The Correlation Anomaly: Return Comovement and Portfolio Choice *

Gordon Alexander[†] Joshua Madsen[‡] Jonathan Ross[§]

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Abstract

Analyzing the correlation matrix of listed stocks, we identify "singletons" that exhibit minimal correlation with the CRSP universe. Portfolios comprised of 100 to 500 singletons *all* have lower betas and standard deviations and, correspondingly, higher average Sharpe and Treynor ratios than the CRSP universe over the sample time period 1950-2015. Portfolios of singletons chosen from subsets of the CRSP universe, including small-value, low-volatility, and momentum stocks, similarly realize lower portfolio standard deviations and higher risk-adjusted returns. These well-diversified portfolios earn the same return as the market portfolio but at much lower levels of risk and suggest that the positive abnormal returns to low-beta portfolios are driven by their component stocks having low average correlation.

Keywords: low correlation, portfolio choice, diversification, return comovement, low-volatility anomaly, betting against beta

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[†]Carlson School of Management, University of Minnesota, gjalex@umn.edu
[‡]Carlson School of Management, University of Minnesota, jmmadsen@umn.edu 612-624-1050
[§]School of Management, State University of New York at Binghamton, jross@binghamton.edu.

1 Introduction

When investing in a portfolio of stocks investors face both idiosyncratic and correlation risk (risk that prices will move in the same direction). Both risks can be addressed by adequate diversification, hence the proverb to “not put all your eggs in one basket.” Portfolio standard deviation, which is a function of portfolio size as well as the individual stocks’ variances and covariances, is one measure of these risks and can be reduced by including a large number of stocks that are not perfectly correlated.[1] However, simply holding a large number of stocks does not guarantee investors that the portfolio will have minimum standard deviation. As pointed out by Goetzmann and Kumar (2008), portfolio standard deviation is more effectively lowered through “active and proper stock selection [which] reflects skill in portfolio composition” (p. 436). Thus selecting *which* stocks are in a portfolio can theoretically have a more significant impact on portfolio standard deviation than the *number* of stocks in the portfolio (Goetzmann et al. (2005)).

Investors also face market risk (beta), or the extent to which a portfolio’s return comoves with the overall market. Unlike idiosyncratic and correlation risk, market risk cannot be reduced by holding more stocks but only by changing the portfolio composition (e.g., adding lower beta stocks to a higher beta portfolio). Thus stock selection affects both market risk (beta) and correlation risk (portfolio standard deviation). Furthermore, as we demonstrate, a portfolio with low correlation risk will also have low market risk but the inverse is not necessarily true.[2] We thus argue that correlation risk is the more salient risk that investors should care about, yet there is limited research on methods to reduce correlation risk. In this paper we empirically explore methods of minimizing portfolio standard deviation when using equal weights by focusing on pairwise correlations and contrast the empirical properties of

[1]It is well known that portfolio standard deviation typically decreases at a decreasing rate with each randomly-added stock (e.g., Statman (1987); Domian et al. (2007)). By including a large number of stocks (using value-weighting) that are not perfectly correlated, the idiosyncratic risk of each stock is minimized and the portfolio’s standard deviation converges to the market portfolio’s standard deviation.

[2]A portfolio of low-beta stocks is not guaranteed to exhibit minimal standard deviation if those stocks are themselves highly correlated, while a portfolio of stocks with low mean correlation will have minimal standard deviation and a low beta.

portfolios with low correlations and with those having low betas.

Because portfolio standard deviation is a function of both variances and covariances, minimizing this parameter for a portfolio of a predetermined size is computationally demanding. Specifically, for a target portfolio of size N' chosen from a universe of size N (where $N' \leq N$) there are $\binom{N}{N'}$ portfolio standard deviations to compute, which is impractical when selecting from the CRSP universe.[3] Searching across various portfolio sizes quickly magnifies the number of computations further.

We propose a simple alternative to form diversified portfolios with minimal correlation which only requires that an investor annually calculate the $n \times n$ correlation matrix of listed stocks. We focus on forming equal-weighted portfolios ranging from 75 to 500 stocks due to practical constraints in optimizing over both portfolio composition and weights.[4] To form a diversified portfolio of size s, we determine a threshold c for each annual correlation matrix such that only s stocks have no individual pairwise correlation with any stock that is greater than or equal to c. We refer to such stocks as "singletons". Thus any stock that has a correlation of c or larger with any other stock is excluded from our portfolio. To prevent our algorithm from methodically identifying penny stocks, we restrict the CRSP universe to stocks with at least 36 months of returns during our 66-year sample period (1950-2015) and a $5 stock price at the beginning of the holding period.

We demonstrate that these low-correlation portfolios have betas significantly less than one and realize approximately a 20% reduction in portfolio standard deviation relative to the market portfolio, consistent with our theoretical derivation that portfolio standard deviation and beta are increasing in a portfolio's average pairwise correlation. Surprisingly, the risk premiums of these diversified portfolios differ insignificantly from the CRSP universe, suggesting that one can minimize portfolio standard deviation without sacrificing return.[5]

[3]For example, there are $\binom{2000}{200} = 6.8 \times 10^{280}$ possible combinations of 200 stocks selected from a 2,000 stock population.

[4]Additional support of using equal-weighted portfolios is provided by DeMiguel et al. (2009), who find that out of 14 models used to estimate the Markowitz (1952) optimal weights, none perform consistently better than equal-weighting due to estimation error regarding the covariance matrix.

[5]Risk premium is defined as a portfolio's return in excess of the risk-free rate.

Portfolios that are long these low-correlated stocks and short high-correlated stocks exhibit even lower average pairwise correlation and generate annualized portfolio standard deviations as low as 4.02%, a 76% reduction in portfolio standard deviation relative to the market portfolio. Our evidence suggests that by focusing on correlations investors can form diversified portfolios with low levels of risk that earn market returns.

Both Sharpe and Treynor ratios of these low-correlation portfolios are *all* significantly larger than the relevant ratio of the CRSP universe from which they were selected. Alphas from the one-factor model, a three-factor market model that includes the Fama and French (1992) SMB and HML factors, and a four-factor model that also includes the Carhart (1997) momentum factor are all statistically significant across various portfolio sizes and range from 12-26 basis points per month. However, when we include the Frazzini and Pedersen (2014) betting-against-beta factor ('BaB'), the coefficient on this factor is positive and statistically significant while the alphas are essentially zero and statistically insignificant. Because beta is also a function of variances and covariances, these insignificant alphas suggest that stock correlations explain much of the betting-against-beta strategy.

Examining associations between these singleton portfolios and common risk factors, we find that the low-correlation portfolios are tilted towards small value stocks (positive loading on SMB and HML factors).[6] Thus, our risk-adjusted returns could be driven by the small-value anomaly. Accordingly, we next investigate the incremental performance of singleton portfolios chosen from various subsets of the CRSP universe. To benchmark the performance of these singleton portfolios, we also form low-beta portfolios chosen from the same subset. Thus, from the CRSP universe of small-value stocks, we form a singleton portfolio of 100 stocks and a separate portfolio of the 100 lowest beta stocks. We repeat this analysis for both various portfolio sizes as well as for different subsets of the CRSP universe (low volatility stocks and high momentum stocks).

[6] Our portfolios exhibit negative exposure to the momentum factor and contain stocks from all NYSE market capitalization quintiles as well as each of the Fama-French 12 industries and are not weighted towards illiquid stocks.

We find that singleton portfolios of size 75 to 175 selected from the universe of small-value stocks *all* realize *lower* average portfolio standard deviations and *higher* Sharpe ratios than the universe of small-value stocks. For example, whereas the small value portfolio (average size 465 stocks) realizes an average portfolio standard deviation of 18.55% and a Sharpe ratio of 0.687, the singleton portfolio containing 125 of these stocks realizes an average portfolio standard deviation of 16.41% and a Sharpe ratio of 0.785 (both statistically significantly different). Furthermore, these low-correlation small-value stocks exhibit significant alphas of 16.8 basis points per month (significant at the 5% level), even after controlling for the SMB, HML, momentum, and betting-against-beta factor. These significant alphas suggest that, at least in the small-value universe of stocks, low-correlation portfolios generate abnormal returns incremental to the betting-against-beta anomaly.

Examining the low-beta portfolio containing the 125 small-value stocks with the lowest betas, we find that these portfolios realize a statistically *higher* portfolio standard deviation of 19.06% and a statistically *lower* Sharpe ratio of 0.669 relative to the low-correlation portfolio. Furthermore, 31.13% of the 125 low-beta stocks are also classified as singletons, suggesting some overlap between the two selection methodologies. The return differences between the singleton and low-beta portfolios are statistically significant over a number of portfolio sizes, indicating that selecting low-correlation stocks from the population of small-value stocks results in a larger improvement in risk-adjusted returns than when selecting from low-beta stocks.

We run similar comparisons between low-correlation and low-beta portfolios selected from the universe of low-volatility and high-momentum stocks. In both cases, we find that the subset of stocks which exhibit minimal correlation realizes a lower portfolio standard deviation and a higher Sharpe ratio than the population from which they were chosen as well as significant alphas. Furthermore, the Sharpe ratios are generally statistically larger than Sharpe ratios of low-beta portfolios chosen from the same population.

Regardless of the population of stocks considered (e.g., CRSP universe, small cap stocks, low-volatility stocks, or momentum stocks), we are thus able to systematically select a much smaller subset that over our 66-year sample period realizes lower portfolio standard deviations, and in some cases higher risk-adjusted returns (as measured by Sharpe ratios and alphas).

Because our diversified portfolios are also low-beta portfolios, higher risk-adjusted returns contradict financial theory of a positive relationship between risk and return. Related research finds that portfolios of either low-volatility or low-beta stocks earn market returns at systematically lower levels of risk (i.e., the low-volatility anomaly).[7] Our singleton portfolios are fundamentally different from low-volatility portfolios in that our approach focuses on minimizing the upper bound of the $\frac{n^2-n}{2}$ correlation parameters, whereas the low-volatility anomaly minimizes only n standard deviations. We empirically find that our singleton portfolios contain equal proportions of both low- and high-volatility stocks, further evidence that our analysis differs from the low-volatility anomaly. Together, the differences in methodology and portfolio composition complement the evidence that low-volatility portfolios earn excess risk-adjusted returns and provide additional evidence of a negative relationship between risk and return.

Portfolio standard deviation tends to decrease at a decreasing rate with portfolio size, ceteris paribus (see footnote 1). However, the relationship between portfolio standard deviation and portfolio size for our singleton portfolios is less clear. Using our methodology, average return correlation *increases* with portfolio size because forming a larger portfolio necessarily requires introducing stocks with higher pairwise return correlations, which mechanically increases the average pairwise return correlation. Since the mean pairwise correlation is not held constant, increasing portfolio size can *increase* portfolio standard deviation when using our methodology. Thus relative to a singleton portfolio of size s, the singleton portfolio of size $s+1$ can have higher or lower standard deviation depending on the relative impact of

[7] See Ang et al. (2006), Ang et al. (2009), Clarke et al. (2006), Blitz and Van Vliet (2007), and Frazzini and Pedersen (2014). According to Baker et al. (2011), this low-volatility/ low-beta anomaly is a candidate for "the greatest anomaly in finance" (p. 40).

the additional stock on the average correlation. We conjecture, given equal weighting, that standard deviation is initially decreasing in portfolio size as the effects of including additional stocks is greatest for small portfolios. Yet because of the well-known decreasing benefit of increasing portfolio size, we also conjecture that at some portfolio size s^* each additional stock results in a *higher* portfolio standard deviation, suggesting a convex relationship between portfolio size and standard deviation. Our methodology is able to pin down the portfolio of s^* stocks that has the minimum portfolio standard deviation of all of our singleton portfolios.

Our trading strategy focuses on identifying a portfolio of N stocks with minimal correlation at the end of each December using the previous three years of returns, holding this diversified portfolio for a year, and then re-forming a new portfolio of N stocks each January. Thus there is no look-ahead bias. For a target portfolio of $N = 250$ stocks, we find a 34.13% (20.23%) likelihood that a singleton selected in year t will also be selected in year $t + 1$ ($t+2$). Thus some stocks are persistently identified as singletons. However, our use of equal-weighted portfolios requires re-balancing the portfolio each year. As a result, the average annual portfolio turnover rate ranges from 66% to 78% for our various-sized low-correlation portfolios.[8]

We contribute to research on portfolio selection. Whereas prior research emphasizes methods to derive Markowitz's theoretical security weights for a given level of expected return, there is limited research on *which* securities to include in a predetermined portfolio of size N', where N' is strictly less than the size of the CRSP universe. Although the reduction in portfolio standard deviation from increasing portfolio size is well-documented (e.g., Statman (1987); Domian et al. (2007)), there is limited guidance on which securities to include in order to optimize portfolio risk. The complex task of identifying which stocks to include in a portfolio by jointly considering portfolio size, standard deviations, and pair-wise return correlations may explain why individual investors in particular hold under-diversified portfolios (e.g., Goetzmann and Kumar (2008)). Practical implementation of portfolio se-

[8]We calculate portfolio turnover rate as $\frac{min(\text{purchases,sales})}{\text{average portfolio value}}$, assuming \$100 was invested in each stock.

lection thus typically focuses on diversifying across stock fundamentals, such as industry membership, despite documented shortcomings of most industry classifications (see Elton and Gruber (1970); Bhojraj et al. (2003)).

Our contribution centers on the application of a new methodology that allows us to group stocks by a characteristic of primary concern to investors, namely, return correlation. We illustrate a novel approach to identifying well-diversified portfolios that optimally trades off the benefits of portfolio size with changes in the underlying assets' correlations, resulting in significant reductions in portfolio standard deviation and higher risk-adjusted returns.

Our evidence that singleton portfolios (i.e., portfolios of varying sizes, each with minimal average correlation) tend to earn abnormal risk-adjusted returns contributes to growing evidence that greater risk (as measured by either portfolio standard deviation or beta) is not compensated with higher expected returns. Our analysis complements research that low-volatility stocks earn excess risk-adjusted returns and highlights that applying our methodology to low-volatility stocks generates even larger risk-adjusted returns. Despite differences in methodologies, the combined evidence presents a challenge to financial models of a positive relationship between risk and return.

In section 2 we show that portfolio standard deviation and beta are increasing in the average pairwise correlation of a portfolio's stocks. In section 3 we outline our methodology to identify stocks with minimal correlation. Section 4 presents our main analysis, and section 5 concludes.

2 Theory

To show that portfolio standard deviation is increasing in the average pairwise correlation of the portfolio's stocks, consider the expression for portfolio variance given in equation (1):

$$\sigma_p^2 = \mathbf{w}'\Sigma\mathbf{w} \tag{1}$$

where $\mathbf{w}$ is a vector of weights, Σ is the variance-covariance matrix of stock returns and $'$ denotes the transpose operator. Without loss of generality assume equal weighting. Expanding equation (1) produces equation (2):

$$\sigma_p^2 = \begin{bmatrix} \frac{1}{N} & \frac{1}{N} & \cdots & \frac{1}{N} \end{bmatrix} * \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1N}\sigma_1\sigma_N \\ \rho_{21}\sigma_2\sigma_1 & \sigma_2^2 & \cdots & \rho_{2N}\sigma_2\sigma_N \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N1}\sigma_N\sigma_1 & \cdots & \cdots & \sigma_N^2 \end{bmatrix} * \begin{bmatrix} \frac{1}{N} \\ \frac{1}{N} \\ \vdots \\ \frac{1}{N} \end{bmatrix} \tag{2}$$

where ρ_{ij} is the correlation between stock i and stock j's return and σ_i is the standard deviation of stock i's return. Matrix multiplication of equation (2) produces the following expression for portfolio variance:

$$\sigma_p^2 = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{ij}\sigma_i\sigma_j. \tag{3}$$

Since the variance of a random variable is always non-negative, the right-hand side of equation (3) is non-negative. Taking square roots of both sides and differentiating with respect to the ρ_{ij} gives:

$$\frac{\partial \sigma_p}{\partial \rho_{ij}} = \frac{\sigma_i\sigma_j}{N\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{ij}\sigma_i\sigma_j}} \;\geq\; 0 \quad \forall \quad i, j. \tag{4}$$

Because equation (4) is non-negative (i.e., $\sigma_i \geq 0$ for all i), the portfolio's standard deviation is increasing in the equal-weighted portfolio's mean pairwise return correlation.[9] Thus, to minimize portfolio standard deviation, investors will have a preference for portfolios with lower average pairwise return correlations, ceteris paribus.

Minimizing a portfolio's mean pairwise return correlation also leads to a lower portfolio beta. To see this note that beta is simply the covariance of the portfolio with the market portfolio divided by the variance of the market portfolio:

[9] We also assume that the denominator of equation (4) is positive.

$$\beta_p = \frac{\rho_{pM}\sigma_p\sigma_M}{\sigma_M^2} = \rho_{pM}\frac{\sigma_p}{\sigma_M} \tag{5}$$

Since it was shown in equation (4) that σ_p is increasing in the portfolio's mean pairwise return correlation, it follows from equation (5) that beta is as well. Note that ρ_{pM} is independent of the mean pairwise correlation of the returns of the portfolio's stocks. The more correlated the returns of the individual stocks are with each other (on average) says nothing regarding how correlated the portfolio is with the market portfolio, ceteris paribus.

In an influential paper, Frazzini and Pedersen (2014) document that "betting against beta" (long low-beta assets and short high-beta assets) produces positive risk-adjusted returns. Based on the definition of beta, these positive risk-adjusted returns must either be due to (1) a low correlation between the portfolio and the market (ρ_{pM}) or (2) a low portfolio standard deviation (σ_p). By analyzing the performance of portfolios with low mean pairwise correlation, we shed insights into the source of the betting-against-beta returns.

Previous research also documents that portfolio standard deviation decreases with size at a decreasing rate.[10] Portfolio standard deviation is increasing in mean pairwise correlation, ceteris paribus, yet for randomly selected portfolios there is no theoretical relationship between portfolio size and mean pairwise correlation. Our contribution lies in creating portfolios such that mean pairwise correlation is an *increasing* function of portfolio size. Furthermore, our methodology allows an investor to form smaller, more diversified portfolios without sacrificing returns.

[10]See p. 59 of Elton et al. (2010) and Table 1 in Statman (1987) for example.

3 Methodology

To identify diversified portfolios of size s, for each year t we create an $n \times n$ indicator matrix, $\mathcal{M}_t$, from the $n \times n$ CRSP correlation matrix. We set the (i, j) non-diagonal entries of $\mathcal{M}_t$ equal to one if $\rho_{ij} \geq c$ but zero otherwise and set the diagonal entries equal to zero, where ρ_{ij} is the correlation in stock returns for stocks i and j over the prior 36 months and c is the threshold level of historical return correlation (hereafter "threshold correlation") for which we determine whether two stocks are "similar". By adjusting c we can find a c^* for each year such that only s stocks have no individual pairwise correlation with any stock that equals or exceeds c^*.[11] We refer to such stocks as "singletons" and each year form equal-weighted singleton portfolios. Thus any stock in a given year that has a correlation of c^* or larger with any other stock is excluded from these portfolios for that year.

Our methodology, while simple to implement, is capable of forming portfolios with low average correlation. Table A1 of our Appendix demonstrates that a portfolio of $s = 250$ singletons (produced using a given value of c), has a notably *lower* average pairwise correlation than random portfolios of the same size that are chosen from the CRSP population, validating that these singleton portfolios contain stocks with minimal correlation.[12]

As discussed earlier in Section 2, portfolio standard deviation is decreasing in n and increasing in mean pairwise correlation (referred to here as comovement). To determine the optimal portfolio size (s^*) with the lowest possible portfolio standard deviation (of all possible "singleton" portfolio sizes), we change the number of identified singletons (s) by changing c. The number of singletons decreases as c decreases because fewer stocks have no correlation with any other stock at the reduced threshold.[13] Furthermore, the mean

[11] This is equivalent to ranking stocks on their maximum pairwise correlation and selecting only those stocks whose maximum correlation is less than c^*.

[12] Specifically, Table A1 displays for each year the average pairwise return correlation (Rand250) for 1,000 portfolios consisting of 250 stocks that are chosen randomly from the CRSP universe and shows that in no year is the average or even the minimum of these 1,000 portfolios lower than COR for the 250 singleton portfolio, S250. We cannot, however, prove that the portfolio of s singletons for a given c always has the lowest average pairwise return correlation.

[13] When $c = 1$ every stock is a singleton. By definition all singletons in a portfolio of size s are also in a portfolio of size $s + \epsilon$ for all integers $\epsilon > 0$.

pairwise correlation (COR) of the singleton portfolio decreases as s decreases because the maximum correlation of any stock in the portfolio is lower.[14] By reducing c we can thus identify portfolios that are *more* diversified (i.e., have lower comovement). In other words, using our methodology, portfolio standard deviation is an *increasing* function of portfolio size due to a higher average pairwise correlation between the resulting singletons. In modern portfolio theory, portfolio standard deviation strictly decreases as portfolio size increases, assuming stocks are chosen randomly. However, by judiciously choosing securities with low correlation, one can select fewer securities and realize a lower portfolio standard deviation. Increasing the size of the singleton portfolio thus trades off the well-known reduction in idiosyncratic risk with the increased risk associated with higher portfolio comovement. Prior research demonstrates that increasing portfolio size decreases portfolio standard deviation at a decreasing rate (Statman (1987); Domian, Louton, and Racine (2007)). We thus conjecture that the diversification benefits of increasing portfolio size from, say 10 to 50 securities outweigh the expected increase in portfolio comovement. However, it is unclear, ex ante, whether the diversification benefits of increasing portfolio size from, say 300 to 400 also outweigh the expected increase in portfolio comovement. Which portfolio size optimally trades off the diversification benefits of increasing portfolio size with the cost of greater portfolio comovement is thus an empirical question that we next address.

4 Results

4.1 Average portfolio statistics

We start with the CRSP universe of stocks with at least 36 months of returns and a \$5 share price at the beginning of year t. We require 36 months of returns to form our similarity matrix each year and require a \$5 share price to ensure our methodology does

[14] For any singleton portfolio p created using c_p, the correlation of that portfolio cannot be larger than c_p because all pairwise correlations are less than or equal to c_p.

not select small, thinly traded stocks as singletons.[15] To focus on common stocks we follow previous research and require a CRSP share code of 10 or 11, thus excluding ADRs, closed-end funds, foreign-domiciled stocks, and real estate investment trusts (Lyon et al. (1999)). Following the method outlined in Section 3, we initially find a c^* each year such that $s = 500$. We then find new values of c^* such that $s = 400$, $s = 350$, $s = 300$, $s = 275$, $s = 250$, $s = 225$, $s = 200$, and $s = 100$ each year.[16] Figure 1 plots the value of c^* chosen to identify 500 and 250 singletons each year of our sample period, as well as the mean pairwise correlation of the market portfolio. The threshold value c^* each year is highly correlated with the annual average market correlation, where larger values of c^* are required to form a portfolio of s singletons in years when individual stocks are more highly correlated.

We next compare (and plot) average portfolio statistics of these various-size singleton portfolios and the market portfolio. Table 1 summarizes the average performance of the equal-weighted CRSP universe (MKT) and singleton portfolios (S) of various sizes over our 66-year sample period (1950 to 2015).[17] We compare the portfolios' mean pairwise return correlation (COR), beta (BETA), standard deviation (STD), portfolio risk premium (RP), Sharpe ratio (SR), Treynor Ratio (TR), and portfolio turnover rate (PTR).[18] COR for the CRSP universe across these 66 years is 0.204, suggesting modest comovement in returns at the market level. However, regardless of the number of singletons included in the portfolios, COR is always significantly less than the CRSP universe.[19] Furthermore, COR exhibits a

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[15] We use holding period returns adjusted for stock splits, dividends, and delistings throughout our analysis.

[16] Results tabulated in Table A2 of our Appendix are robust to selecting a fixed percentage of stocks from the market each year rather than keeping the portfolio size constant across years.

[17] Table A3 in the Appendix displays a comparison of the S250 portfolio with the equal-weighted and value-weighted CRSP return indices as well as the average performance of 1,000 random 250-stock portfolios that were selected using both equal and value weights. Our results are robust to requiring only 24 months of prior returns.

[18] We calculate BETA as the covariance of the 12 monthly returns of portfolio i with the market portfolio's 12 monthly returns divided by the variance of the 12 market portfolio monthly returns in year t. Risk premium is the annual portfolio return less the risk-free rate. The Sharpe ratio is defined as the portfolio's return less the risk-free rate, divided by portfolio standard deviation. The Treynor ratio is defined as portfolio return less risk-free rate, divided by portfolio beta.

[19] Our null hypothesis is that COR, BETA, and STD are greater than or equal to the market's corresponding statistic while SR and TR are less than or equal to the corresponding statistic. Therefore we use one-tailed t-tests for COR, BETA, STD, SR, and TR, and two-tailed t-tests for RP.

positive relationship with portfolio size, increasing monotonically from 0.124 for the singleton portfolio of 100 stocks to 0.156 for the singleton portfolio of 500 stocks.[20] This relationship is consistent with our methodology. To generate larger singleton portfolios we must increase our similarity benchmark c^*, which results in a larger number of singletons but also singletons that are more correlated with each other on average.

The second row of Table 1 displays betas for these singleton portfolios. As predicted, the singleton portfolio betas are all significantly less than the market portfolio beta of 1 and are monotonically increasing in portfolio size, from a beta of 0.742 for the S100 portfolio to 0.854 for the S500 portfolio. As outlined in section 2, systematically selecting stocks with low pairwise correlation results in low-beta portfolios. Furthermore, by construction the portfolio's mean pairwise correlation is an increasing function of portfolio size, and thus the portfolio beta is also an increasing function of portfolio size.[21]

The combined effect of a significantly smaller correlation and beta for these singleton portfolios relative to the market portfolio (MKT) results in standard deviations that are also significantly smaller than the standard deviation of the market portfolio. Specifically, the average portfolio annual standard deviation (STD) for the CRSP universe is 16.62%, which is significantly larger than the average standard deviation of all our singleton portfolios (which monotonically increase in portfolio size from 13.13% to 14.37%).

Figure 2 plots the empirical relationship between portfolio standard deviation and portfolio size. From the CRSP universe we randomly (with replacement) form 1,000 portfolios of size n and calculate their average portfolio standard deviation. We plot these average portfolio standard deviations using a dashed line, labeled Random Portfolios, in Figure 2 for n ranging from 1 to 800 stocks. For small sample sizes, the average random portfolio standard

[20]This realized comovement over the holding period, although small, is always larger than the stock's historical comovement over the previous three years, suggesting some persistence in comovement but also an upward drift.

[21]The overlap of the composition of the S250 group drawn from the CRSP universe with the composition of the lowest beta quintile is 61%, consistent with our derivation in equation (5) that beta is increasing in a portfolio's mean pairwise return correlation. The overlap with the other four quintiles is 22%, 10%, 5% and 2% respectively (Q2 through Q5).

deviation is quite high (e.g., 20.4% for portfolios of size $n = 10$). However, by increasing the sample size the random portfolio converges to the market portfolio standard deviation of 16.62% documented in Table 1. In no instance is the average standard deviation of these random portfolios less than the market standard deviation, shown with a dotted horizontal line. This confirms the theoretical predictions of the Sharpe model that diversification only addresses idiosyncratic risk, but leaves investors exposed to systematic market risk.

In Figure 2 we also plot the realized singleton portfolio standard deviations using a solid line. These portfolios' low pairwise return correlation and betas initially shift their portfolio standard deviation downward. We find that portfolios comprised of only 10 singletons realize an average standard deviation of 16.4%, approximately equal to the market's standard deviation of 16.6%. Portfolio standard deviation continues to decrease and is minimized at 100 stocks with a standard deviation of 13.1%, at which point it begins to increase and slowly converge toward the market portfolio standard deviation. The initially convex relationship suggests that for portfolios larger than 100 stocks, the diversification benefits of adding additional singleton stocks fail to offset the increase in higher average pairwise correlation. Thus Figure 2 visually shows that an investor can reduce portfolio systematic risk to a level below the market by holding singleton portfolios.

Returning to Table 1, we find that the average equal-weighted CRSP annual risk premium (10.49%) insignificantly differs from each of the singleton portfolios. Although our methodology is agnostic with respect to returns and only seeks to minimize comovment (as discussed in section 2 and shown in Table 1), portfolios with low comovement will also be low-beta portfolios, and thus the classic risk-return trade-off in financial theory suggests that these portfolios' average returns should be lower than the market portfolio. However, the combination of similar returns and lower portfolio risk (as measured by both beta and portfolio standard deviation) produces Sharpe ratios (SR) and Treynor ratios (TR) for portfolios containing at least 100 singletons that are statistically larger than the corresponding market statistics.[22] Thus our methodology produces portfolios that on average realize significantly higher risk-adjusted returns, evidence that contradicts a positive relationship between risk and return.

We plot the distributions of COR, BETA, STD, RP, SR, and TR in Figure 3, extending the range of singleton portfolio sizes displayed in Table 1 to be from 10 to 800 stocks. Consistent with the general trends observed in Table 1, mean pairwise correlation and beta (Panels A and B) are monotonically increasing with portfolio size, and always significantly less than the market. Panel C replicates the standard deviation line from Figure 2. Singleton portfolios of 250-800 stocks have risk premiums equal to or slightly greater than the market portfolio risk premium (Panel D). The resulting Sharpe and Treynor ratios (Panels E and F) exceed the market return for sample sizes of at least 100 and 10 stocks, respectively.

The last row of Table 1 displays turnover rates for the singleton portfolios. We implement our methodology using an annual strategy that requires forming a new portfolio each January using historical returns. However, some stocks are identified as singletons over consecutive years. For a target portfolio of 250 stocks, we find a 34.13% (20.23%) likelihood that a stock identified in year t as a singleton will also be selected in year $t + 1$ ($t + 2$).[23] However, because we form equal-weighted portfolios, annual rebalancing results in higher turnover rates even though some stocks are persistently identified as singletons. The S250 portfolio, for example, has a turnover rate of 71.99%. Furthermore, turnover rates are monotonically decreasing with portfolio size, with a turnover rate of 77.68% for the S100 portfolio decreasing monotonically to a turnover ate of 66.1% for the S500 portfolio.

In untabulated analysis we calculate the proportion of years the various singleton portfolios' average pairwise correlation, beta, and standard deviation were lower than their corresponding market portfolio summary statistics over our 66-year time period. For each year, our methodology *always* identifies a singleton portfolio with lower COR than the CRSP

[22] Empirically, the singleton portfolio consisting of 250 (100) stocks has the highest Sharpe (Treynor) ratio.
[23] The likelihoods are 43.22% and 29.21% for year $t + 1$ and $t + 2$, respectively, for a portfolio of 500 stocks and 25.14% and 12.94% for a portfolio of 100 stocks.

population. This lower COR results in a lower portfolio standard deviation in 88% to 91% of our sample years and lower beta in 94% to 98% of our sample years, depending on the size of the singleton portfolio. We similarly calculate the proportion of years the singleton portfolios' risk premiums, Sharpe ratios and Treynor ratios were higher than their corresponding market portfolio over our sample period. Our portfolios have a higher risk premium in 52% to 55% of our sample years, but have a higher average Sharpe ratio in 70% to 77% and higher Treynor ratio in 65% to 74% of our sample years. Given the consistency of our results across portfolio sizes, we focus in subsequent tests on the S250 portfolio for parsimony because it has the highest Sharpe ratio and second highest Treynor ratio, but note that our results our quantitatively unchanged using portfolios ranging from 100 to 500 stocks. The S250 portfolio always has a lower COR as well as a lower beta and portfolio standard deviation in 97% and 94% of our sample years, respectively. Furthermore, it has a higher average risk premium, Sharpe ratio, and Treynor ratio in 52%, 77%, and 74% of our sample years, respectively.

Tables A4 and A5 in the Appendix displays the performance of the S250 portfolio relative to the market portfolio over various non-overlapping and consecutive year subsets of our 66-year sample period. Out of the 13 distinct five-year time periods in our sample (e.g., 1951-55), the S250 portfolio had a higher Sharpe ratio nine times and a higher Treynor ratio ten times. Sharpe and Treynor ratios are higher in all six ten-year periods as well as in both the first and second half of our sample. Results are robust for the various consecutive-year overlapping subsets of our 66-year sample time period tabulated in Table A4.

4.2 Long-Short Portfolio

In addition to identifying stocks that exhibit low mean pairwise correlation, our algorithm ranks the CRSP universe by each stock's maximum correlation, and thus can also identify stocks with high mean pairwise correlation by selecting stocks with the highest maximum correlation. In this subsection we examine properties of a risk-neutral portfolio that is long our low-correlated stocks and short our high-correlated stocks.

Table 2 tabulates results for the long side of the portfolio (low-correlated stocks, replicated from Table 1 for convenience), the short side of the portfolio (high-correlated stocks), and the corresponding long-short portfolio. All portfolios are equal-weighted and contain an equal number of low- and high-correlated stocks. We examine three long-short portfolios: one that contains 100 low and high-correlated stocks, one with 250 low and high-correlated stocks, and one that each year divides the entire CRSP universe into two equal portfolios of low and high-correlated stocks (LMed and HMed, respectively).[24]

Consistent with our algorithm successfully sorting stocks by correlation, the low correlated stocks (L100, L250, and LMed) exhibit significantly lower mean pairwise correlation over the subsequent one-year holding period than the corresponding high correlated stocks (H100, H250, and HMed). The long-short portfolio realizes an even lower mean correlation ranging from 0.040 to 0.009, a substantial reduction in the portfolio's mean correlation. Betas for the high-correlation stocks are all significantly greater than one while the low-correlation stocks have betas significantly less than one, resulting in negative betas for the long-short portfolios. Portfolio standard deviations of the high-correlated stocks are all significantly greater than the standard deviations of our low-correlated stocks. Due to the low mean correlation of the long-short portfolio, these long-short portfolios realize even further reductions in portfolio standard deviation, ranging from 9.39% for the portfolio of 100 low- and high-correlated stocks each to 4.02% for the median split portfolio. Risk premiums are positive but small and statistically insignificant for the long-short portfolios, producing insignificant Sharpe ratios. Treynor ratios are all negative due to the absolute value of the short beta being larger than the long beta.

Overall, the results in Table 2 suggest that a long-short portfolio generated using our algorithm to identify low- and high-correlated stocks generates portfolios with substantial reduction in risk as measured by portfolio standard deviation.

[24]Each year this portfolio is thus long half the market and short the other half.

4.3 Risk Factors and Portfolio Composition

We next examine the exposure of the singleton portfolios to known risk factors. Each month we calculate the raw return for each singleton portfolio and, after subtracting the risk-free rate, regress these excess returns on the Fama and French (1992) mimicking portfolios (i.e., market, SMB, and HML), Carhart (1997) momentum factor, and Frazzini and Pedersen (2014) betting-against-beta factor (BaB).[25] In Table 3 we display factor loadings for the S250 singleton portfolio using the market model (column 1), three-factor model (column 2), four-factor model (column 3), and a five-factor model (column 4). Since results are similar across all portfolio sizes from Table 1, we focus on the S250 portfolio for conciseness. The alpha on this singleton portfolio using the market model is 26.7 basis points per month, and decreases to 12 basis points when using the three-factor model and 14.2 basis points when using the four-factor model (all statistically significant). Thus these results are consistent with the statistically larger Sharpe and Treynor ratios in Table 1 and suggest that this diversified, low-correlation portfolio earns abnormal risk-adjusted returns. However, after including the BaB factor in column 4, the alphas become economically and statistically insignificant. Because betas are a function of both variances and correlations, these insignificant alphas suggest that correlations are the primary driver of the BaB factor.

Examining the factor loadings in Table 3, we find that market betas are all statistically less than 1 (F-statistics tabulated at the bottom of the table). The significantly positive loadings on the SMB, HML, and BaB factors indicate that this singleton portfolio is tilted towards small-cap value stocks with low betas. Interestingly, the S250 portfolio has negative exposure to the momentum factor that is significant only when the BaB factor is included.

To better understand the characteristics of these singleton stocks we next analyze the distribution of their market capitalization. Each year we sort all NYSE stocks into quintiles and determine annual cutoffs for each group. We then assign each S250 stock (based on its

[25] The Pastor and Stambaugh (2003) liquidity factor is not available for our entire 66-year period. Coefficients on this liquidity factor are consistently insignificant when we restrict our sample period to the period when this factor is available.

market capitalization at the beginning of the holding period) into one of these five quintiles where Q1 is the smallest quintile. We display the percentage of stocks belonging to each quintile in Figure 4. The equal-weighted S250 portfolios comprise a relatively large number of stocks from the smallest NYSE size quintile (particularly since 1976) but also contain a significant number of stocks from each size quintile. Across our 66-year sample period, 50.1% of our stocks are from size quintile 1, 17.0% from quintile 2, 13.2% from quintile 3, 10.7% from quintile 4, and 9.1% from quintile 5.

The significant number of small stocks is consistent with our use of equal-weighted portfolios. Our initial restriction to stocks with a price of at least $5 suggests that these small stocks are unlikely to be highly illiquid. Furthermore, unreported factor loadings on Pastor and Stambaugh's (2003) liquidity factor are insignificant. However, to understand whether the large risk-adjusted returns we document are due to holding illiquid stocks, we next plot a similar distribution of Amihud (2002) illiquidity ratios. Specifically, each year we estimate annual illiquidity ratios for the CRSP universe and form annual quintiles. We then assign each S250 stock to one of these quintiles and plot the distribution of illiquidity quintiles where Q1 is the most liquid quintile. As depicted in Figure 5, our singleton stocks have roughly equal representation in each of the five illiquidity quintiles. Across the 66-year period, on average 15.3% of our stocks are from the illiquidity quintile 1, 18.5% from quintile 2, 20.8% from quintile 3, 24.8% from quintile 4, and 21.0% from quintile 5.[26] Combined, the size and illiquidity distribution suggests that our results are not a result of systematically selecting small illiquid stocks.

Increasing the number of securities in a portfolio reduces portfolio standard deviation asymptotically to a positive bound as long as the proportion invested in each security is reduced and there is no systematic relationship between the stocks that are included.[27] One

[26] Between 1978 and 1982 our methodology identified on average 154 NASDAQ stocks. Because NASDAQ volume is unavailable on CRSP prior to 1982, for these years we plot the illiquidity distribution just for the universe of stocks for which we can calculate the Amihud illiquidty ratio.

[27] Forming a large portfolio of, for example, energy stocks would thus do relatively little to reduce either systematic or idiosyncratic risk.

standard way to create a diversified portfolio involves diversifying across industry membership. Our methodology is agnostic with respect to fundamental characteristics. Thus the low betas and standard deviations we document could derive from our methodology systematically selecting stocks with certain fundamental characteristics and industry membership. The factor loadings in Table 3 control for several fundamental characteristics (e.g., size), but provide no evidence on the industry composition of these stocks.

To understand the composition of stocks selected by the our methodology, we next display the annual distribution of the S250 stocks in the Fama-French 12 industries in Figure 6 over our 66-year sample period. The trends suggest that the proportion of manufacturing stocks declines over our sample period, while the proportion of financial and health-care stocks increases. The only industries not represented every year are the Telephone (represented in 57 years), Consumer Durables (represented in 63 years), and Energy (represented in 65 years) industries. Furthermore, the proportion of stocks from each of these 12 industries is roughly equal to the proportion of the total number of stocks in each industry. In Table 4 we display the average number of singletons from the S250 portfolio in each of the 12 industries as well as the average percent of singletons belonging to each industry in the CRSP universe. Although the average number of stocks included from each industry varies considerably (e.g., 3 stocks from the "Telephone and TV" industry and 41 stocks from the "Finance" industry), this variation is driven by the variation in the underlying population of stocks. The average percent of stocks included from each industry ranges from 7% to 15%. The fact that these average percentages are fairly similar despite a large variation in the number of stocks from each industry suggests that our singleton portfolio sample reflects the natural variation in these industries within the CRSP universe.

4.4 Subsets of the Market

4.4.1 Small Value

As noted earlier, in Table 3 all the coefficients on SMB and HML are statistically significant, suggesting that the singleton portfolio is tilted towards small value stocks. Accordingly, we next examine whether the lower risk and higher risk-adjusted returns (alphas) associated with these singleton portfolios are directly related to differences in size. We apply our methodology to subsets of the CRSP universe, focusing on small-value stocks. Specifically, at the beginning of each year we independently sort stocks by their market cap and book-to-market (b/m) ratio. We select all stocks with a market capitalization less than the NYSE median market cap (as of the previous June) and a b/m ratio above the 70th percentile of NYSE b/m ratios. We then apply our methodology to each of the resulting groups and form portfolios comprised of 75, 100, 125, 150, and 175 singletons by varying our similarity parameter c. Due to a limited number of observations in the earliest years of our sample, we restrict this analysis to the years 1967 to 2015 when there are at least 175 small-value stocks each year.

To benchmark these singleton portfolios, we also examine the low-beta subset of small-value stocks. As mentioned earlier, our algorithm tends to identify low-beta stocks. However, because we require that each singleton exhibits no correlation with any other stock above the threshold c each year, not all low-beta stocks will be singletons. The focus on minimal pairwise correlations, rather than correlation with the market, suggests that the composition of our singleton portfolios will differ from low-beta portfolios. Thus, from the universe of small-value stocks, we also select the n stocks with the lowest historical betas and compare characteristics of these low-beta portfolios with our low-correlation portfolios.

Table 5 Panel A displays results for the singleton portfolios selected from the universe of small-value stocks and Panel B results for the comparable low-beta portfolios. Similar to the analysis in Table 1, in Panel A correlation, beta, and standard deviations for all five singleton portfolios are significantly lower than the equal-weighted portfolio of all small-cap value stocks. Risk premiums are insignificantly different, but Sharpe and Treynor ratios are significantly larger for all five small-cap singleton portfolios.

These results differ (often significantly) from results tabulated in Panel B. In contrast to the singleton portfolios, the low-beta portfolios do not realize any significant reduction in correlation or even beta, and standard deviations are statistically greater for this low-beta subset of small-value stocks. Even though stocks in Panel B are those with the lowest historical betas (measured using the previous 36 months of returns), the realized betas over the holding period are significantly *higher* than the beta of portfolios formed using our algorithm. This pattern suggests a strong reversion in beta for the extreme low-beta stocks and that one can obtain a lower beta portfolio by focusing on pairwise correlations than by focusing directly on beta. While Sharpe ratios for each of these low-beta portfolios are insignificantly smaller than the corresponding small-value portfolio, the portfolios containing 75, 100, and 125 low-beta stocks realize significantly smaller Sharpe ratios than the corresponding singleton portfolios. This observation suggests that the application of our methodology to small-value stocks results in a greater increase in risk-adjusted return than the selection of a similar number of low-beta stocks.

We next estimate five-factor models for our small-value singleton portfolios in Table 5 Panel C using the market, SMB, HML, momentum, and BaB factors. As before, we examine portfolios containing 75, 100, 125, 150, and 175 small-value singletons. Inconsistent with results in Table 3, alphas are statistically significant for four of these five portfolios and range from 16.4 to 19.6 basis points per month.[28] Betas are significantly less than one and the SMB and HML coefficients are positive and statistically significant, consistent with the underlying portfolio composition of small value stocks. The BaB factor is also positive and significant, consistent with the selection of low-beta stocks, yet it does not fully explain the risk-adjusted returns earned by these low-correlation portfolios. The momentum

[28]Only the S100 portfolio has an insignificant alpha.

factor is negative for all portfolio sizes, suggesting that these portfolio tend to go against the momentum strategy.

4.4.2 Low-Volatility

A growing body of research suggests that low-volatility and low-beta stocks outperform high-volatility and high-beta stocks (Ang et al. (2009); Baker et al. (2011); Frazzini and Pedersen (2014)). This puzzling negative relationship between risk and return seemingly contradicts the risk-return trade-off of modern financial theory, and suggests that higher returns can be realized by holding less-risky portfolios. Our evidence suggests that diversified portfolios formed by minimizing average correlation also generate larger risk-adjusted returns than the market portfolio. Because these diversified portfolios are also low-beta portfolios, we investigate the extent to which our methodology mirrors the low-volatility anomaly. Each year we sort the CRSP universe into quintiles using the historical standard deviation of each stock's returns (using the past 36 months). We then investigate the overlap between our S250 stocks (chosen from the CRSP universe) and these volatility quintiles. If our results mirror the low-volatility anomaly, then we would expect a high proportion of our S250 stocks to also be included in the bottom volatility quintile. In contrast, we find across our 66 years that 27.04% of our S250 singletons (on average) are in the bottom volatility quintile. Furthermore, 17.95% of our singletons are in the top volatility quintile.[29] These initial summary statistics suggest that the larger risk-adjusted returns realized by our singleton portfolios are distinct from the low-volatility anomaly.

We next examine whether our methodology can improve upon the abnormal returns identified by the low-volatility anomaly. The first column in both panels of Table 6 displays summary statistics for the low-volatility quintile (rebalanced annually). The average correlation of this portfolio is 0.201, significantly higher than any of the singleton portfolios displayed in Table 1 (the highest was 0.156 for the S500 portfolio). This higher average

[29]The remaining three quintiles Q2, Q3, and Q4 overlap percentages are: 19.88%, 17.70%, and 17.44%.

correlation suggests, using average correlation as a benchmark for a diversified portfolio, that low-volatility portfolios are less diversified. In contrast, the beta for the low-volatility portfolio is 0.561, lower than any of the singleton portfolios in Table 1 and indicative of a portfolio with minimal market risk. The contrasting relationship between average correlation and beta for this low-volatility portfolio highlight that low market risk does not ensure a diversified portfolio.

The low-volatility portfolio realized an average standard deviation of 11.2% (which is roughly 68% of the standard deviation of the market portfolio) and an average risk premium of 9.63%. The low beta and standard deviation result in an economically significant Sharpe ratio of 1.035 and Treynor ratio of 0.206 that are notably greater than the market statistics of 0.649 and 0.105 in Table 1, consistent with previous evidence of large risk-adjusted returns for these minimally risky portfolios (see, e.g., Frazzini and Pedersen (2014)). To determine whether our results are distinct from this low-volatility anomaly, we apply our methodology to the portfolio of low-volatility stocks, adjusting our similarity parameter c to identify between 75 and 175 low-volatility singletons. Summary statistics for these singleton low-volatility portfolios are displayed in the remaining columns of Table 6 Panel A. For all portfolio sizes, our methodology is able to identify a subset of stocks which exhibit statistically lower average correlation than the low-volatility portfolio (correlations range from 0.122 to 0.149). Market betas and portfolio standard deviations are also statistically lower for these singleton portfolios, evidence that our methodology can effectively lower market risk by minimizing return correlation even for this subset of low-volatility stocks. Sharpe and Treynor ratios are statistically larger for each of the five singleton portfolios that we consider. Together, the evidence suggests that the singleton portfolio can improve upon the already significant returns of the low-volatility anomaly.

In Panel B we contrast the effect of selecting singletons from the universe of low-volatility stocks with the effect of selecting low-beta stocks from the same universe. Similar to the results in Table 5 Panel B, we find that each of these low-beta portfolios realize standard deviations and Sharpe ratios that are not statistically different than the universe of low-volatility stocks. The evidence suggests that application of low-beta to these low-volatility stocks does not result in any significant improvement in risk-adjusted return. Furthermore, we document significant differences between the singleton and low-beta portfolios. Portfolio standard deviations are significantly lower for the singleton portfolio chosen from the low-volatility population relative to low-beta stocks chosen from the same population, and Sharpe ratios are larger for all five portfolios and statistically significant for the portfolios of 125 and 150 singletons. We observe similar results when using Treynor ratios.

We also estimate five-factor models for our low-volatility singleton portfolios in Panel C, analogous to Table 5 Panel C. Four of the five portfolios produce statistically significant alphas at the 10% confidence level or better, and range from 10.9 to 14.7 basis points. Together, our results suggest that our methodology for identifying low-correlated stocks generates larger improvements in risk-adjusted returns than selection of only low-beta stocks.

4.4.3 Momentum

In our final analysis we repeat our horse race, contrasting the performance of low-correlation and low-beta portfolios chosen from high momentum stocks. Specifically, we rank stocks based on their prior 36-month return and select the highest quintile of stocks as high momentum stocks. From this group, we again apply our methodology to select between 75 and 175 singletons, and also rank these stocks by their individual beta to form low-beta subsets. Consistent with the results in Tables 5 and 6, the low-correlation singleton portfolios realize significantly lower standard deviations and betas as well as significantly higher Sharpe and Treynor ratios and alphas (although two of the Sharpe ratios and one of the alphas are statistically insignificant), relative to both the universe of momentum stocks and the subset of low-beta stocks.

5 Conclusion

This paper introduces a new methodology which enables us to identify a more diversified subset of the market than the market itself. We identify "singleton" stocks that exhibit limited correlation with every other stock in the CRSP universe. We demonstrate that an equally-weighted portfolio of singletons beats the CRSP universe on a risk-adjusted return basis over our sample time period 1950-2015. The standard deviation of the singleton portfolio declines rapidly with portfolio size but then slowly increases. At an optimal portfolio size of $N \approx 100$ stocks the singleton portfolio has minimal portfolio standard deviation. Our methodology systematically identifies portfolios whose mean return comovement decreases monotonically with N. Our results are robust to using various sub-periods of our sample time period.

Our paper contributes to the extensive literature on portfolio choice by identifying a method to parameterize portfolio mean pairwise return correlation as an increasing function of portfolio size. This enables us to directly reduce portfolio standard deviation as well as beta below the overall market's standard deviation and beta. Since our method is agnostic with respect to returns, our singleton portfolios do not exhibit statistically different returns than the market from which they are chosen. As a result, these portfolios have significantly higher Sharpe and Treynor ratios than their benchmarks. Furthermore, when coupled with low-volatility stocks, these resulting portfolios have even higher risk-adjusted returns.

To our knowledge, our paper is the first to propose a systematic method of choosing a well-diversified subset of stocks from the market (or subsets of the market such as low-volatility or small cap groups) that focuses on minimizing return comovement. We show that the risk-adjusted returns of these portfolios significantly beat their respective market over a 66-year sample period as well as over sub-periods as short as five years.

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Figures

Figure 1

Similarity Threshold Values and Mean Market Correlation and Size of Market (1950-2015)

This figure plots the threshold value c^*'s used by our methodology to identify 500 and 250 singletons each year for 1950–2015, as well as the average pairwise correlation (over the previous 36 months) of CRSP stocks.

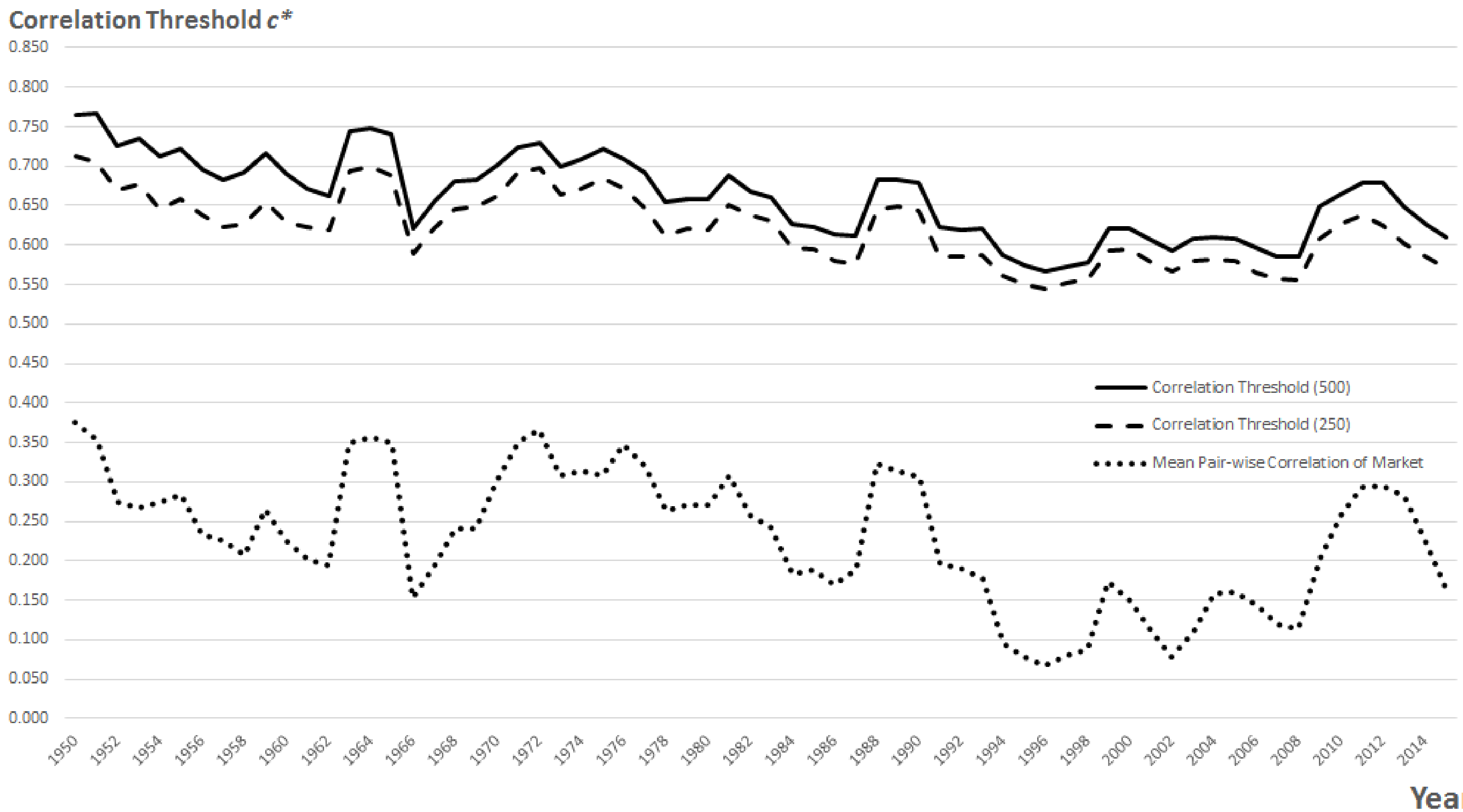


Figure 2
Singleton Portfolio Minimum Standard Deviation Frontier (1950-2015)

This figure plots the average standard deviation of 1,000 random portfolios of various sizes, the realized market portfolio standard deviation, and the portfolio standard deviation of singleton portfolios of various sizes. Our sample period for realized market portfolio and singleton portfolios is 1950 to 2015.

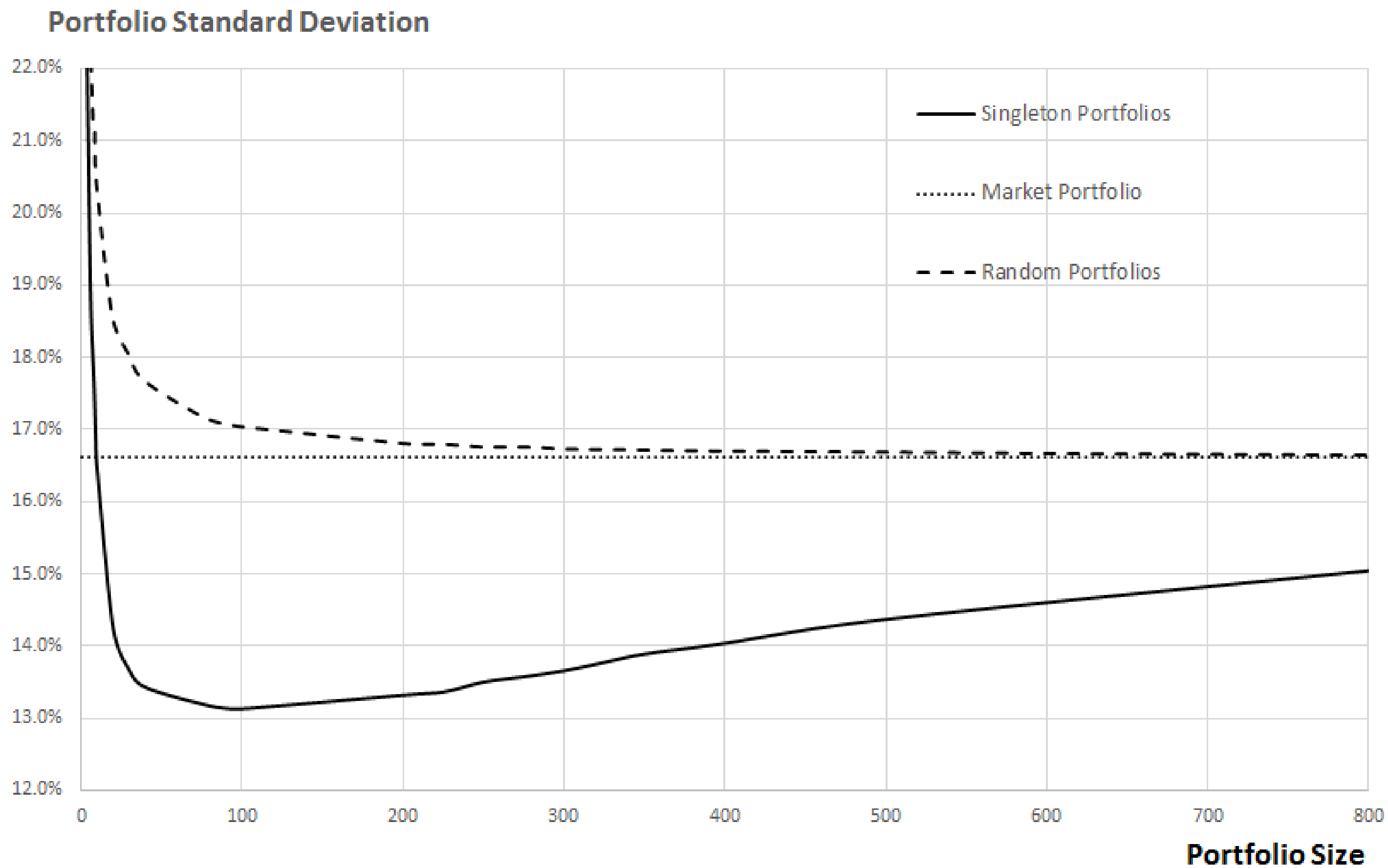


Figure 3
Singleton Portfolios vs MKT (1950-2015)

This figure depicts average portfolio statistics for six singleton portfolios of size 10 to 800. The dark and dashed lines represent the singleton and market portfolios, respectively; portfolio size is on the horizontal axis.

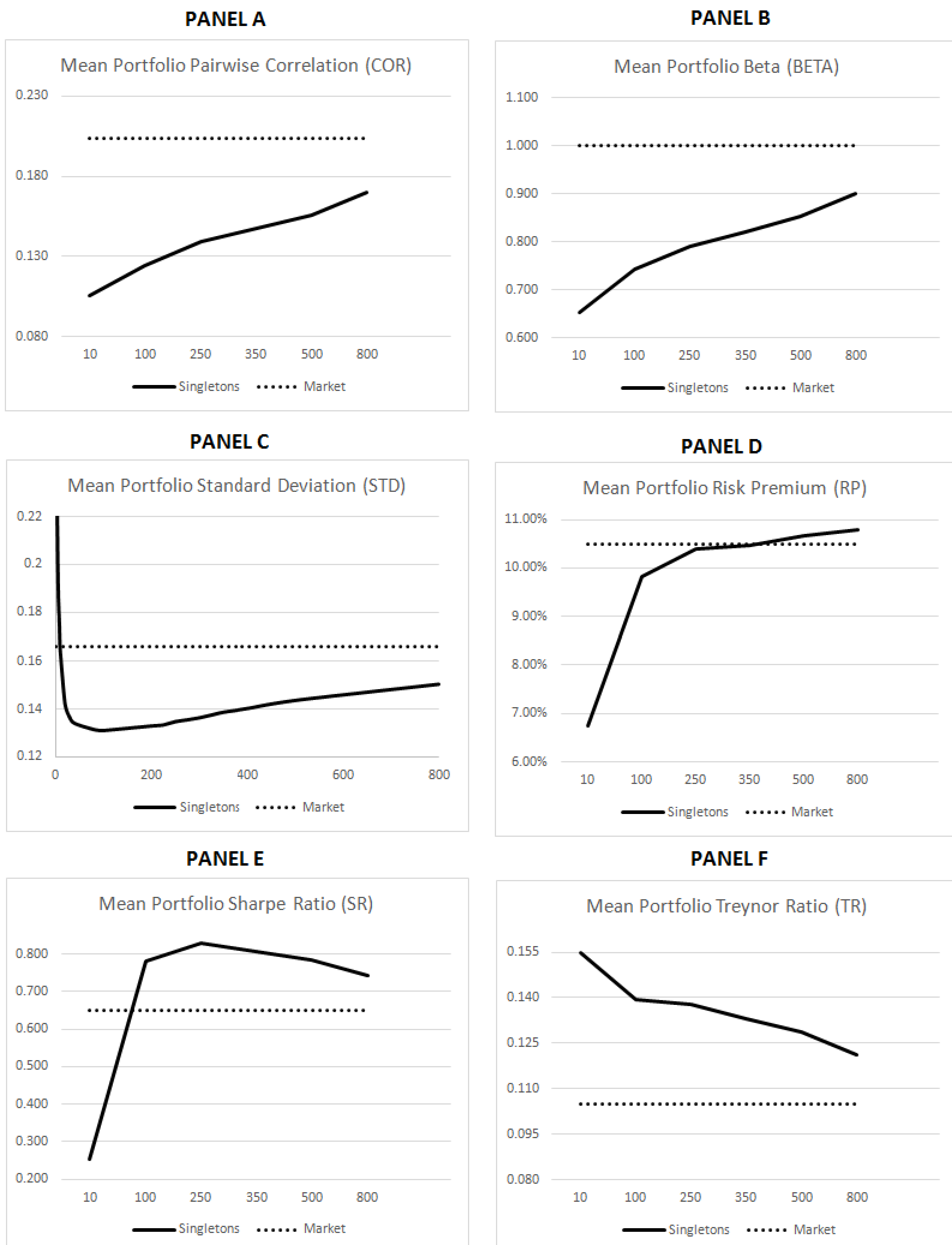


Figure 4

This figure depicts the proportion of stocks in the 250 singleton portfolio from five NYSE market capitalization quintiles for the years 1950-2015. Each year we rank the market capitalization of NYSE stocks into quintiles and plot the percentage of singleton stocks belonging to each quintile where "Q1" represents the smallest size quintile.

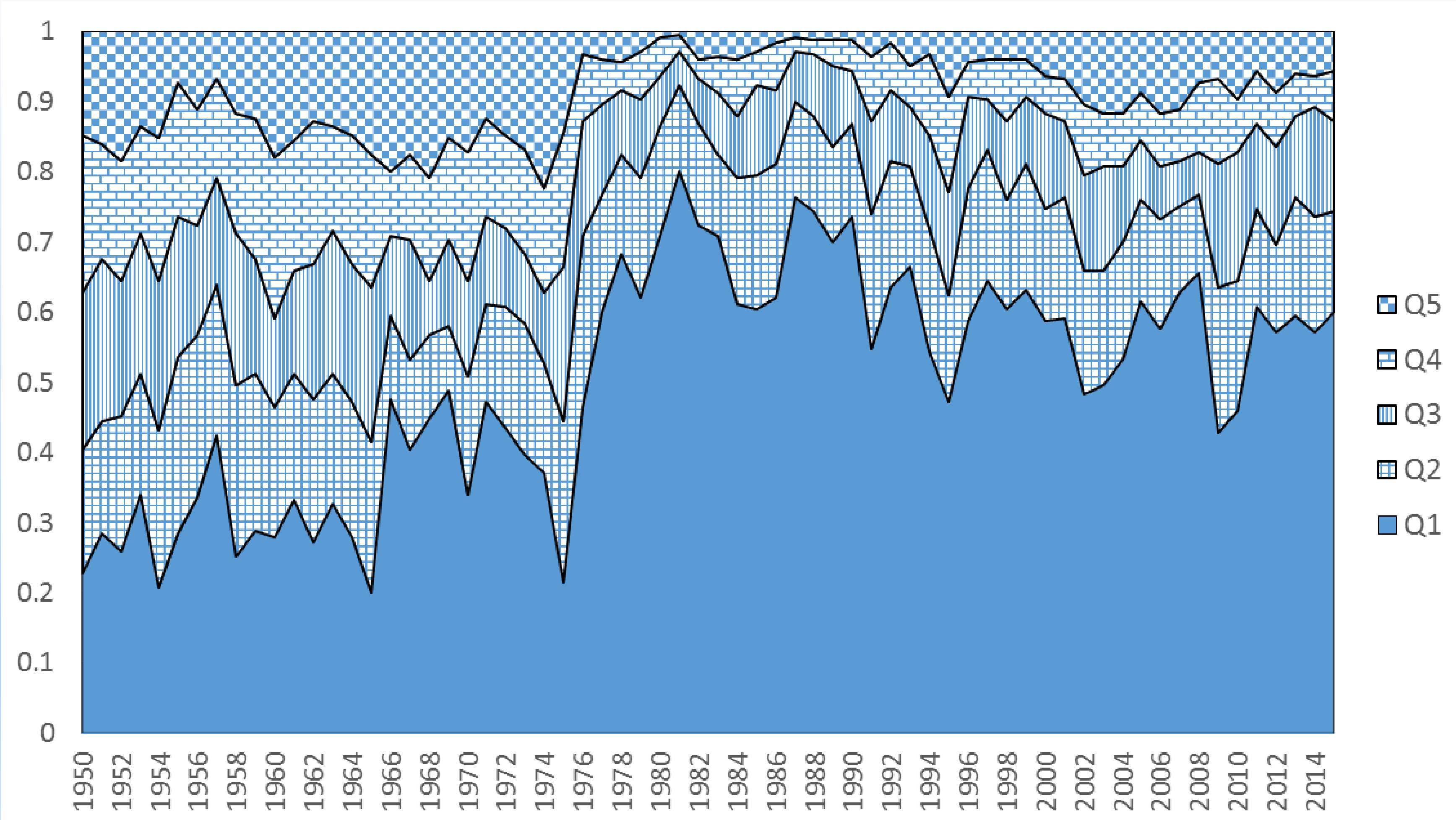


Figure 5

This figure depicts the proportion of stocks in the 250 singleton portfolio from five market-level Amihud illiquidity quintiles. Each year we calculate an annual Amihud ratio for all CRSP stocks with non-missing data and then these stocks are sorted into illiquidity quintiles. We plot the percentage of 250 singleton stocks belonging to each quintile. For the years 1976 to 1982 volume data is unavailable for NASDAQ stocks (indicated with vertical lines). Because NASDAQ stocks were selected during these years by our methodology, approximately 100 of the 250 stocks were assigned to an illiquidity quintile. "Q1" represents the most liquid quintile.

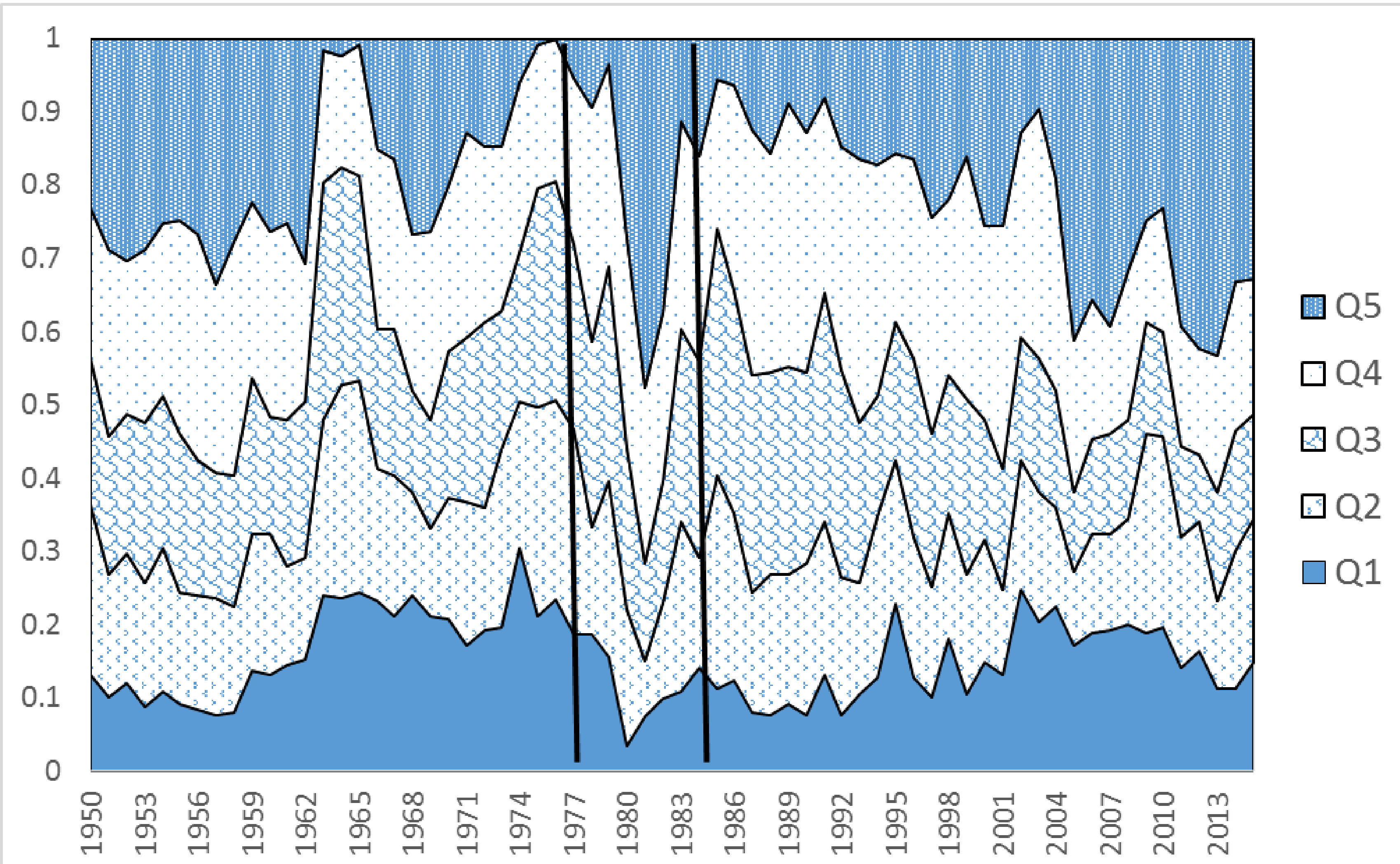
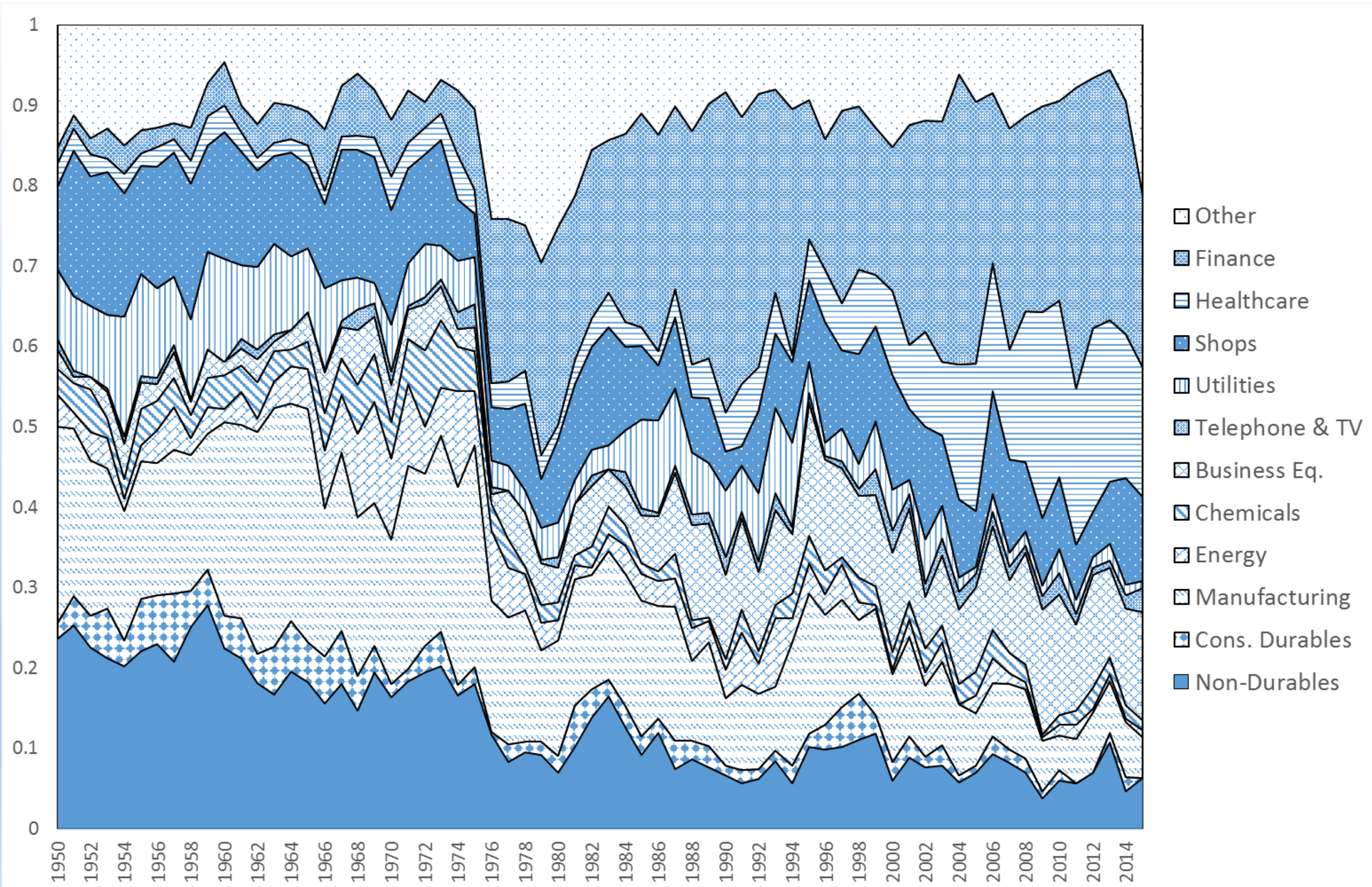


Figure 6

This figure depicts the proportion of stocks in the 250 singleton portfolio from each of the Fama-French 12 industries over the 66-year sample period. There are stocks from each industry in all sample years except for the Consumer Durables, Energy, and Telphone and TV industries which have 63, 65, and 57 years, respectively.



Tables

Table 1
Low Correlation Portfolios vs CRSP Universe (1950-2015)

This table summarizes the average performance of the equal-weighted CRSP universe between 1950 and 2015, as well as low-correlation portfolios drawn from this universe. We start with all CRSP stocks with at least 36 months of prior returns, stock price $\geq$ \$5 as of last trading day of year $t-1$, and a share code of 10 or 11 (MKT). From this universe, each year we form portfolios of size $n = \{100, 200, 250, 300, 350, 400, 500\}$ comprised of low-correlation stocks. To identify n low-correlation stocks (i.e., singletons) each year, we find the cutoff value c^* such that only n stocks have all pairwise correlations (based on prior 36 months) less than c^* (i.e., $\rho_{i,j} < c^* \ \forall\ j \neq i$). We tabulate average pairwise return correlations (COR), beta (BETA), standard deviation (STD), risk premium (RP), Sharpe ratios (SR), Treynor ratios (TR), portfolio turnover rates (PTR), and average number of stocks (N) for each of these portfolios. The Sharpe ratio is the portfolio annual return less the risk-free rate (t-bill monthly rate) divided by the portfolio's standard deviation and the Treynor ratio is the portfolio annual return less the risk-free rate divided by the portfolio's beta (calculated using the portfolio's 12 monthly returns). Portfolio turnover rate is calculated as [min(purchases, sales) $\div$ average portfolio value], assuming \$100 is invested in each of the portfolio stocks. T-statistics are reported in parentheses; ***(**)(*) denotes a statistically significant one-tailed t-test (two-tailed for RP) of the null hypothesis that the portfolio mean is less than or equal to (for COR, BETA, and STD) or greater than or equal to (for SR and TR) the corresponding CRSP market mean at the 1%(5%)(10%) level, respectively.

STAT	**MKT**	**S500**	**S400**	**S350**	**S300**	**S250**	**S200**	**S100**
COR	0.204	0.156	0.150	0.147	0.143	0.139	0.135	0.124
		(11.8)***	(12.3)***	(12.2)***	(12.6)***	(13.2)***	(13.3)***	(14.1)***
BETA	1	0.854	0.830	0.820	0.804	0.790	0.776	0.742
		(12.2)***	(13.1)***	(13.0)***	(13.4)***	(13.7)***	(15.1)***	(14.3)***
STD	16.62%	14.37%	14.04%	13.89%	13.65%	13.50%	13.32%	13.13%
		(7.76)***	(8.21)***	(8.29)***	(8.35)***	(8.45)***	(8.89)***	(8.25)***
RP	10.49%	10.67%	10.55%	10.46%	10.30%	10.38%	10.07%	9.84%
		(0.38)	(0.12)	(0.29)	(0.06)	(0.35)	(0.17)	(0.67)
SR	0.649	0.785	0.806	0.807	0.806	0.830	0.808	0.783
		(3.54)***	(3.73)***	(3.67)***	(3.11)***	(3.32)***	(2.77)***	(2.10)**
TR	0.105	0.129	0.132	0.133	0.134	0.138	0.135	0.140
		(3.78)***	(3.88)***	(3.85)***	(3.47)***	(3.70)***	(3.19)***	(3.05)***
PTR	38.10%	66.10%	68.23%	69.31%	70.63%	71.99%	73.66%	77.68%

Table 2
Long-Short Correlation Factor (1950-2015)

This table examines the average performance of equal-weighted portfolios containing the lowest and highest correlated stocks as well as the portfolio long low correlated and short high correlated stocks. Our sample period is 1950 to 2015. The lowest (highest) correlated stocks are the first (last) n singletons produced by our algorithm. We summarize the results for portfolios of size $n = \{100, 250, \text{median split}\}$. We tabulate average pairwise return correlations (COR), beta (BETA), standard deviation (STD), risk premium (RP), Sharpe ratios (SR) and Treynor ratios (TR) for each of these portfolios. Sharpe ratio is portfolio annual return less the risk-free rate (t-bill monthly rate) divided by portfolio standard deviation and Treynor ratio is portfolio annual return less risk-free rate divided by portfolio beta (calculated using the portfolio's 12 monthly returns). T-statistics are reported in parentheses; ***(**)(*) denotes a statistically significant two-tailed t-test of the null hypothesis that the long-short portfolio mean is equal to zero at the 1%(5%)(10%) level, respectively.

STAT	**L100**	**H100**	**L100 - H100**	**L250**	**H250**	**L250 - H250**	**LMed**	**HMed**	**LMed - HMed**
COR	0.124	0.273	0.040	0.139	0.265	0.030	0.174	0.241	0.009
			(9.95)***			(10.02)***			(8.45)***
BETA	0.742	1.070	-0.333	0.790	1.080	-0.290	0.919	1.081	-0.162
			(-9.53)***			(-9.35)***			(-9.95)***
STD	13.13%	18.51%	9.39%	13.50%	18.53%	7.84%	15.40%	18.06%	4.02%
			(22.35)***			(14.35)***			(15.03)***
RP	9.84%	9.19%	0.34%	10.38%	9.62%	0.41%	10.66%	10.33%	0.20%
			(0.25)			(0.38)			(0.34)
SR	0.783	0.437	0.036	0.830	0.509	0.083	0.919	1.081	0.110
			(0.22)			(0.52)			(0.74)
TR	0.140	0.084	-0.039	0.138	0.090	0.012	0.119	0.094	-0.025
			(-0.15)			(0.11)			(-0.22)

Table 3
Low Correlation Portfolios Factor Loadings (1950-2015)

This table examines the exposure of the S250 portfolio to known risk factors. Specifically, each month we calculate the raw return for the singleton portfolio containing 250 stocks, and after subtracting the risk-free rate, regress these excess returns on the market (CRSP value-weighted market portfolio less risk-free rate), SMB, HML, momentum, and betting-against-beta (BaB) factors over the 792 month sample time period. Standard errors are robust to heteroskedasticity and t-statistics are reported in parentheses. ***(**)(*) indicate two-tailed statistical significance that the coefficient is not equal to zero at the 1%(5%)(10%) level respectively.

	Market (1)	3 Factor (2)	4 Factor (3)	BaB (4)
Alpha (%)	0.267***	0.120**	0.142***	-0.001
	(3.39)	(2.38)	(2.60)	(-0.02)
Market Factor	0.800***	0.735***	0.731***	0.729***
	(30.28)	(41.08)	(42.72)	(53.81)
SMB Factor		0.603***	0.603***	0.603***
		(17.76)	(18.24)	(23.35)
HML Factor		0.270***	0.263***	0.158***
		(8.15)	(8.26)	(6.66)
Momentum Factor			-0.023	-0.070***
			(-0.88)	(-3.17)
BaB Factor				0.271***
				(13.83)
Observations	792	792	792	792
Portfolio Size	250	250	250	250
Adj R-Squared	0.710	0.889	0.890	0.924
F-stastistic Market=1	57.217	220.235	246.278	399.879

Table 4
S250 Industry Composition

This table examines the composition of the 250 singleton portfolio. We group stocks by the Fama–French 12 industries and tabulate the number of years at least one stock from each industry was included in the singleton portfolio, as well as the average number of stocks included and the average percentage of stocks included from each industry.

	No. Years	Avg. Stocks	Avg. Percent
Business Equipment	66	17	0.07
Telephone and TV	57	3	0.09
Chemicals and Allied Products	66	7	0.09
Manufacturing	66	36	0.09
Consumer Durables (cars, TVs, furniture)	63	7	0.10
Energy (oil, gas, coal)	65	9	0.10
Finance	66	41	0.11
Other	66	28	0.12
Utilities	66	14	0.12
Healthcare, Medical Equipment, Drugs	66	15	0.14
Shops (wholesale, retail, services)	66	26	0.15
Non-Durables (food, tobacco, textiles, etc.)	66	31	0.16

Table 5
Small-Value Stocks: Low Correlation vs. Low Beta

This table summarizes the average performance of the equal-weighted small-value market (SMVAL) portfolio between 1967 (the first year with at least 175 small-value stocks) and 2015 as well as low-correlation and low-beta portfolios drawn from this market. We start with all CRSP stocks with at least 36 months of prior returns, stock price $\geq$ \$5 as of last trading day of year $t-1$, and a share code of 10 or 11 (MKT), and then select all stocks with a market cap below the NYSE median market cap as of the previous June and a book-to-market ratio above the 70th percentile NYSE book-to-market ratio as of the previous December (small value stocks). From this universe, each year we form portfolios of size $n = \{75, 100, 125, 150, 175\}$ comprised of either low-correlation (Panel A) or low-beta (Panel B) stocks. Panel C examines the exposure of the low-correlation portfolios (chosen from the small-value market) to known risk factors by regressing the excess monthly return of these low-correlation portfolios on the market (CRSP value-weighted market portfolio less risk-free rate), SMB, HML, momentum, and betting-against-beta (BaB) factors. To identify n low-correlation stocks (i.e., singletons) each year, we find the cutoff value c^* such that only n stocks have all pairwise correlations (based on prior 36 months) less than c^* (i.e., $\rho_{i,j} < c^* \forall\ j \neq i$). To identify n low-beta stocks, we calculate the beta of each stock as of the beginning of year t using the prior 36-months of returns and choose the n stocks with the lowest historical betas. We tabulate average pairwise return correlations (COR), beta (BETA), standard deviation (STD), risk premium (RP), Sharpe ratios (SR), Treynor ratios (TR), and average number of stocks (N) for each of these portfolios. The Sharpe ratio is portfolio annual return less the risk-free rate (t-bill monthly rate) divided by portfolio standard deviation and the Treynor ratio is portfolio annual return less risk-free rate divided by portfolio beta (calculated using the portfolio's 12 monthly returns). Overlap % (reported in Panel B) is the percentage of low-beta stocks that are also low-correlation stocks. T-statistics are reported in parentheses; ***(**)(*) denotes a statistically significant one-tailed t-test (two-tailed for RP) of the null hypothesis that the portfolio mean is less than or equal to (for COR, BETA, and STD) or greater than or equal to (for SR and TR) the corresponding CRSP market mean at the 1%(5%)(10%) level, respectively. †††(††)(†) denotes a statistically significant two-tailed t-test of the null hypothesis that the low correlation singleton portfolio mean summary statistic in Panel A is different than the corresponding low beta portfolio mean in Panel B at the 1%(5%)(10%) level, respectively.

Panel A: Low-Correlation Portfolios vs. Small-Value Market Portfolio

STAT	SMVAL	S175	S150	S125	S100	S75
COR	0.178	0.151	0.147	0.142	0.141	0.134
		(8.63)***	(8.68)***	(9.07)***	(8.76)***	(9.63)***
BETA	0.927	0.821	0.803	0.782	0.766	0.746
		(7.56)***	(7.79)***	(7.62)***	(7.87)***	(8.52)***
STD	18.55%	16.93%	16.72%	16.41%	16.26%	16.23%
		(5.53)***	(5.63)***	(5.68)***	(5.80)***	(5.45)***
RP	13.62%	13.24%	13.28%	13.25%	12.95%	13.33%
		(-0.97)	(-0.69)	(-0.65)	(-0.99)	(-0.38)
SR	0.682	0.758	0.773	0.785	0.794	0.817
		(2.16)**	(2.20)**	(2.08)**	(2.04)**	(2.09)**
TR	0.139	0.158	0.163	0.168	0.171	0.183
		(3.03)***	(3.10)***	(2.96)***	(2.82)***	(2.73)***
Avg. N	465	175	150	125	100	75

Panel B: Low-Beta Portfolios vs. Small-Value Market Portfolio

STAT	SMVAL	B175	B150	B125	B100	B75
COR	0.178	0.177	0.179†††	0.179†††	0.179†††	0.178†††
		(0.22)	(-0.64)	(-0.43)	(-0.38)	(-0.09)
BETA	0.927	0.931†††	0.931†††	0.936†††	0.933†††	0.931†††
		(-0.40)	(-0.43)	(-0.77)	(-0.44)	(-0.23)
STD	18.55%	18.83%†††	18.86%†††	19.06%†††	19.11%†††	19.43%†††
		(-1.61)*	(-1.78)**	(-2.36)**	(-1.99)**	(-2.36)**
RP	13.62%	13.91%	13.82%	13.92%	13.78%	14.05%
		(0.76)	(0.45)	(0.54)	(0.24)	(0.55)
SR	0.682	0.692	0.691	0.669††	0.659††	0.664††
		(0.31)	(0.31)	(-0.39)	(-0.54)	(-0.34)
TR	0.139	0.147	0.148	0.149†	0.151	0.154
		(1.69)**	(1.91)**	(1.57)*	(1.58)*	(1.65)*
OVLP %	–	41.68%	36.07%	31.13%	25.47%	19.02%
Avg. N	465	175	150	125	100	75

Panel C: Alphas for Singleton Portfolios of Small Value Stocks

	S175	S150	S125	S100	S75
Alpha (%)	0.164**	0.176**	0.168**	0.138	0.196**
	(2.19)	(2.22)	(2.02)	(1.60)	(2.16)
Market Factor	0.721***	0.703***	0.682***	0.676***	0.657***
	(37.12)	(35.86)	(33.26)	(30.34)	(28.74)
SMB Factor	0.780***	0.770***	0.759***	0.752***	0.752***
	(21.72)	(21.69)	(21.04)	(19.52)	(19.96)
HML Factor	0.359***	0.344***	0.316***	0.323***	0.319***
	(9.78)	(9.32)	(8.22)	(7.85)	(7.89)
Momentum Factor	-0.187***	-0.196***	-0.186***	-0.177***	-0.183***
	(-6.82)	(-7.03)	(-6.70)	(-6.11)	(-5.68)
BaB Factor	0.284***	0.296***	0.313***	0.315***	0.299***
	(10.56)	(10.83)	(10.89)	(10.06)	(9.18)
Observations	588	588	588	588	588
Avg Portfolio Size	175	150	125	100	75
Adj R-Squared	0.90	0.89	0.88	0.86	0.84
F-stat Market=1	206.59	230.03	240.27	211.53	224.17

Table 6
Low-Volatility Stocks: Low Correlation vs. Low Beta

This table summarizes the average performance of the low volatility portfolio (LowVol) between 1967 (to be consistent with Table 5) and 2015 as well as low-correlation and low-beta portfolios drawn from this market. We start with all CRSP stocks with at least 36 months of prior returns, stock price $\geq$ \$5 as of last trading day of year $t-1$, and a share code of 10 or 11 (MKT), and then select stocks in the lowest quintile based on their prior 36-month historical standard deviation of returns. From this universe, each year we form portfolios of size $n = \{75, 100, 125, 150, 175\}$ comprised of either low-correlation (Panel A) or low-beta (Panel B) stocks. Panel C examines the exposure of the low-correlation portfolios (chosen from the low-volatility market) to known risk factors by regressing the excess monthly return of these low-correlation portfolios on the market (CRSP value-weighted market portfolio less risk-free rate), SMB, HML, momentum, and betting-against-beta (BaB) factors. To identify n low-correlation stocks (i.e., singletons) each year, we find the cutoff value c^* such that only n stocks have all pairwise correlations (based on prior 36 months) less than c^* (i.e., $\rho_{i,j} < c^* \; \forall \; j \neq i$). To identify n low-beta stocks, we calculate the beta of each stock as of the beginning of year t using the prior 36-months of returns and choose the n stocks with the lowest historical betas. We tabulate average pairwise return correlations (COR), beta (BETA), standard deviation (STD), risk premium (RP), Sharpe ratios (SR), Treynor ratios (TR), and average number of stocks (N) for each of these portfolios. The Sharpe ratio is portfolio annual return less the risk-free rate (t-bill monthly rate) divided by portfolio standard deviation and the Treynor ratio is portfolio annual return less risk-free rate divided by portfolio beta (calculated using the portfolio's 12 monthly returns). Overlap % (reported in Panel B) is the percentage of low-beta stocks that are also low-correlation stocks. T-statistics are reported in parentheses; ***(**)(*) denotes a statistically significant one-tailed t-test (two-tailed for RP) of the null hypothesis that the portfolio mean is less than or equal to (for COR, BETA, and STD) or greater than or equal to (for SR and TR) the corresponding CRSP market mean at the 1%(5%)(10%) level, respectively. †††(††)(†) denotes a statistically significant two-tailed t-test of the null hypothesis that the low correlation singleton portfolio mean summary statistic in Panel A is different than the corresponding low beta portfolio mean in Panel B at the 1%(5%)(10%) level, respectively.

Panel A: Low-Correlation Portfolios vs. Low-Volatility Market Portfolio

STAT	LowVol	S175	S150	S125	S100	S75
COR	0.201	0.149	0.142	0.137	0.131	0.122
		(9.96)***	(10.13)***	(10.06)***	(10.08)***	(10.49)***
BETA	0.561	0.519	0.505	0.496	0.484	0.470
		(3.62)***	(4.38)***	(4.56)***	(4.85)***	(5.16)***
STD	11.20%	10.33%	10.10%	9.95%	9.84%	9.67%
		(3.98)***	(4.55)***	(4.69)***	(4.93)***	(5.11)***
RP	9.63%	9.81%	9.85%	9.71%	9.76%	9.83%
		(0.38)	(0.43)	(0.15)	(0.23)	(0.32)
SR	1.035	1.158	1.185	1.202	1.177	1.217
		(1.72)**	(1.86)**	(1.86)**	(1.41)**	(1.68)**
TR	0.206	0.224	0.234	0.237	0.237	0.249
		(1.50)*	(1.79)**	(1.81)*	(1.49)*	(1.86)**
Avg. N	546	175	150	125	100	75

Panel B: Low-Beta Portfolios vs. Low-Volatility Market Portfolio

STAT	LowVol	B175	B150	B125	B100	B75
COR	0.201	0.202†††	0.203†††	0.201†††	0.201†††	0.200†††
		(-0.21)	(-0.71)	(-0.04)	(-0.00)	(0.16)
BETA	0.561	0.556†††	0.561†††	0.561†††	0.559†††	0.561†††
		(0.90)	(-0.09)	(-0.03)	(0.25)	(0.01)
STD	11.20%	11.20%†††	11.32%†††	11.29%†††	11.32%†††	11.48%†††
		(-0.07)	(-1.17)	(-0.80)	(-0.81)	(-1.64)*
RP	9.63%	9.36%	9.35%	9.22%	9.26%	9.26%
		(0.06)	(0.04)	(-0.70)	(-0.41)	(-0.36)
SR	1.035	1.058	1.047†	1.025†	1.044	1.077
		(0.73)	(0.41)	(-0.28)	(0.20)	(0.88)
TR	0.206	0.217	0.215	0.208	0.213	0.227
		(1.35)*	(1.25)	(0.30)	(0.71)	(1.74)**
Overlap	–	35.42%	30.37%	25.18%	20.39%	15.35%
Avg. N	546	175	150	125	100	75

Panel C: Alphas for Singleton Portfolios of Low Volatility Stocks

	S175	S150	S125	S100	S75
Alpha (%)	0.095	0.114*	0.109*	0.121*	0.147**
	(1.60)	(1.88)	(1.82)	(1.93)	(2.40)
Market Factor	0.563***	0.544***	0.530***	0.517***	0.499***
	(33.97)	(31.86)	(30.58)	(28.66)	(26.95)
SMB Factor	0.274***	0.275***	0.281***	0.273***	0.276***
	(10.58)	(10.29)	(9.91)	(9.08)	(8.87)
HML Factor	0.176***	0.170***	0.152***	0.142***	0.129***
	(4.97)	(4.78)	(4.23)	(3.92)	(3.51)
Momentum Factor	-0.086***	-0.086***	-0.089***	-0.086***	-0.091***
	(-3.62)	(-3.73)	(-4.03)	(-3.69)	(-4.18)
BaB Factor	0.322***	0.320***	0.327***	0.324***	0.319***
	(12.14)	(12.02)	(12.34)	(12.28)	(12.10)
Observations	588	588	588	588	588
Avg Portfolio Size	175	150	125	100	75
Adj R-Squared	0.88	0.87	0.86	0.84	0.83
F-stat Market=1	693.03	712.50	737.05	715.27	733.71

Table 7
High Momentum Stocks: Low Correlation vs. Low Beta

This table summarizes the average performance of the high momentum portfolio (HIMOM) between 1967 (to be consistent with Table 5) and 2015 as well as low-correlation and low-beta portfolios drawn from this market. We start with all CRSP stocks with at least 36 months of prior returns, stock price $\geq \$5$ as of last trading day of year $t-1$, and a share code of 10 or 11 (MKT), and then select stocks in the highest quintile based on their prior 36-month cumulative return. From this universe, each year we form portfolios of size $n = \{75, 100, 125, 150, 175\}$ comprised of either low-correlation (Panel A) or low-beta (Panel B) stocks. Panel C examines the exposure of the low-correlation portfolios (chosen from the high momentum market) to known risk factors by regressing the excess monthly return of these low-correlation portfolios on the market (CRSP value-weighted market portfolio less risk-free rate), SMB, HML, momentum, and betting-against-beta (BaB) factors. To identify n low-correlation stocks (i.e., singletons) each year, we find the cutoff value c^* such that only n stocks have all pairwise correlations (based on prior 36 months) less than c^* (i.e., $\rho_{i,j} < c^* \ \forall \ j \neq i$). To identify n low-beta stocks, we calculate the beta of each stock as of the beginning of year t using the prior 36-months of returns and choose the n stocks with the lowest historical betas. We tabulate average pairwise return correlations (COR), beta (BETA), standard deviation (STD), risk premium (RP), Sharpe ratios (SR), Treynor ratios (TR), and average number of stocks (N) for each of these portfolios. The Sharpe ratio is portfolio annual return less the risk-free rate (t-bill monthly rate) divided by portfolio standard deviation and the Treynor ratio is portfolio annual return less risk-free rate divided by portfolio beta (calculated using the portfolio's 12 monthly returns). Overlap % (reported in Panel B) is the percentage of low-beta stocks that are also low-correlation stocks. T-statistics are reported in parentheses; ***(**)(*) denotes a statistically significant one-tailed t-test (two-tailed for RP) of the null hypothesis that the portfolio mean is less than or equal to (for COR, BETA, and STD) or greater than or equal to (for SR and TR) the corresponding CRSP market mean at the 1%(5%)(10%) level, respectively. †††(††)(†) denotes a statistically significant two-tailed t-test of the null hypothesis that the low correlation singleton portfolio mean summary statistic in Panel A is different than the corresponding low beta portfolio mean in Panel B at the 1%(5%)(10%) level, respectively.

Panel A: Low-Correlation Portfolios vs. High Momentum Market Portfolio

STAT	HIMOM	S175	S150	S125	S100	S75
COR	0.203	0.156	0.153	0.149	0.143	0.134
		(9.35)***	(9.13)***	(9.12)***	(9.53)***	(9.38)***
BETA	0.913	0.828	0.821	0.809	0.790	0.767
		(5.73)***	(5.44)***	(6.01)***	(6.55)***	(6.53)***
STD	16.71%	15.40%	15.32%	15.21%	15.04%	14.95%
		(4.70)***	(4.27)***	(4.54)***	(4.58)***	(3.87)***
RP	10.27%	10.94%	11.13%	11.00%	10.98%	11.81%
		(1.52)	(1.64)	(1.26)	(1.03)	(1.77)*
SR	0.670	0.775	0.787	0.782	0.794	0.814
		(2.65)**	(2.72)**	(2.45)**	(2.28)**	(2.37)***
TR	0.119	0.143	0.148	0.149	0.154	0.164
		(3.47)**	(3.65)**	(3.57)***	(3.37)***	(3.55)***
Avg. N	546	175	150	125	100	75

Panel B: Low-Beta Portfolios vs. High Momentum Market Portfolio

STAT	HIMOM	B175	B150	B125	B100	B75
COR	0.203	0.203†††	0.203†††	0.203†††	0.202†††	0.203†††
		(0.37)	(0.26)	(0.08)	(0.43)	(0.06)
BETA	0.913	0.909†††	0.916†††	0.915†††	0.909†††	0.916†††
		(0.40)	(-0.31)	(-0.20)	(0.30)	(0.18)
STD	16.71%	16.87%†††	17.05%†††	17.12%†††	17.02%†††	17.31%†††
		(-1.02)	(-1.89)**	(-1.95)**	(-1.31)*	(-1.95)**
RP	10.27%	10.46%	10.49%	10.59%	10.60%	10.44%
		(0.55)	(0.61)	(0.93)	(0.77)	(0.28)
SR	0.670	0.695†	0.684††	0.695	0.688	0.646††
		(1.10)	(0.54)	(0.70)	(0.46)	(-0.52)
TR	0.119	0.124††	0.123†††	0.124†††	0.123†††	0.121†††
		(1.11)	(0.74)	(0.90)	(0.80)	(0.31)
Overlap %	–	34.60%	29.95%	24.62%	19.65%	15.56%
Avg. N	546	175	150	125	100	75

Panel C: Alphas for Singleton Portfolios of Momentum Stocks

	S175	S150	S125	S100	S75
Alpha (%)	0.109*	0.111*	0.103	0.114*	0.172**
	(1.94)	(1.88)	(1.60)	(1.72)	(2.48)
Market Factor	0.766***	0.755***	0.746***	0.718***	0.689***
	(51.45)	(47.86)	(43.51)	(39.96)	(36.65)
SMB Factor	0.562***	0.567***	0.551***	0.554***	0.566***
	(19.70)	(21.56)	(17.93)	(17.73)	(20.96)
HML Factor	0.099***	0.106***	0.117***	0.083***	0.075**
	(3.90)	(3.99)	(4.07)	(2.82)	(2.40)
Momentum Factor	-0.050***	-0.054***	-0.064***	-0.071***	-0.070***
	(-2.85)	(-3.02)	(-3.23)	(-3.43)	(-3.57)
BaB Factor	0.234***	0.246***	0.251***	0.275***	0.291***
	(10.49)	(10.37)	(9.50)	(10.68)	(11.34)
Observations	588	588	588	588	588
Avg Portfolio Size	175	150	125	100	75
Adj R-Squared	0.93	0.92	0.91	0.90	0.88
F-stat Market=1	245.79	241.93	218.75	245.87	273.90

Appendix A

Table A1
Comovement: Random Portfolios vs Low Correlation

This table reports the results from randomly choosing 1,000 portfolios of 250 stocks at the beginning of each year from the CRSP universe and calculating the mean pairwise return correlation over the previous 36 months (RAND 250). *Prob* is the percentage of times a random portfolio had a lower mean historical pairwise return correlation than the 250 singleton portfolio in a given year and S250 is the mean pairwise correlation of the 250 singleton portfolio.

Yr	Rand250	Prob	S250	Yr	Rand250	Prob	S250	Yr	Rand250	Prob	S250
1950	0.376	0	0.227	1972	0.366	0	0.216	1994	0.096	0	0.012
1951	0.353	0	0.215	1973	0.309	0	0.163	1995	0.079	0	0.005
1952	0.275	0	0.163	1974	0.314	0	0.151	1996	0.066	0	0.003
1953	0.268	0	0.155	1975	0.307	0	0.160	1997	0.080	0	0.006
1954	0.275	0	0.126	1976	0.348	0	0.107	1998	0.087	0	0.006
1955	0.282	0	0.128	1977	0.318	0	0.080	1999	0.172	0	0.029
1956	0.233	0	0.100	1978	0.265	0	0.040	2000	0.148	0	0.034
1957	0.226	0	0.093	1979	0.270	0	0.052	2001	0.111	0	0.030
1958	0.209	0	0.086	1980	0.270	0	0.051	2002	0.078	0	0.019
1959	0.262	0	0.134	1981	0.309	0	0.089	2003	0.110	0	0.027
1960	0.224	0	0.108	1982	0.255	0	0.091	2004	0.158	0	0.025
1961	0.203	0	0.104	1983	0.242	0	0.076	2005	0.161	0	0.029
1962	0.194	0	0.102	1984	0.183	0	0.043	2006	0.143	0	0.016
1963	0.349	0	0.215	1985	0.186	0	0.029	2007	0.119	0	0.007
1964	0.355	0	0.219	1986	0.171	0	0.027	2008	0.116	0	0.007
1965	0.351	0	0.200	1987	0.191	0	0.022	2009	0.202	0	0.045
1966	0.153	0	0.060	1988	0.323	0	0.087	2010	0.260	0	0.062
1967	0.194	0	0.091	1989	0.313	0	0.093	2011	0.291	0	0.074
1968	0.242	0	0.111	1990	0.305	0	0.082	2012	0.295	0	0.063
1969	0.239	0	0.117	1991	0.197	0	0.027	2013	0.281	0	0.035
1970	0.301	0	0.161	1992	0.189	0	0.023	2014	0.227	0	0.027
1971	0.351	0	0.209	1993	0.180	0	0.028	2015	0.167	0	0.018
								MEAN:	**0.230**	**0**	**0.082**

Table A2

Robustness: Low Correlation Portfolios vs CRSP Universe (1950-2015)

This table summarizes the average performance of the equal-weighted CRSP universe between 1950 and 2015, as well as low-correlation portfolios drawn from this universe. We start with all CRSP stocks with at least 36 months of prior returns, stock price $\geq$ \$5 as of last trading day of year $t-1$, and a share code of 10 or 11 (MKT), resulting in an average sample of 2,274 stocks per year. From this universe, each year we find the cutoff value c^* such that only $X\%$ of these stocks have all pairwise correlations (based on prior 36 months) less than c^* (i.e., $\rho_{i,j} < c^* \ \forall \ j \neq i$). We tabulate results for the values of X that produce an average portfolio size of $\{500, 400, 350, 300, 250, 200, 100\}$ across our 66-year sample to match the portfolios chosen from Table 1. We tabulate average pairwise return correlations (COR), beta (BETA), standard deviation (STD), risk premium (RP), Sharpe ratios (SR), Treynor ratios (TR) and average number of stocks (N) for each of these portfolios. Sharpe ratio is portfolio annual return less the risk-free rate (t-bill monthly rate) divided by portfolio standard deviation and Treynor ratio is portfolio annual return less risk-free rate divided by portfolio beta (calculated using the portfolio's 12 monthly returns). T-statistics are reported in parentheses; ***(**)(*) denotes a statistically significant one-tailed t-test (two-tailed for RP) of the null hypothesis that the portfolio mean is less than or equal to (for COR, BETA, and STD) or greater than or equal to (for SR and TR) the corresponding CRSP market mean at the 1%(5%)(10%) level, respectively.

STAT	**MKT**	**22.0%**	**17.6%**	**15.4%**	**13.2%**	**11.0%**	**8.8%**	**4.4%**
COR	0.204	0.152	0.147	0.143	0.139	0.134	0.131	0.124
		(13.1)***	(13.2)***	(13.6)***	(14.0)***	(14.4)***	(14.2)***	(13.4)***
BETA	1	0.848	0.833	0.817	0.801	0.787	0.775	0.740
		(13.1)***	(12.9)***	(13.8)***	(14.4)***	(14.5)***	(13.5)***	(14.3)***
STD	16.62%	14.35%	14.15%	13.91%	13.70%	13.51%	13.44%	13.30%
		(7.98)***	(7.83)***	(8.29)***	(8.46)***	(8.66)***	(8.44)***	(7.84)***
RP	10.49%	10.57%	10.61%	10.52%	10.36%	10.34%	10.33%	9.31%
		(0.14)	(0.22)	(0.04)	(0.22)	(0.26)	(0.25)	(1.60)
SR	0.649	0.793	0.805	0.813	0.826	0.832	0.818	0.698
		(3.74)***	(3.61)***	(3.65)***	(3.65)***	(3.55)***	(2.77)***	(0.73)
TR	0.105	0.130	0.134	0.135	0.137	0.141	0.143	0.134
		(3.95)***	(4.03)***	(4.06)***	(4.07)***	(4.13)***	(3.58)***	(2.42)***
Mean N	2274	500	400	350	300	250	200	100

Table A3
S250 vs Random Portfolio Summary Stats (1950-2015)

This table summarizes the average performance over years 1950 to 2015 of the equal-weighted CRSP universe, value-weighted CRSP universe, 250 singleton portfolio, and 1,000 randomly selected portfolios of 250 stocks using uniform and value-weighted probability weights (RAND 250ew and RAND 250vw, respectively). We tabulate average pairwise return correlations (COR), betas (BETA), standard deviations (STD), portfolio risk premiums (RP), Sharpe ratios (SR), and Treynor ratios (TR). A statistically significant one-tailed t-test (two-tailed for RP and BETA) of the null hypothesis that the singleton portfolio mean is less than or equal to (for COR and STD) or greater than or equal to (for SR and TR) the corresponding random mean at the 1%, 5%, and 10% level is denoted by ***, **, and *, respectively (t-stat in parentheses).

STAT	**MKTew**	**MKTvw**	**S250**	**RAND 250ew**	**RAND 250vw**
COR	0.204	0.204	0.139	0.204 (13.2)***	0.259 (13.1)***
BETA	1.000	1.000	0.790	1.000 (13.8)***	0.882 (3.49)***
STD	16.6%	14.2%	13.5%	16.8% (8.89)***	15.1% (3.37)***
RP	10.5%	7.87%	10.4%	10.5% (0.190)	8.58% (1.48)*
SR	0.649	0.671	0.830	0.638 (3.49)***	0.671 (1.77)**
TR	0.105	0.079	0.138	0.105 (3.69)***	0.106 (2.09)**

Table A4
S250 vs Market over Various-Length Non-Overlapping Time Periods

This table reports the mean portfolio summary statistics of the market and singleton portfolios over non-overlapping 5, 10 and 33-year time periods throughout our 1950-2015 sample period. Checkmarks (✓) indicate whether the market or singleton portfolio wins in terms of having a higher Sharpe Ratio (SR) and Treynor Ratio (TR) for a given period.

		MKT					S250					
	Time Period	**COR**	**STD**	**RP**	**SR**	**TR**	**COR**	**BETA**	**STD**	**RP**	**SR**	**TR**
5 yr	1951-55	0.236	12.14%	18.05%	1.295	0.181	0.167	0.769	9.39%	15.35%	1.459✓	0.192✓
	1956-60	0.187	10.71%	9.96%	0.781	0.099	0.144	0.868	9.64%	12.09%	1.117✓	0.146✓
	1961-65	0.250	13.63%	13.76%	1.307	0.138	0.210	0.896	12.90%	14.70%	1.494✓	0.166✓
	1966-70	0.287	20.68%	5.52%	0.023✓	0.055	0.238	0.884	18.60%	6.43%	0.009	0.073✓
	1971-75	0.302	22.00%	0.33%	-0.282	0.003	0.260	0.954	21.18%	0.64%	-0.178✓	0.015✓
	1976-80	0.269	22.38%	20.33%	0.935	0.203	0.161	0.782	17.74%	20.39%	1.305✓	0.267✓
	1981-85	0.170	16.32%	9.29%	0.479	0.093	0.081	0.703	12.23%	11.24%	0.858✓	0.159✓
	1986-90	0.221	16.88%	-1.38%	0.045✓	-0.014✓	0.133	0.718	12.78%	-3.06%	-0.187	-0.039
	1991-95	0.079	11.46%	16.96%	1.410	0.170	0.049	0.802	9.72%	17.58%	1.672✓	0.219✓
	1996-00	0.108	16.45%	7.45%	0.516✓	0.074✓	0.085	0.816	13.90%	5.70%	0.399	0.063
	2001-05	0.139	17.85%	14.47%	0.815	0.145	0.079	0.630	12.35%	16.36%	1.343✓	0.252✓
	2006-10	0.205	20.59%	4.80%	-0.034✓	0.048✓	0.120	0.710	13.66%	-0.09%	-0.195	0.018
	2011-15	0.188	14.62%	12.04%	0.894	0.120	0.075	0.745	11.37%	14.28%	1.463✓	0.218✓
10 yr	1951-60	0.211	11.42%	14.00%	1.038	0.140	0.156	0.819	9.52%	13.72%	1.288✓	0.169✓
	1961-70	0.268	17.16%	9.64%	0.665	0.096	0.224	0.890	15.75%	10.56%	0.752✓	0.120✓
	1971-80	0.285	22.19%	10.33%	0.327	0.103	0.211	0.868	19.46%	10.51%	0.564✓	0.141✓
	1981-90	0.196	16.60%	3.96%	0.262	0.040	0.107	0.711	12.51%	4.09%	0.335✓	0.060✓
	1991-00	0.094	13.95%	12.20%	0.963	0.122	0.067	0.809	11.81%	11.64%	1.036✓	0.141✓
	2001-10	0.172	19.22%	9.64%	0.391	0.096	0.099	0.670	13.00%	8.14%	0.574✓	0.135✓
33 yr	1950-82	0.252	17.03%	11.63%	0.682	0.116	0.191	0.851	15.26%	11.28%	0.870✓	0.146✓
	1983-15	0.156	16.20%	9.36%	0.616	0.094	0.088	0.731	12.18%	9.09%	0.790✓	0.130✓

Table A5
S250 vs Market over Various-Length Overlapping Time Periods

This table reports the mean portfolio summary statistics of the market and singleton portfolios over various length overlapping year subsets of our 66 year sample time period (i.e., 62 five-year periods). Periods for which singleton portfolio Sharpe and Treynor ratios (SR and TR, respectively) are greater than their respective market counterparts are indicated with a checkmark (✓).

Time Period (1950-2015)	N	MKT					S250					
		COR	**STD**	**RP**	**SR**	**TR**	**COR**	**BETA**	**STD**	**RP**	**SR**	**TR**
5 yr	62	0.203	16.79%	10.20%	0.629	0.102	0.140	0.791	13.66%	10.20%	0.805✓	0.135✓
10 yr	57	0.201	16.85%	9.76%	0.603	0.098	0.140	0.795	13.83%	9.90%	0.771✓	0.130✓
15 yr	52	0.201	17.07%	9.63%	0.590	0.096	0.141	0.797	14.09%	9.83%	0.753✓	0.128✓
20 yr	47	0.203	17.23%	9.43%	0.568	0.094	0.142	0.800	14.28%	9.65%	0.728✓	0.126✓
25 yr	42	0.205	17.37%	9.38%	0.556	0.094	0.142	0.798	14.38%	9.60%	0.718✓	0.126✓
30 yr	37	0.206	17.48%	9.47%	0.557	0.095	0.142	0.795	14.41%	9.71%	0.723✓	0.128✓
35 yr	32	0.206	17.48%	9.47%	0.555	0.095	0.141	0.794	14.41%	9.72%	0.723✓	0.128✓
40 yr	27	0.205	17.43%	9.44%	0.557	0.094	0.142	0.796	14.40%	9.66%	0.721✓	0.127✓
45 yr	22	0.204	17.28%	9.37%	0.563	0.094	0.142	0.799	14.32%	9.60%	0.723✓	0.126✓
50 yr	17	0.202	17.14%	9.55%	0.580	0.096	0.142	0.799	14.18%	9.74%	0.739✓	0.127✓
55 yr	12	0.201	16.94%	9.73%	0.599	0.097	0.141	0.796	13.94%	9.91%	0.767✓	0.130✓
60 yr	7	0.202	16.84%	9.93%	0.610	0.099	0.140	0.792	13.73%	9.94%	0.779✓	0.131✓
65 yr	2	0.204	16.64%	10.42%	0.647	0.104	0.139	0.790	13.52%	10.37%	0.830✓	0.138✓
66 yr	1	0.204	16.62%	10.49%	0.649	0.105	0.139	0.790	13.50%	10.38%	0.830✓	0.138✓