,t)}{\partial t^2} - \frac{\partial^2 V(x,t)}{\partial x^2} = \sum_{i=0}^{s} M_{ig}\delta(x - v_{it})\,[1 - \frac{1}{g}\Big(\frac{\partial^2 V(x,t)}{\partial t^2} + 2v_i\frac{\partial^2 V(x,t)}{\partial x\partial t} + v_i^2\frac{\partial^2 V(x,t)}{\partial x^2}\Big)] \qquad (4)$$

The corresponding boundary at x=0 is one of the

$$\begin{aligned} &V(0,t) = V'(0,t)=0\\ &V(0,t) = V''(0,t) =0\\ &V'''(0,t) = V''(0,t) = 0\\ &V'(0,t)= V'''(0,t) =0 \end{aligned} \qquad (5)$$

And at x=L we have

$$\begin{aligned} &EIV''(L,t) - \omega^2 JV'(L,t) = 0\\ &EIV'''(L,t) + \omega^2 M_1 V(L,t) = 0 \end{aligned} \qquad (6)$$

The initial conditions are given as
$V(x,0) = V_t(x,0) =0$

Note that M_1 and J are the mass of the attached mass and mass moment of inertia respectively.

3. SOLUTION TECHNIQUE

The exact solution of the above initial boundary value problem cannot be obtained. Hence, we resort to employing an approximate analytical technique which involves taking the generalized finite integral transform of the initial-boundary value problem and then solving the resulting ordinary differential equations with the initial conditions using modified struble's technique. This technique is discussed in detail in [9, 10, 11].

To this end, the generalized integral transform is defined over the closed interval [0,L] with the kernel $y_n(x)$ which is defined as

$$y_n(x) = Sin\frac{\theta_n}{L}x + A_n Cos\frac{\theta_n}{L}x + B_n Sinh\frac{\theta_n}{L}x + C_n Cosh\frac{\theta_n}{L}x$$

where θ_n is the mode frequency and A_n, B_n and C_n are constants to be determined using the boundary conditions.

Transforming the partial differential equation (4) we have

$$\ddot{Y}(j,t) + \omega_j^2 Y(j,t) - \varepsilon_1 \sum_{r=1}^{\infty} \frac{\mu N_0}{a_0(r)} D_j Y(j,t) + \frac{M}{\mu L} \sum_{s=1}^{\infty} \frac{1}{a_0(s)} [Q_1(vt) + 2v\, Q_2(vt) + v^2 Q_3(vt)] = \frac{Mg}{\mu L}\, \phi_j(vt) \quad (7)$$

Where Y(j,t) is the lateral transformed and

$$Q_1(vt) = \int_0^L Y_s(x) Y_j(x) dx + 2 \sum_{m=1}^{\infty} Cos \frac{m\pi t}{L} [\int_0^L Cos \frac{m\pi t}{L} Y_s(x) Y_j(x)] dx$$

$$Q_2(vt) = \int_0^L \frac{dY_s(x)}{dx} Y_j(x) dx + 2 \sum_{m=1}^{\infty} Cos \frac{m\pi t}{L} [\int_0^L Cos \frac{m\pi t}{L} \frac{dY_s(x)}{dx}\ Y_j(x)] dx$$

$$Q_3(vt) = \int_0^L \frac{d^2 Y_s(x)}{dx^2} Y_j(x) dx + 2 \sum_{m=1}^{\infty} Cos \frac{m\pi t}{L} [\int_0^L Cos \frac{m\pi t}{L} \frac{d^2 Y_s(x)}{dx^2}\ Y_j(x)] dx$$

$$a_0(j) = \frac{\mu L}{2}(1 - B_j^2 + \frac{1}{a_j}\, 9B_j^2 Sinha_j Cos\, a_j + 2\, B_j\, Cosha_j Sina_j - 2\, B_j\, Sinha_j Cosa_j - \frac{1}{2} Sina_j)$$

$$\varepsilon_1 = \frac{N}{N_0}$$

N_0 is the pre-stressed term in a particular direction

From here two cases are considered, namely, (i) Moving force and (ii) Moving mass. Note that equation (7) is obtained by considering only one mass. The equation describes the moving mass pre-stressed Bernoulli-Euler problem. However it can only be resolved if we first know the modified frequency of a related simplified problem, which is known as the moving force pre-stressed Bernoulli-Euler problem. Hence, we proceed to obtain the required modified frequency by solving the moving force problem as follows.

The equation governing the moving force problem obtained from (7) is

$$\ddot{Y}(j,t) + \omega_j^2 (1 - \tau_1 \beta^*(j,j)) Y(j,t) - \tau_1 \sum_{\substack{r=1 \\ r \neq n}}^{\infty} \beta^*(j,r)\, Y(r,t) = \frac{Mg}{\mu L}\, \phi_j(vt) \quad (8)$$

Note that

$$\tau_1 = \frac{\varepsilon_1}{1+\varepsilon} \quad (9)$$

and

$$\beta^*(j,r) = \mu \frac{N_0}{a_0(r)} D_r$$

In order to solve equation (8) we assume the following form of solution.

$$Y(j,r) = \lambda_j(j,t)\, Cos(\omega_j t - \alpha(j,t)) \quad (10)$$

Where $\lambda_j(j,t)$ and $\alpha(j,t)$ are slowly varying functions. Putting equation (10) into equation (8) we obtain [11]

$$Y(j,r) = \lambda_0 Cos(\gamma_{jn} t - \alpha_j) \tag{11.5}$$

Where $\lambda_0$and α_j are constants and

$$\gamma_{jn} = \frac{\omega_j}{2}(2 - \tau_1 \beta^*(j,j) \tag{12}$$

is the desired modified frequency of the moving force problem and it is due to the effect of the pre-stressed constant.

Using equation (12) the entire equation (8) can now be replaced by the free system operator defined in terms of the modified frequency γ_{jn} so as to obtain

$$\ddot{Y}(j,t) + \gamma_{jn} Y(j,t) = Q_{pf} \phi_j(vt) \tag{13}$$

Where

$$Q_{pf} = \frac{Mg}{\mu L}$$

solving (13) via method of Laplace transform we have

$$Y_f(x,t) = \sum_{j=1}^{\infty} \frac{Q_{pf} C_j}{a_0(j)\gamma_{pj}(\gamma_{pj}^4 - a_j^4)} \{(\gamma_{pj}^2 - a_j^2)\left[\gamma_{pj}\left(Cosh a_j t - Cos\gamma_{pj} t\right) + \frac{B_j}{C_j}\left(\gamma_{pj} Sinh\, a_j t - a_j Sin\gamma_{pj} t\right)\right] + (\gamma_{pj}^2 + a_j^2)[\frac{A_j}{C_j}\gamma_{pj}\left(Cas\, a_j t - Cos\gamma_{pj} t\right] \frac{1}{C_j}\left(\gamma_{pj} Sin\, a_j t - a_j Sin\gamma_{pj} t\right)]\}[Sin\frac{a_j}{L}x + A_j Cos\frac{a_j}{L}x + B_j Sinh\frac{a_j}{L}x + C_j Cosh\frac{a_j}{L}x] \tag{14}$$

Equation (14) is the transverse displacement of a pre-stressed Bernoulli -Euler beam with an attached mass at x=L and traversed by moving force.

We are now in the best position to solve the moving mass pre-stressed Bernoulli-Euler beam problem described by equation (8). To this end we note that using equation (11), equation (8) can be rewritten as

$$\ddot{Y}(j,t) + \tau_2 q_{PB}(t)\dot{Y}(j,t) + \left(\gamma_{ps}^2 - \tau_2 q_{PC}(t)\right)(1 - \tau_2 q_{PA}(t))Y(j,t) + \tau_2\left(1 - \tau_2^2 q_{PB}(t)\right)\sum_{\substack{s=1 \\ s\neq n}}^{\infty} \frac{1}{\alpha_{(0)}(s)}[q_{PB}(t)\ddot{Y}(s,t) + q_{PB}(t)\dot{Y}(s,t) + q_{PC}(t)Y(s,t) = \tau_2\left(1 - \tau_2 q_{PA}(t)\right)g\, \phi_j(vt) \tag{15}$$

Where

$$q_{PA1} = \frac{1}{\alpha_0(j)}\alpha_1(j,j) + 2\sum_{m=1}^{\infty} Cos\frac{m\pi vt}{L}\,\alpha_{11}(j,j)$$

$$q_{PB1} = 2\frac{v}{\alpha_0(j)}\alpha_2(j,j) + 2\sum_{m=1}^{\infty} Cos\frac{m\pi vt}{L}\,\alpha_{22}(j,j)$$

$$q_{PC1} = \frac{v^2}{\alpha_0(j)}\alpha_3(j,j) + 2\sum_{m=1}^{\infty} Cos\frac{m\pi vt}{L}\,\alpha_{33}(j,j)$$

and

$$\alpha_1(j,j) = \alpha_1(j,s)/s = j$$
$$\alpha_2(j,j) = \alpha_2(j,s)/s = j$$
$$\alpha_3(j,j) = \alpha_3(j,s)/s = j$$

Here

$\tau_2 = \frac{\varepsilon}{1+\varepsilon}$

and

$$\varepsilon = \tau_2 + o(\tau_2^2) \tag{16}$$

using argument similar to that of the above for moving force problem, we obtain

$$Y(j,r) = A_0 e^{r_0 t} Cos(\beta_s t - \Gamma_j) \tag{17}$$

Where

$r_0 = \frac{\tau_2}{\alpha_0(j)} v\alpha_2(j,j)$

$$\beta_{ps} = \frac{\gamma_{ps}}{2}(2 - \frac{\tau_2}{\alpha_0(j)}(\alpha_1(j,j) - \frac{v^2}{\gamma_{ps}^2}\alpha_3(j,j) \tag{18}$$

Equation (18) is the desired extended modified frequency due to the effect of moving mass. Replace the free vibration mode with the extended modified frequency in equation (15) we have

$$\ddot{Y}(j,t) + \beta_{ps}^2 Y(j,t) = Q_{ps}\phi_j(vt) \tag{19}$$

Where

$Q_{ps} = \tau_2 g$

And

$\phi_j(vt) = Sin\frac{\alpha_s}{L}t + A_j Cos\frac{\alpha_s}{L}t + B_j Sinh\frac{\alpha_s}{L}t + C_j Cosh\frac{\alpha_s}{L}t$

Solving equation (19) using method of Laplace transformation and its inversion we have.

$$Y_m(x,t) = \sum_{j=1}^{\infty} \frac{Q_{ps}C_j}{\alpha_0(j)\beta_j(\beta_j^4 - a_j^4)}\{(\beta_j^2 - a_j^2)\left[\beta_j\left(Cosha_j t - Cos\beta_j t\right) + \frac{B_j}{C_j}\left(\beta_j Sinh\, a_j t - a_j Sin\beta_j t\right)\right] + (\beta_j^2 + a_j^2)[\frac{A_j}{C_j}\beta_j(Cas\, a_j t - Cos\beta_j t] + \frac{1}{C_j}(\beta_j Sin\, a_j t - a_j Sin\beta_j t)]\}[Sin\frac{a_j}{L}x + A_j Cos\frac{a_j}{L}x + B_j Sinh\frac{a_j}{L}x + C_j Cosh\frac{a_j}{L}x] \tag{20}$$

Equation (20) is the desired lateral displacement of the pre-stressed Bernoulli beam which has an arbitrary support at x=0 with an attached mass at x=L and traversed by a moving mass.

4. A PARTICULAR CONFIGURATION

Considering a pre-stressed Bernoulli-Euler beam which is simply supported at the end x=0 carrying an attached mass at the other end. The boundary condition in this case, are

$Y(0,t) = Y''(0,t) = 0$

$$EIY''(L,t) - \omega^2 JY'(L,t) = 0$$

$$EIY'''(L,t) + \omega^2 M_1 Y(L,t) = 0 \tag{21}$$

The corresponding Kernel is

$$Y_j(x) = Sin\frac{a_j}{L}x - B_j Sinh\frac{a_j}{L}x \tag{22}$$

$$B_j = \frac{Sina_j + \frac{\omega^2 JL}{EIa_j}Cosa_j}{Sinha_j - \frac{\omega^2 JL}{EIa_j}Cosha_j} \tag{23}$$

Hence the transformed governing equation is

$$\ddot{Y}(j,t) + \omega_j^2 Y(j,t) - \varepsilon_1 \sum_{r=1}^{\infty} \beta^*(r,j)\, Y(j,t) + \varepsilon \sum_{s=1}^{\infty} \frac{L}{a_0(s)} [Q_1(j,s) + 2v\, Q_2(j,s) +$$
$$v^2 Q_3(j,s)]\, Y(r,t) = \varepsilon L g \phi_j(vt) \qquad (24)$$

Where

$$\beta^*(j,j) = \frac{N_0 D_r^2}{L a_0(j)}(-K_1 + B_r K_3 - B_r K_9 + B_r^2 K_{11}) \qquad (25)$$

$$Q_1(j,s) = K_1 + B_j(K_3 - K_9) + B_j^2 K_{11} +$$
$$2\sum_{m=1}^{\infty} Cos\frac{m\pi vt}{L}(K_{17} + B_j(K_{19} - K_{25}) + B_j^2 K_{27}) \qquad (26)$$

$$Q_2(j,s) = \frac{a_j}{L}(K_1 + B_j(K_3 - K_9) - B_j^2 K_{11} +$$
$$2\sum_{m=1}^{\infty} Cos\frac{m\pi vt}{L}(K_{17} + B_j(K_{19} - K_{25}) + B_j^2 K_{27}) \qquad (27)$$

$$Q_3(j,s) = \frac{a_j}{L^2}(K_1 + B_j(K_3 - K_9) + B_j^2 K_{11} +$$
$$2\sum_{m=1}^{\infty} Cos\frac{m\pi vt}{L}(K_{17} + B_j(K_{19} - K_{25}) + B_j^2 K_{27}) \qquad (28)$$

a_j, ε and ε_1 are as defined earlier and the K's are as in the appendix[11]

For moving force problem the transformed equation reduce to

$$\ddot{Y}(j,t) + \omega_j^2 Y(j,t) = \tau_0 L g \phi(vt) \qquad (29)$$

which when solved using Extended Modified Strubble's Technique(EMST) discussed in the previous section, yields

$$Y_{fs}(x,t) = \sum_{j=1}^{\infty} \frac{Q_{psf}}{a_0(j)\omega_j(\omega_p{}^4 - \gamma_p^4)} \{B_j(\omega_p^2 - \gamma_p^2)[\omega_p Sinh\, \gamma_p t - \gamma_p Sin\omega_p t] + (\omega_p^2 -$$
$$\gamma_p^2)[\omega_p Sin\, \gamma_p t - \gamma_p Sin\omega_p t]\}[Sin\frac{a_j}{L}x + B_j Sinh\frac{a_j}{L}x] \qquad (30)$$

$$\gamma_p = \frac{\omega_j}{2}(2 - \tau_0 \beta^*(j,j) \qquad (31)$$

Equation (30) is the equation of transverse displacement of the pre-stressed Bernoulli- Euler beam which has a simple support at x=0 with an attached mass at x=L and is traversed by moving force. Solving the entire equation (24) the solution for moving mass problem is obtained as

$$Y_{MS}(x,t) = \sum_{p=1}^{\infty} \frac{Q_{psm}}{a_0(p)\gamma_{pm}(\gamma_{pm}{}^4 - \Omega_p^4)} \{B_p(\gamma_p^2 - \Omega_p^2)[\gamma_{pm} Sinh\, \Omega_p t - \Omega_p Sin\gamma_{pm} t] +$$
$$(\gamma_p^2 + \Omega_p^2)[\gamma_{pm} Sin\, \Omega_p t - \Omega_p Sin\gamma_{pm} t]\}\, [Sin\frac{a_p}{L}x + B_p Sinh\frac{a_p}{L}x] \qquad (32)$$

$$\gamma_{pm} = \frac{\gamma_p}{2}(2 - \frac{\tau_2}{a_0(j)}(\alpha_1(j,j) - \frac{v^2}{\gamma_p^2}\alpha_3(j,j)) \qquad (33)$$

Equation (32) is the lateral displacement of the pre-stressed Bernoulli -Euler beam which has a simple support at x=0 with lumped mass at x=L and traversed by moving mass.

5. RESULTS AND DISCUSSION

The numerical analysis is carried out for simply supported Bernoulli beam with an attached mass at the end x=L using the same dimensions as in [10] for comparison of results in addition the value of N_0= 20lb/s and $\beta = \frac{N}{N_0}$ =0.5 or 0.75

Table 1: t=0.5 when pre-stressed constant is tensile ($i.e. \beta > 0$)

	1a				1b			
W/t	W_{FL}	W_{FL}	W_{ML}	W_{ML}	W_F	W_F	W_M	W_M
	M=3	M=15	M=3	M=15	M=3	M=15	M=3	M=15
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.025	$5.00X10^{-6}$	$2.71X10^{-5}$	$2.00X10^{-8}$	$1.039X10^{-7}$	$1.21X10^{-4}$	$6.07X10^{-4}$	$4.10X10^{-7}$	$2.04X10^{-6}$
0.050	$4.00X10^{-5}$	$1.99X10^{-4}$	$1.80X10^{-7}$	$8.871X10^{-7}$	$9.68X10^{-4}$	$4.84X10^{-4}$	$3.26X10^{-6}$	$1.63X10^{-5}$
0.075	$1.23X10^{-4}$	$6.14X10^{-4}$	$6.40X10^{-7}$	$3.182X10^{-6}$	$3.26X10^{-7}$	$1.63X10^{-3}$	$1.10X10^{-5}$	$5.48X10^{-5}$
0.100	$2.64X10^{-4}$	$1.32X10^{-3}$	$1.64X10^{-6}$	$7.871X10^{-6}$	$7.69X10^{-3}$	$3.84X10^{-2}$	$2.59X10^{-5}$	$1.29X10^{-4}$
0.125	$4.64X10^{-4}$	$2.32X10^{-3}$	$3.29X10^{-6}$	$1.646X10^{-5}$	$1.50X10^{-2}$	$7.48X10^{-2}$	$5.01X10^{-5}$	$2.51X10^{-4}$
0.150	$7.21X10^{-4}$	$3.58X10^{-3}$	$5.98X10^{-6}$	$2.99X10^{-5}$	$2.58X10^{-2}$	$1.29X10^{-1}$	$8.58X10^{-5}$	$2.29X10^{-4}$

1(a) Variation of lateral deflection (W_{FL}, W_{WL}) pre- stressed simply supported Bernoulli beam carrying a lumped mass at x=L, against x for (i) Moving force (W_{FL}) and (ii) Moving mass (W_{ML}) at various time t for a Tensile pre stressed constant (i.e β>0).

1(b). Is as in (a) without lumped mass.
Table 1 shows the values of the deflection of the simply supported pre stressed beam.

a) i Moving force and moving mass problems
b) ii The beam with or without lumped mass at x=L against time t. The associated pre-stress mass is tensile.

It is found out that the values of deflection increases as t increases for both moving force and moving mass problems. This also holds for simply supported pre-stressed Bernoulli beam with or without an added mass. Also as M increases the value of the deflection of both the beam with or without attached mass as well as for both moving force and moving mass problems increase.

Table 2 shows the values of the deflection of both the simply supported pre-stressed Bernoulli beam with or without an added mass, as well as, case of moving force or moving mass problems against the vales of distance x. The pre-stress which is tensile is also considered. It is noted that for a moving force problem, irrespective of whether the beam under consideration has an attached mass or not, the values of the deflection decreases as x increases. on the other hand, for the moving mass problem, regardless of whether the beam being considered is with a lumped mass or not he values of the deflection increase as x increases.

Table 2

		2a				2b			
X		W_{FL}	W_{FL}	W_{ML}	W_{ML}	W_F	W_F	W_M	W_M
		M=3	M=15	M=3	M=15	M=3	M=15	M=3	M=15
T=0.30	0	$1.01X10^{-2}$	$5.31X10^{-5}$	$3.31X10^{-5}$	$1.65X10^{-4}$	$2.105X10^{-1}$	1.05227	$5.90X10^{-4}$	$2.95X10^{-3}$
	50	$9.41X10^{-3}$	$4.71X10^{-2}$	$3.54X10^{-5}$	$1.77X10^{-4}$	$2.098X10^{-1}$	1.0490	$5.92X10^{-4}$	$2.96X10^{-3}$
	100	$8.68X10^{-3}$	$4.34X10^{-2}$	$3.78X10^{-5}$	$1.89X10^{-4}$	$2.091X10^{-1}$	1.0453	$5.94X10^{-4}$	$2.97X10^{-3}$
	150	$1.01X10^{-3}$	$3.98X10^{-2}$	$4.02X10^{-5}$	$2.01X10^{-4}$	$2.083X10^{-1}$	1.0416	$5.96X10^{-4}$	$2.98X10^{-3}$
	200	$7.96X10^{-3}$	$3.62X10^{-2}$	$4.25X10^{-5}$	$2.13X10^{-4}$	$2.076X10^{-1}$	1.0380	$5.98X10^{-4}$	$2.99X10^{-3}$
	250	$6.52X10^{-3}$	$3.26X10^{-2}$	$4.49X10^{-5}$	$2.24X10^{-4}$	$2.069X10^{-1}$	1.0343	$6.00X10^{-4}$	$3.00X10^{-3}$
	300	$5.79X10^{-3}$	$2.90X10^{-2}$	$4.72X10^{-5}$	$2.36X10^{-4}$	$2.061X10^{-1}$	1.0306	$6.03X10^{-4}$	$3.01X10^{-3}$
T=0.5	0	$4.77X10^{-2}$	$2.39X10^{-1}$	$1.50X10^{-4}$	$7.50X10^{-4}$	$9.75X10^{-1}$	4.8736	$2.003X10^{-3}$	$1.002X10^{-2}$
	50	$4.44X10^{-2}$	$2.22X10^{-1}$	$1.60X10^{-4}$	$8.02X10^{-4}$	$9.71X10^{-1}$	4.8566	$2.010X10^{-3}$	$1.005X10^{-2}$
	100	$4.11X10^{-2}$	$2.06X10^{-1}$	$1.71X10^{-4}$	$8.53X10^{-4}$	$9.68X10^{-1}$	4.8396	$2.017X10^{-3}$	$1.009X10^{-2}$
	150	$3.78X10^{-2}$	$1.89X10^{-1}$	$1.81X10^{-4}$	$9.05X10^{-4}$	$9.65X10^{-1}$	4.8226	$2.024X10^{-3}$	$1.012X10^{-2}$
	200	$3.45X10^{-2}$	$1.73X10^{-1}$	$1.91X10^{-4}$	$9.75X10^{-3}$	$9.61X10^{-1}$	4.8056	$2.031X10^{-3}$	$1.016X10^{-2}$
	250	$3.12X10^{-2}$	$1.56X10^{-1}$	$2.02X10^{-4}$	$1.01X10^{-3}$	$9.58X10^{-1}$	4.7886	$2.038X10^{-3}$	$1.019X10^{-2}$
	300	$2.79X10^{-2}$	$1.40X10^{-1}$	$2.12X10^{-4}$	$1.06X10^{-3}$	$9.54X10^{-1}$	4.7715	$2.045X10^{-3}$	$1.023X10^{-2}$
T=1.0	0	$4.13X10^{-1}$	2.0628	$1.10X10^{-3}$	$5.49X10^{-3}$	7.7992	3.900X10	$4.125X10^{-3}$	$2.063X10^{-2}$
	50	$3.88X10^{-1}$	1.9410	$1.16X10^{-3}$	$5.81X10^{-3}$	7.7720	3.886X10	$4.140X10^{-3}$	$2.070X10^{-2}$
	100	$3.64X10^{-1}$	1.8192	$1.23X10^{-3}$	$6.12X10^{-3}$	7.7448	3.872X10	$4.154X10^{-3}$	$2.077X10^{-2}$
	150	$3.40X10^{-1}$	1.6973	$1.29X10^{-3}$	$6.44X10^{-3}$	7.7176	3.859X10	$4.169X10^{-3}$	$2.084X10^{-2}$
	200	$3.16X10^{-1}$	1.5755	$1.35X10^{-3}$	$6.75X10^{-3}$	7.6904	3.845X10	$4.183X10^{-3}$	$2.091X10^{-2}$
	250	$2.91X10^{-1}$	1.4537	$1.41X10^{-3}$	$7.07X10^{-3}$	7.6631	3.832X10	$4.197X10^{-3}$	$2.099X10^{-2}$
	300	$2.66X10^{-1}$	1.3319	$1.48X10^{-3}$	$7.39X10^{-3}$	7.6359	3.818X10	$4.212X10^{-3}$	$2.106X10^{-2}$

(a) This shows the variation of the lateral beam deflection of (W_{FL}, W_{ML}) of a pre-stressed Simply Supported Bernoulli beam carrying an attached mass at x=L, against x for (i) Moving force (W_{FL}) and (ii) Moving mass (W_{ML}) at various time t for a Tensile pre-stressed constant (i.e $\beta>0$).
(b) Is as in (a) but without an attached mass.

6. CONCLUSION

The dynamic analysis of a pre-stressed Bernoulli beam carrying an attached mass at x=L and traversed by an arbitrary number of concentrated moving masses is considered. It is found that for the moving force problem, irrespective of whether the simply supported pre-stressed Bernoulli beam is with an attached mass or not, the values of the corresponding deflection decreases as x increases while the deflection increases as x increases for moving mass problems. This shows that an attached mass has a significant effect on a moving mass problem for a tensile pre-stressed beam. Thus, axial length of the beam must be considered in such a case of moving mass problem when mass is attached and the load has arbitrary concentration for a pre-stressed (tensile) beam.

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