RISK, RETURN, AND HEDGING OF FIXED-RATE MORTGAGES

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Mortgages comprise 31% of the U.S. credit market outstanding, compared to only 21% in U.S. Treasury and agency securities and 13% in corporate bonds. Bank loans, municipal bonds, and consumer credit each represent only 7% of the credit market. In recent years, the supply of new mortgage securities has been more than twice the supply of new corporate bonds issued. The mortgage market today is one of the most liquid markets in the world, despite the fact that only about one-third of total residential mortgages have been securitized (as of 1989).

Such observations demonstrate the importance of understanding the risks and returns of mortgage securities. Because many savings and loans have failed for lack of risk management of mortgage portfolios, it is also important to understand the hedging of mortgage securities. Hedging allows banking institutions to take risks they would otherwise consider unacceptable and pass those risks to other market participants who specialize in taking them.

The risk, return, and hedging aspects of mortgages are more complicated than for most fixed-income securities, because mortgages give the borrower the option to prepay the loan at par at any time during the life of the loan. While our understanding of risk, return, pricing, and hedging of securities with options has been propelled by the seminal work of Black and Scholes [1973] and the literature that has built upon that work, mortgages are much more complicated option-like securities than those dealt with by Black and Scholes.

Black and Scholes assume, for example, 1) that interest rates are constant, 2) that the option is "European" in that it could be exercised only on the final maturity date, 3) that the exercise price is known and

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fixed, and 4) that the returns on the underlying asset are normally distributed. Of course, the volatility of interest rates is the reason the prepayment option in a mortgage has value and is of interest. Furthermore, it may well be optimal for different people to exercise their prepayment options at different times, particularly as they have different effective costs of exercising their options. Finally, the underlying asset for the prepayment option is a bond, which certainly does not have normally distributed returns.

In this article, mortgages are viewed as far too complicated to value precisely and rigorously, even with the Black-Scholes model and the many improvements developed in the eighteen subsequent years. Given this view, my goal here is limited and data-oriented.

Data on interest rates, prepayment rates, and mortgage prices are used to develop risk and return properties and hedging methods that reflect the information in market prices. Assuming that market prices reflect the information of well-informed investors, mortgage prices for different coupons should reflect some of the most up-to-date values and models of the complicated prepayment options. The article uses both the cross section of mortgage prices by coupon and the time series of prices and prepayment rates to develop risk and return estimates and hedging strategies. It is shown that many of the risk functions inferred empirically do have characteristics that would be expected from the option pricing theory of Black and Scholes.

The flow of the paper is as follows. Section I examines mortgage payments and the prepayment option. Section II examines historical interest rate volatility and the yield spread between mortgages and Treasuries. Section III analyzes mortgage prepayment rates and the factors that move them. Section IV examines historical mortgage prices and their sensitivity to bond prices. Section V examines the market's implicit price elasticities or "effective durations" for different mortgage coupons and relates them to mortgage price volatility. Section VI develops effective dynamic hedging strategies for mortgages. Section VII presents data series on monthly rates of return for different mortgage coupons and summary statistics of risk and return for them. Section VIII summarizes the paper's principal results.

I. MORTGAGE PAYMENTS AND THE PREPAYMENT OPTION

Consider a borrower who takes out a standard thirty-year fixed-rate mortgage for $100,000 to buy a house worth $125,000. One can calculate that the mortgage must have level monthly payments of $952.32 to amortize the loan and reflect an annual interest rate of 11%, compounded monthly.

As this is a fixed-rate, fixed-payment loan, if market rates increase to 12%, the present value of the borrower's fixed payments declines to $92,583. A smaller debt is good for the borrower, which reflects the now below-market rate of 11%. On the other hand, if rates decrease to 10%, the present value of the borrower's payments increases to $108,518, reflecting the fact that the borrower's loan is now above the current market rate.

Table 1 shows the entire schedule of payments and their present values for three different mortgages with 9%, 11%, and 13% rates, discounted at current market rates from 7% to 18%.

A typical fixed-rate mortgage provides the borrower with the option to pay off the mortgage at the unpaid outstanding balance ($100,000 initially), usually with no prepayment penalty. Continuing with the 11% fixed-rate mortgage example, if rates decrease to 10%, the borrower's same monthly payments would amortize a new loan for $108,518, which could be obtained from another bank to pay off the old loan. By prepaying, the borrower pockets $8,518 in present value (8.52 "points" or % of par), less any refinancing costs not included in the rate (usually assumed to be two to five points).

The gross amounts of the refinancing gains from prepaying mortgages with 9%, 11%, and 13% coupons are also in Table 1. Note that the gains from refinancing and prepayment can be quite large; a 13% mortgage refinanced at 9%, for example, results in a $37,481 gain per $100,000 of loan balance, less refinancing costs.

Looking at the column labeled "Prepayment Profits" for the 11% mortgage, one can see that they are equal to a call option on the present value of the mortgage's fixed cash flows without prepayments, with an exercise price of par. The call option is "in the money" if rates have fallen below the fixed mortgage rate, and many models assume that the mortgage will be refinanced and prepaid. Thus, in theory, whenever the current market mortgage rate is below the fixed coupon rate (or far enough below to offset refinancing costs), the bank or mortgage investor will be paid off and receive $100,000 (or 100% par). If rates have risen above the mortgage's fixed rate, then the borrower is assumed to keep the mortgage outstanding, resulting in values below par for the bank or mortgage investor.

TABLE 1 ■ Value of the Option to Prepay a $100,000 Mortgage

			Borrower's Fixed Mortgage Rate								
			9.00%			11.00%			13.00%		
Monthly Loan Payment			$804.62			$952.32			$1,106.20		
Current Mortgage Rate (%)	20-Year TBond Yield (%)	TBond Futures Price	PV (Payment)	Prepayment Profit	T-Bond Call X = 105.14 Q = 0.84	PV (Payment)	Prepayment Profit	TBond Call X = 86.68 Q = 0.99	PV (Payment)	Prepayment Profit	TBond Call X = 72.82 Q - 1.16
7.00	5.50	130.10	120,941	20.94	20.94	143,141	43.14	43.14	166,270	66.27	66.27
8.00	6.50	116.66	109,657	9.66	9.66	129,786	29.79	29.79	150,757	50.76	50.72
9.00	7.50	105.14	100,000	0.00	0.00	118,357	18.36	18.34	137,481	37.48	37.39
10.00	8.50	95.23	91,687	0.00	0.00	108,518	8.52	8.50	126,052	26.05	25.93
11.00	9.50	86.68	84,490	0.00	0.00	100,000	0.00	0.00	116,158	16.16	16.03
12.00	10.50	79.27	78,224	0.00	0.00	92,583	0.00	0.00	107,543	7.54	7.46
13.00	11.50	72.82	72,738	0.00	0.00	86,090	0.00	0.00	100,000	0.00	0.00
14.00	12.50	67.19	67,908	0.00	0.00	80,374	0.00	0.00	93,360	0.00	0.00
15.00	13.50	62.25	63,634	0.00	0.00	75,316	0.00	0.00	87,485	0.00	0.00
16.00	14.50	57.90	59,834	0.00	0.00	70,818	0.00	0.00	82,260	0.00	0.00

Notes: X = Exercise Price.
Q = Quantity of Call Options Purchased.

Thus, from the mortgage investor's point of view, the prepayment option is "heads (rates up) I lose, as my bond falls in value," and "tails (rates down), I don't win, as the mortgage is prepaid at par." This asymmetric situation (called "negative convexity" from its payoff graph's curvature) is not attractive unless there is compensation for this option risk. Insured mortgage investors are compensated in fact by a positive spread of about 1.25% above Treasury rates of comparable duration.

A mortgage investor's theoretical payoff pattern is shown in Figures 1A and 1B. The investor's profits are identical to buying a straight long-term, fixed-rate bond that cannot be prepaid (which has interest rate risk) and shorting a call option on that bond to the borrower. Alternatively, and equivalently through the "put and call parity" relationship, the investor's position has the risk of investing in riskless short-term Treasury bills and having written a put option on the long-term, fixed-rate bond having the level payments promised on the mortgage. If rates decrease, the investor receives a certain amount (called at par), while if rates increase, the mortgage remains outstanding (put to the investor) at a loss in market value.

The values of prepayment options on mortgages with different coupons are very similar to Treasury bond options with different exercise prices (see Table 1). For example, the prepayment option on a 9%, thirty-year mortgage has similar payoffs to those of 0.84 call options on twenty-year T-bond futures with an exercise price of 105.14. Payoffs on the prepayment options of 11% and 13% mortgages are likewise similar to those of 0.99 and 1.16 T-bond options with exercise prices of 86.68 and 72.82, respectively.

As simple as this analysis is, it is remarkably useful in understanding hedging and pricing for fixed-rate mortgages. To hedge these risks, we create offsetting positions through dynamic trading strategies or option purchases. A mortgage may be viewed either as long a bond and short a call, or just short a put. Thus, to hedge a mortgage security, one can either 1) short a straight bond and purchase or dynamically create a call option, or 2) purchase or dynamically create a put option. Whether a mortgage is priced properly depends upon whether the interest rate spread between the mortgage's yield and that on comparable risk non-callable bonds is too large or too small in relation to the costs of option purchase or option creation.

FIGURE 1A ■ Mortgage = Bond - Call Option

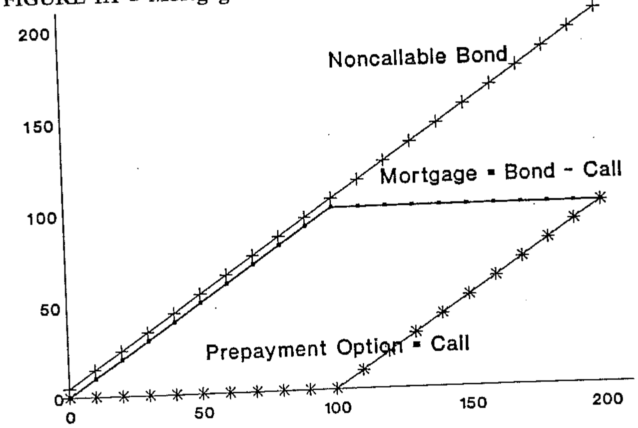


FIGURE 1B ▪ Mortgage = Riskless Bills - Put Option

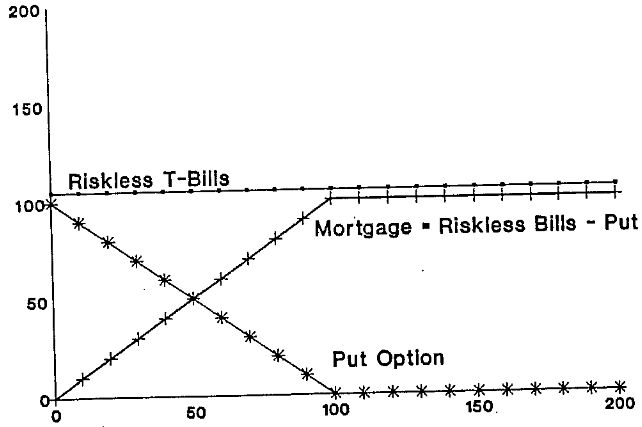
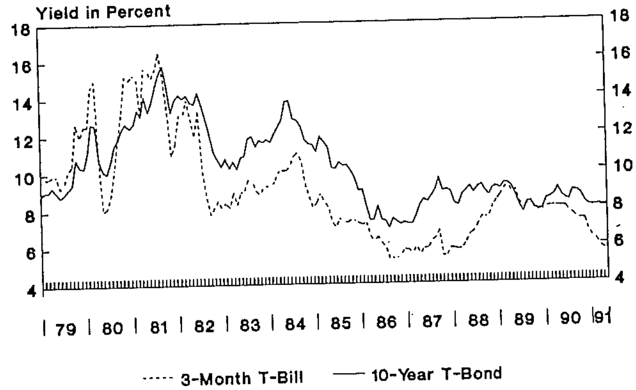


FIGURE 2 ▪ 3-Month and 10-Year Treasury Rates: 1979 to May 1991



II. INTEREST RATE VOLATILITY AND THE MORTGAGE-TREASURY YIELD SPREAD

Most data sets on mortgage-backed securities (MBS) start in the mid-1980s, as that is when the MBS market grew most rapidly and when data became more generally available. Early work on hedging MBS was done by Breeden and Giarla [1987], using data for the three and a half-year period from 1984 to mid-1987.

For this current article, data for the thirteen-year period from December 31, 1977, through December 31, 1990, were collected. The length of the period gives us more power to examine the effects of recession (1980, 1981-1982), rate volatility (1979-1982), and burnout on mortgage prepayments, pricing, and hedging (1988-1990).

Price data were obtained from the *Wall Street Journal* for the entire period. Prepayment data are from Salomon Brothers (monthly for 1983-1990) and from Drexel, Burnham, Lambert (annually for 1978-1982). The price data from the *Wall Street Journal* are known to contain several errors, so the data were carefully checked by examining the time series of price changes for the various coupons, as well as the time series of price spreads across coupons. Apparent errors in the data were corrected prior to the analysis. As the prices examined are for MBS guaranteed by the Government National Mortgage Association (GNMA), carrying a "full faith and credit" guarantee of the U.S. government, credit risk is not a serious issue and is ignored throughout the article.

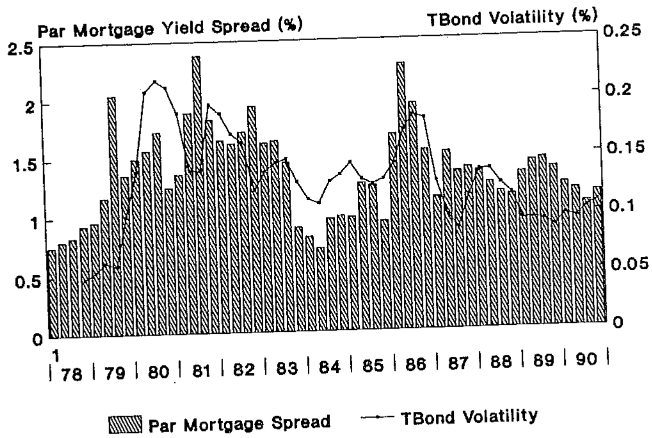
Interest rate volatility increased dramatically in 1979 when the Federal Reserve changed its monetary policy. Figure 2 shows the levels of both short-term and long-term interest rates for 1979-1991. Both short-term (three-month) and long-term (ten-year) rates exceeded 15% in 1981 and then dropped to as low as 5% and 7%, respectively. These huge interest rate movements caused correspondingly huge price movements in mortgages, Treasury bonds, and futures prices.

Note also that the slope of the yield curve changes quite significantly during the period examined. It was significantly downward-sloping in 1979, turns to steeply upward-sloping in 1983-1984, to downward-sloping in 1989, and back to a sharp upward slope in 1991. As McConnell and Singh [1991] have shown, the slope of the yield curve has significant implications for prepayment rates on adjustable-rate mortgages (ARMs) versus fixed-rate mortgages. It is plausible that the slope of the yield curve also affects prepayments on fixed-rate mortgages.

Figure 3 shows the spread in yields between current market rates for par GNMA mortgage securities and seven-year Treasury securities, which have similar price elasticity. Given that the securities have similar credit risks (none) and similar interest rate risks, spread fluctuations should reflect primarily the value of the prepayment option described in Section I. The borrower's prepayment option has greater value in times of greater rate volatility, and lenders and investors should require greater spreads of promised mortgage yields over Treasury yields at those times.

As interest rate volatility was greatest from 1979 to 1982 and in late 1985 to 1986, it is comforting that Figure 3 shows that mortgage-Treasury spreads are widest during those periods. Prior to late 1979 and from 1987-1990, rate volatility was comparatively low, which made the borrower's prepayment option less valuable and the equilibrium mortgage-Treasury spread low. Thus, the

FIGURE 3 ▪ Par Mortgage Spread to 7-Year Treasury



TBond volatility is annualized and based upon the last 12 months' returns.

general pattern of movements is broadly consistent with our simple prepayment option analysis.

III. PREPAYMENT DATA AND ANALYSIS

Prepayment rates on fixed-rate mortgages respond primarily to several factors: 1) the mortgage's coupon rate in relation to current refinancing opportunities; 2) the age of the mortgage; 3) the season of the year; 4) the degree of "burnout"; and 5) the growth/recession state of the macroeconomy.

The effects of the major variables on prepayments are as follows. First, higher-coupon mortgages should have higher prepayment rates, as Table 1 shows that the benefit from prepayments is larger. Second, few people refinance immediately after they enter into a mortgage loan. As a mortgage ages, people become more mobile and more inclined to refinance, which leads to increases in prepayment rates. After about two years, FHA mortality series show that prepayments on par mortgages flatten out at about an annual prepayment rate of 6%, and the mortgages are viewed as well-seasoned.

Next, mobility and prepayments are related to weather for homebuilding and school year timing, both of which lead to increases in prepayments during the summer and decreases during the winter. Richard and Roll [1989] show that the longer mortgage rates have been below the mortgage's coupon rate, and the longer it has been optimal to refinance, the more the composition of the pool's borrowers changes toward those who, for whatever reason, tend to be less inclined to refinance. The slowing of the prepayment rate of a pool of mortgages as the quick payers depart is called the burnout effect.

Finally, economic growth affects job mobility and therefore prepayment rates, as mortgages are often prepaid when the borrower moves to another location. Slower economic growth means less mobility and slower prepayments. The tables and figures to follow illustrate all these effects.

Table 2 shows the annualized percentage paydowns from prepayments quarterly from 1978 to 1990 for GNMAs with coupons from 8% to 15%. Note that these prepayment rates are for mortgages with fixed maturities, so the time series reflect mortgages that are aging. In contrast to FNMA and FHLMC mortgage-backed securities, all the mortgages underlying a GNMA MBS have initial maturities of thirty years, so one can be assured that a thirty-year GNMA MBS that matures in 2010 was issued in 1980. Also note that no data exist for high-coupon mortgages prior to 1980, as rates had never before been high enough for those high-coupon MBS to be issued!

Look at mortgages with coupons of 10% or higher that were recently issued, and the aging effect is apparent. Immediately after the mortgages were issued, prepayment rates were very low (usually <1%). Then, despite the very high interest rates of 1981-1982, prepayment rates increased on these new mortgages, while prepayments of seasoned discounts such as GNMA 8s and 9s slowed because of the very high interest rates.

The general sensitivity of prepayments to interest rate movements is also easy to see. When rates were high in 1981-1982, prepayments were low. When rates dropped sharply in 1985-1986, prepayment rates accelerated dramatically from the 5% to 10% range to as high as 45% on high coupons. Subsequent to that 1986-1987 period of rapid prepayments, the burnout effect described becomes apparent. Despite the fact that interest rates generally remained low in the 1988-1990 period, prepayment rates on high-coupon mortgages dropped by half or more. The expected seasonal pattern of prepayments is shown in Figure 4, with prepayments lowest in the winter and highest in summer.

A problem with examining the risks of investments in mortgage-backed securities is that the volatile movements in interest rates often dominate the effects of other variables and lead to apparent non-stationarities in the relationships. For example, when GNMA 13s sold at a discount to par in 1981-1982, it was not optimal to prepay them. Yet during most of the remaining period

TABLE 2 ▪ GNMA Annualized Paydowns From Prepayments
Quarterly 1978–1990

Date	Par Mtg Yield (%)	3 Month Treasury (%)	10 Year Treasury (%)	Slope 10Yr-3M (%)	TBond Futures Price	8%	9%	10%	11%	12%	13%	14%	15%
						Mortgage Coupon / Annualized Percentage Paydown							
33178	8.73	6.65	8.12	1.47	95.94	6.4	6.7						
63078	9.33	7.21	8.59	1.38	93.00	6.4	6.7						
93078	9.32	8.07	8.50	0.43	93.31	6.4	6.7						
123178	10.13	9.60	9.12	-0.48	90.34	6.4	6.7						
33179	10.06	9.77	9.08	-0.69	90.19	7.1	2.2	0.1					
63079	9.88	9.25	8.76	-0.49	91.81	7.1	2.2	0.1					
93079	11.50	10.44	9.42	-1.02	87.78	7.1	2.2	0.1					
123179	11.67	12.53	10.31	-2.22	82.19	7.1	2.2	0.1	0.8				
33180	14.08	14.98	12.60	-2.38	67.94	3.4	2.2	0.7	0.8				
63080	11.39	8.18	9.98	1.80	81.19	3.4	2.2	0.7	0.8				
93080	13.57	11.89	11.83	-0.06	70.69	3.4	2.2	0.7	0.8				
123180	13.72	15.02	12.43	-2.59	71.38	3.4	2.2	0.7	0.8				
33181	14.57	13.00	13.10	0.10	67.06	1.4	1.3	1.2	1.4		0.7	0.2	0.4
63081	15.94	15.08	13.84	-1.24	64.38	1.4	1.3	1.2	1.4		0.7	0.2	0.4
93081	18.42	15.15	15.76	0.61	56.00	1.4	1.3	1.2	1.4		0.7	0.2	0.4
123181	15.95	11.54	13.93	2.39	61.91	1.4	1.3	1.2	1.4		0.7	0.2	0.4
33182	16.01	13.90	14.17	0.27	61.94	1.1	1.1	1.3	1.9		2.6	2.3	4.2
63082	16.14	13.32	14.32	1.00	60.69	1.1	1.1	1.3	1.9		2.6	2.3	4.2
93092	13.38	7.79	11.93	4.14	71.06	1.1	1.1	1.3	1.9		2.6	2.3	4.2
123182	12.21	8.13	10.31	2.18	76.63	1.3	1.6	1.5	2.1	0.4	3.5	4.2	19.8
33183	12.13	8.95	10.59	1.64	75.97	2.3	3.5	2.0	3.1	0.7	5.8	16.4	46.4
63083	12.49	9.04	10.89	1.85	74.44	3.3	5.2	2.7	4.0	1.1	9.1	22.3	35.7
93083	12.79	9.00	11.39	2.39	72.72	3.7	5.8	3.0	4.4	1.8	8.4	15.6	21.0
123183	12.69	9.26	11.76	2.50	70.03	2.8	4.4	2.4	3.6	1.7	7.4	12.3	19.9
33184	13.20	9.98	12.43	2.45	66.22	2.6	2.5	2.9	3.7	3.1	7.8	11.6	22.1
63084	14.52	10.26	13.83	3.57	59.63	3.5	2.8	3.2	4.3	4.1	8.9	16.1	19.8
93084	13.46	10.58	12.40	1.82	67.22	2.5	2.3	2.5	3.4	3.8	7.1	11.5	12.7
123184	12.45	8.08	11.45	3.37	71.06	2.3	2.0	2.4	3.3	3.6	6.6	9.5	14.3
33185	12.65	8.44	11.63	3.19	69.72	2.5	2.3	2.5	3.4	4.0	7.6	13.4	22.3
63085	11.40	7.01	10.15	3.14	77.06	3.5	3.0	3.1	4.2	5.7	9.6	17.9	24.0
93085	11.41	7.27	10.33	3.06	75.59	4.7	4.1	4.2	6.0	9.1	16.1	31.8	35.8
123185	9.76	7.24	8.98	1.74	85.22	4.7	4.6	4.3	5.7	9.6	17.1	30.7	30.8
33186	9.00	6.51	7.38	0.87	102.31	4.4	5.1	4.4	7.2	16.2	25.0	34.8	39.8
63086	9.58	6.13	7.42	1.29	99.56	7.3	7.3	7.0	14.7	38.1	41.0	50.4	41.5
93086	9.24	5.31	7.49	2.18	96.56	10.0	9.7	10.3	21.2	49.3	47.2	51.9	46.7
123186	8.61	5.79	7.25	1.46	98.19	9.6	9.0	10.4	21.9	43.3	41.8	45.2	37.5
33187	8.53	5.95	7.62	1.67	98.47	7.7	8.0	12.2	24.4	41.2	36.1	37.2	33.4
63087	9.75	5.86	8.37	2.51	91.50	10.6	10.2	16.1	29.4	42.9	35.6	40.6	37.5
93087	10.80	6.88	9.65	2.77	81.69	8.5	8.8	10.6	16.7	27.4	27.8	31.7	29.2
123187	10.01	5.84	8.86	3.02	87.97	6.4	7.2	8.0	11.2	18.8	19.1	22.3	22.4

TABLE 2 ▪ Continued

Date	Par Mtg Yield (%)	3 Month Treasury (%)	10 Year Treasury (%)	Slope 10Yr-3M (%)	TBond Futures Price	8%	9%	10%	11%	12%	13%	14%	15%
						Mortgage Coupon Annualized Percentage Paydown							
33188	9.80	5.86	8.65	2.79	90.09	5.3	5.9	6.5	9.9	19.7	17.2	22.6	24.3
63088	9.90	6.74	8.80	2.06	88.75	7.9	8.5	10.6	15.3	27.5	24.9	29.0	26.2
93088	9.90	7.47	8.83	1.36	88.75	8.1	7.6	9.9	12.9	21.2	21.3	24.3	21.2
123188	10.34	8.36	9.13	0.77	89.13	6.8	7.9	8.3	10.7	18.8	17.0	21.6	19.6
33189	10.69	9.18	9.27	0.09	88.41	5.3	5.5	6.4	8.2	15.6	14.7	22.0	21.2
63089	9.51	8.24	8.08	-0.16	97.94	5.9	6.6	7.4	8.9	15.6	16.6	22.0	20.0
93089	9.80	8.15	8.28	0.13	95.84	7.1	6.3	9.1	11.6	19.1	16.1	20.4	20.7
123189	9.33	7.82	7.91	0.09	98.66	6.4	7.1	8.6	12.7	20.6	17.0	20.2	18.5
33190	9.92	8.03	8.63	0.60	91.88	5.7	6.2	6.9	10.9	18.2	14.6		17.8
63090	9.63	7.98	8.41	0.43	94.34	6.7	7.3	7.7	10.8	17.3	15.9		16.6
93090	9.77	7.35	8.81	1.46	89.38	6.9	7.2	8.0	10.2	16.4	13.5		13.5
123190	9.14	6.33	8.06	1.43	95.72	5.9	6.0	6.4	8.7	13.8	13.1		13.8

Source: 1979-1982: Drexel, Burnham, Lambert (annual), 1983-1990: Salomon Brothers (monthly).
Prices and rates are end of quarter. Prepayment rates are averages for the quarter.

they were premiums, and it was optimal to prepay. As will be shown, mortgage price volatilities should be and are very different in those different circumstances.

A simple transformation of the data that is used often in this paper (as it generally works very well) is to examine data series for investment strategies sorted by a given spread of the mortgage's coupon to the coupon of the current par mortgage. Thus, instead of examining the prepayment rates, risks, and returns of GNMA 13s, we often look at those series for the changing (but well-defined) set of mortgages that have coupons that are, say, 2% to 3% above the current par mortgage rate, i.e., "premium mortgages." This transformation mitigates the interest rate effect and allows for better examination of other effects.

FIGURE 4 ▪ GNMA Seasonal Prepayment Multipliers: 1983-1990

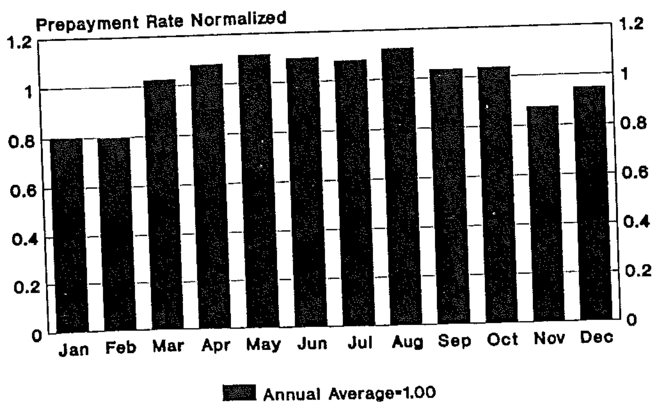


Table 3 shows annual averages of prepayment rates for securities sorted by their spreads to the par mortgage rate. The higher prepayment rates of high coupons are apparent from examining this table, as is the burnout effect.

The annual growth rate of GNP, the number of housing starts, and the national unemployment rate are all displayed in Table 3 as indicators of macroeconomic performance and mobility. The recession periods of mid-1980 and late-1981 through 1982 have very low prepayment rates, even on seasoned discounts sorted so as to hold interest rates approximately constant. This shows the expected macroeconomic effect. Studies that start with data from the mid-1980s often cannot detect this effect, as their tests have little power because of the consistency of growth from 1983 through 1990. Studies that use older data that cover recessions typically find a strong macroeconomic effect, particularly related to housing starts.

Figure 5 graphs the relationship of prepayment rates by degree of premium or discount for the entire time period 1978-1990. Prepayments form an "S-curve" as they flatten out at both ends of the graph. For very high premiums, it is optimal for most people to refinance and prepay, and they are probably doing so with as much haste as ever, whether the rate saving is 3% or 5%. For deep discounts, it is not optimal for most borrowers to prepay, and such people are probably minimizing prepayments.

TABLE 3 ■ Average Annual GNMA Prepayments by Coupon–Par Mortgage Rate

					Discounts						Premiums								
	Average	House	GNP	Unem-	-4.99	-3.99	-2.99	-1.99	-0.99	-0.49	0.00	0.50	1.00	1.50	2.00	2.50	3.00	4.00	5.00
	Par	Starts	Growth	ployment	-4.00	-3.00	-2.00	-1.00	-0.50	0.00	0.49	0.99	1.49	1.99	2.49	2.99	3.99	4.99	5.99
Year	Mortgage	(000)	(%)	(%)	Annualized Paydown from Prepayments (%)														
1978	9.14	2,020	5.3	6.1				7.7	8.0	8.9	6.7								
1979	10.65	1,745	2.5	5.8			3.9	4.1	1.0	0.3									
1980	12.98	1,292	-0.2	7.1		1.7	1.2	1.1	2.1	3.2	2.5	3.7	3.2	4.8	4.8				
1981	15.71	1,084	1.9	7.6	1.3	1.2	1.3	1.2	0.4	1.4	0.3								
1982	14.77	1,062	-2.5	9.7	1.5	2.4	2.4	2.7	3.4	2.1	3.4	3.5		4.8	4.2	12.0			
1983	12.52	1,703	3.6	9.6	3.7	3.6	2.2	3.4	3.9	4.5	7.1	10.3		19.8	18.9	40.7	37.3		
1984	13.27	1,750	6.8	7.5	2.5	2.7	3.2	4.9	7.2	6.9	11.2	10.2	12.7	15.7	15.9	16.0			
1985	11.64	1,742	3.4	7.2			3.4	3.7	4.1	5.4	6.5	8.6	11.1	14.0	18.8	25.1	26.3	26.8	30.8
1986	9.21	1,805	2.7	7.0					8.0	8.0	7.7	8.0	10.4	15.6	23.6	31.7	37.4	41.6	40.5
1987	9.50	1,621	3.4	6.2					8.6	9.9	12.1	16.0	18.6	23.6	28.8	27.2	30.2	32.3	34.1
1988	9.84	1,488	4.5	5.5				7.2	8.0	8.3	10.0	10.9	14.1	17.7	17.8	20.7	21.0	22.3	21.9
1989	9.84	1,376	2.5	5.3				6.5	6.8	7.5	8.2	9.7	11.8	13.2	15.3	18.2	17.6	19.9	19.7
1990	9.67	1,193	1.0	5.5				6.5	6.8	7.0	7.6	9.2	10.4	12.0	15.2	14.4	14.8	15.5	15.0
Averages																			
1978-1982		1,441	1.4	7.3	1.4	1.8	2.2	3.4	3.0	3.2	3.2	3.6	3.2	4.8	4.5	12.0			
1983-1987		1,724	4.0	7.5	3.1	3.2	2.9	4.0	6.3	6.9	8.9	10.6	13.2	17.7	21.2	28.1	32.8	33.6	35.1
1988-1990		1,352	2.7	5.4				6.7	7.2	7.6	8.6	9.9	12.1	14.3	16.1	17.7	17.8	19.2	18.8
1978-1990		1,529	2.7	6.9	2.3	2.3	2.5	4.5	5.2	5.6	6.9	9.0	11.5	14.1	16.3	22.9	26.4	26.4	27.0

Notes: GNP Growth is in real terms.
Economic data are from the U.S. Department of Commerce

Still, there is always a base level of prepayments due to forced house sales, nuisance mortgage balance levels, refinancing against higher collateral values, and the actions of fiscally conservative borrowers who simply wish to reduce debt. Most of these effects are relatively insensitive to rates and form the flat base of minimal core prepayments.

Figures 6A, 6B, and 6C show average prepayment rate curves for three subperiods that are chosen to illustrate the major effects. The first subperiod, 1978-1982, has many new mortgages originated when rates surged from 9% to 16% during the period. Prepayments are generally low during this subperiod because of the lack of seasoning (aging) of these mortgages. During the 1983–

FIGURE 5 ■ GNMA Prepayment Rates versus Coupon–Par: Average 1978-1990

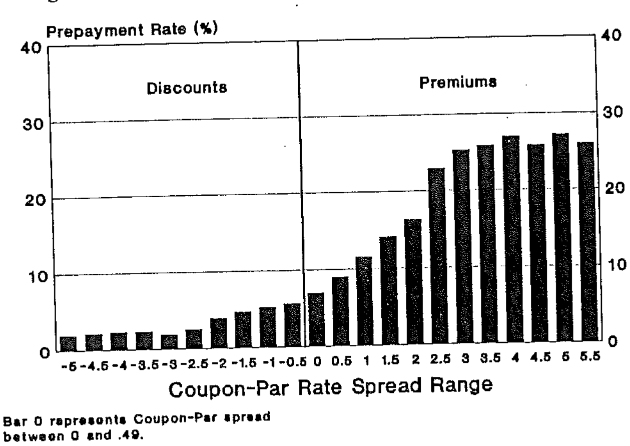


FIGURE 6A ■ GNMA Prepayment Rates versus Coupon–Par: Average 1978–1982

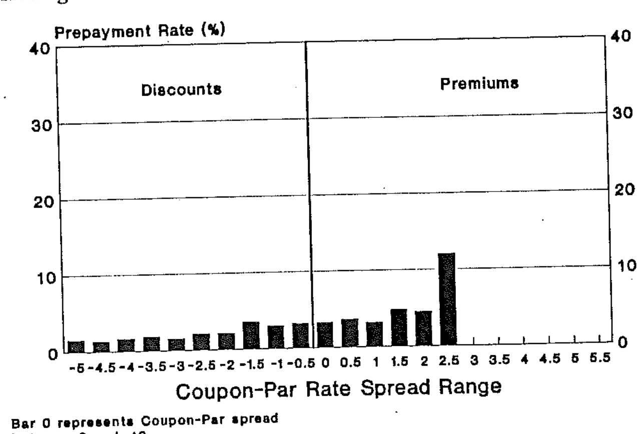


FIGURE 6B ▪ GNMA Prepayment Rates versus Coupon–Par: Average 1983–1987

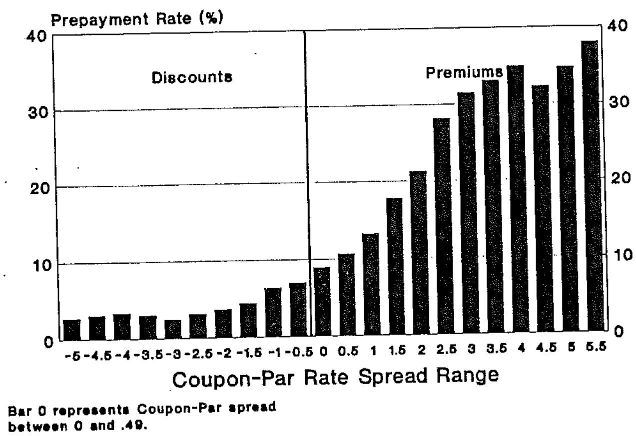
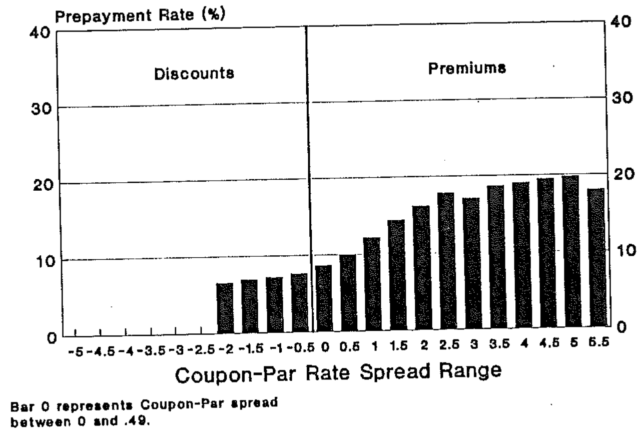


FIGURE 6C ▪ GNMA Prepayment Rates versus Coupon–Par: Average 1988–1990



1987 period, these same mortgages become well-seasoned, and the expected S-curve of prepayments is quite dramatic. Following those rapid paydowns, prepayment rates slow in the 1988-1990 period due to burnout effects.

Figure 7 displays clearly the change in the prepayment function for GNMA 13s pre- and postburnout.

IV. MORTGAGE PRICE CURVES

Here we analyze historical mortgage prices. End-of-month bid price data come from the *Wall Street Journal* for all coupons, including half coupons. Table 4 presents end-of-quarter prices (expressed as a percent of par) for mortgages with coupons ranging from 8% to 15%.

A number of observations can be made from Table 4. First, as interest rates increased from the 7.5% level at the end of 1977 to over 15% in September 1981, twenty-year Treasury bond futures prices dropped as expected from 99% to 56% of par. Prices for GNMA 8s dropped from 97 to 56 over the same period, moving very much in step with T-bond futures. From September 1981 to December 1990, rates fell back to the 8% level, the near-bond futures price increased to 96, and GNMA 8s increased to 94.

As we noted earlier, a mortgage is like a fixed-rate bond less a call option. As bond prices fell, the call option for GNMA 8s became almost worthless, so the GNMA 8's price behavior should be very similar to that of long-term T-bonds. Looking across different coupons, one sees that all mortgage prices move up and down generally with T-bond futures prices and opposite movements in interest rates, reflecting their fixed rates.

To see the effect of the prepayment option, examine the movements of GNMA 13 prices in relation to T-bond futures prices. In contrast to the GNMA 8s, which were always discounts, the GNMA 13s sold for discounts to par in 1981, 1982, and 1984, but sold for premiums during all other years (when mortgage rates were below 13%). As interest rates increased from March 1981 to September 1981, T-bond futures dropped eleven points from 67 to 56, and GNMA 13s dropped fourteen points from 92 to 78. From then until the end of 1982, T-bond futures increased by twenty points to 76, and GNMA 13s increased by twenty-five points to 103.

Thus, during this period of high rates and discount prices, GNMA 13s were just as volatile as T-bond futures. As interest rates continued to drop from December 1982 to December 1986, however, and T-bond futures rose by twenty-two points from 76 to 98, GNMA 13s increased only by six points to 109. The reason for

FIGURE 7 ▪ Prepayment Burnout of GNMA 13s

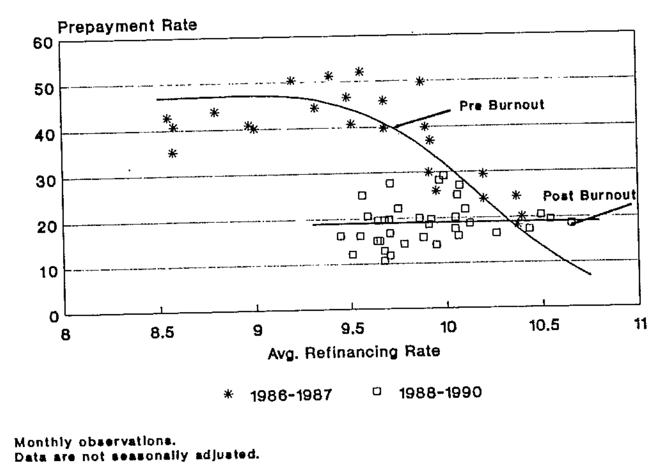


TABLE 4 ■ GNMA Fixed Rate Mortgage Prices

	Market Yields, Spreads, and Futures Prices						GNMA Prices By Morgage Coupon							
Date	Par Mtg Yield	7-Year Treasury	Par Mtg– Treasury	3 Month LIBOR	TED Spread	TBond Futures	8%	9%	10%	11%	12%	13%	14%	15%
123177	8.50%	7.61%	0.89%	7.31%	1.03%	99.50	96.88	103.44						
33178	8.73	7.98	0.75	7.56	0.91	95.94	94.66	101.75						
63078	9.33	8.53	0.80	8.75	1.54	93.00	91.50	98.00						
93078	9.32	8.49	0.83	9.56	1.49	93.31	92.34	98.16						
123178	10.13	9.20	0.93	11.81	2.21	90.34	89.75	94.56						
33179	10.06	9.10	0.96	10.69	0.92	90.19	89.75	94.72						
63079	9.88	8.71	1.17	10.69	1.44	91.81	91.13	95.53						
93079	11.50	9.45	2.05	12.88	2.44	87.78	86.00	90.44						
123179	11.67	10.31	1.36	14.56	2.03	82.19	81.56	85.69	90.31	96.13				
33180	14.08	12.58	1.50	20.00	5.02	67.94	70.78	74.34	78.38	83.75				
63080	11.39	9.82	1.57	9.69	1.51	81.19	84.44	88.84	93.81	98.59				
93080	13.57	11.84	1.73	13.75	1.86	70.69	74.94	77.69	82.19	86.94				
123180	13.72	12.47	1.25	17.75	2.73	71.38	71.16	75.81	80.38	85.97				
33181	14.57	13.21	1.36	14.94	1.94	67.06	68.75	72.25	77.00	81.75		91.94		
63081	15.94	14.05	1.89	17.69	2.61	64.38	64.69	67.56	72.06	76.78		86.84	91.31	
93081	18.42	16.04	2.38	17.75	2.60	56.00	56.06	59.75	64.13	68.56		78.00	82.06	
123181	15.95	14.12	1.83	13.80	2.26	61.91	62.25	66.34	71.03	76.38		86.53	91.91	96.09
33182	16.01	14.36	1.65	15.30	1.40	61.94	62.75	67.25	71.72	76.75		86.84	91.56	95.06
63082	16.14	14.52	1.62	15.95	2.63	60.69	63.13	67.19	71.53	76.69		86.19	90.63	94.75
93082	13.38	11.66	1.72	11.05	3.26	71.06	73.63	78.25	82.91	88.59		98.63	101.47	104.22
123182	12.21	10.27	1.94	9.31	1.18	76.63	78.72	83.72	88.78	94.28		103.38	105.69	107.50
33183	12.13	10.51	1.62	9.63	0.68	75.97	80.16	85.13	89.53	94.53		103.34	105.47	105.91
63083	12.49	10.85	1.64	9.75	0.71	74.44	77.81	82.78	87.75	92.66		102.31	105.16	106.41
93083	12.79	11.34	1.45	9.56	0.56	72.72	76.69	81.09	86.06	91.06		101.03	105.13	107.41
123183	12.69	11.80	0.89	9.94	0.68	70.03	75.31	80.13	85.41	90.91	96.50	101.47	105.44	107.94
33184	13.20	12.39	0.81	10.81	0.83	66.22	72.63	77.28	82.47	88.22	93.84	98.97	104.16	107.44
63084	14.52	13.81	0.71	12.13	1.87	59.63	66.66	70.69	75.63	81.13	86.81	92.34	98.00	101.84
93084	13.46	12.50	0.96	11.50	0.92	67.22	72.16	76.66	81.41	87.50	93.12	98.16	102.41	105.53
123184	12.45	11.47	0.98	8.75	0.67	71.06	76.97	81.72	87.13	93.25	98.16	101.97	106.03	108.81
33185	12.65	11.68	0.97	9.06	0.62	69.72	76.63	80.81	85.94	91.56	96.56	101.63	106.25	108.69
63085	11.40	10.14	1.26	7.81	0.80	77.06	83.50	88.28	94.03	98.75	102.16	106.13	108.97	112.41
93085	11.41	10.17	1.24	8.06	0.79	75.59	83.88	88.28	93.16	97.88	102.88	106.84	109.63	111.69
123185	9.76	8.83	0.93	8.00	0.76	85.22	91.22	96.09	101.31	104.81	107.44	108.25	109.59	112.03
33186	9.00	7.32	1.68	7.44	0.93	102.31	95.72	100.00	104.22	106.91	108.25	108.34	108.81	114.06
63086	9.58	7.30	2.28	6.88	0.75	99.56	93.69	97.28	102.19	105.25	105.78	106.56	107.28	114.25
93086	9.24	7.30	1.94	6.13	0.82	96.56	95.41	98.72	104.28	107.25	107.91	108.47	109.06	113.44
123186	8.61	7.07	1.54	6.38	0.59	98.19	98.22	101.63	106.53	107.75	108.13	109.13	110.06	114.31
33187	8.53	7.40	1.13	6.63	0.68	98.47	97.31	102.19	106.69	107.84	108.91	110.06	112.53	115.00
63087	9.75	8.23	1.52	7.25	1.39	91.50	91.00	96.09	101.31	105.88	108.72	111.34	113.03	115.00
93087	10.80	9.45	1.35	8.31	1.43	81.69	83.78	89.38	95.41	101.19	106.38	109.44	112.00	114.00
123187	10.01	8.63	1.38	7.44	1.60	87.97	88.16	94.00	99.97	104.56	108.00	109.78	112.00	114.00

TABLE 4 ▪ Continued

	Market Yields, Spreads, and Futures Prices						GNMA Prices By Morgage Coupon							
Date	Par Mtg Yield	7-Year Treasury	Par Mtg– Treasury	3 Month LIBOR	TED Spread	TBond Futures	8%	9%	10%	11%	12%	13%	14%	15%
123177	8.50%	7.61%	0.89%	7.31%	1.03%	99.50	96.88	103.44						
33188	9.80	8.45	1.35	6.94	1.08	90.09	89.31	95.97	101.00	107.91	110.31	111.94	114.13	115.00
63088	9.90	8.65	1.25	7.94	1.20	88.75	89.00	94.88	100.56	105.97	109.47	112.00	113.75	115.50
93088	9.90	8.73	1.17	8.75	1.28	88.75	89.13	94.94	100.53	105.19	108.25	110.31	113.41	114.63
123188	10.34	9.20	1.14	9.38	1.02	89.13	87.44	93.03	98.31	103.06	105.88	108.38	111.81	112.56
33189	10.69	9.36	1.33	10.31	1.13	88.41	86.25	91.59	96.72	101.25	104.25	107.91	110.69	111.22
63089	9.51	8.08	1.43	9.31	1.07	97.94	93.16	97.81	102.09	105.50	108.44	110.50	110.75	111.00
93089	9.80	8.35	1.45	9.19	1.04	95.84	92.13	96.69	100.94	104.47	108.84	110.16	111.25	111.38
123189	9.33	7.96	1.37	8.38	0.56	98.66	94.25	98.63	102.78	105.72	108.84	110.69	111.94	112.31
33190	9.92	8.69	1.23	8.50	0.47	91.88	90.66	95.69	100.38	104.06	107.50	109.38		
63090	9.63	8.45	1.18	8.38	0.40	94.34	92.13	97.09	101.72	105.13	108.13	111.00		
93090	9.77	8.70	1.07	8.31	0.96	89.38	90.63	96.06	101.22	105.25	109.13	111.94		
123190	9.14	7.98	1.16	7.63	1.00	95.72	94.38	99.41	103.56	106.94	111.50	113.50	115.03	116.53

Notes: Bid, % Par
Price data from the *Wall Street Journal.*
The TED Spread is 3-month LIBOR minus the 3-month T-bill yield.
Yield for 3-month LIBOR and 3-month T-bill from Salomon Brothers.

this limiting of price increases is the surge in GNMA 13 prepayments shown in Table 2, from 3% annually in 1982 to over 40% annually in late 1986.

According to Table 1, given no prepayments and a current mortgage rate of 9%, the value of the GNMA 13 would exceed 137% of par. The rapid prepayments that occurred in 1986 caused the GNMA 13s to sell for 109 rather than at the 137 price that would have occurred with no prepayments.

Not all borrowers prepay as soon as refinancing rates appear attractive, so mortgages do sell at prices significantly above par, "capping out" at about 110-115 according to the data in Table 4. Theoretical pricing models that derive mortgage prices that never significantly exceed par, because optimal prepayments are assumed, are not very realistic.

Figures 8A through 8D give historical relationships of GNMA 9s, 11s, 13s, and 15s to Treasury bond futures prices. The fits are statistically quite significant, with the high-coupon 15s having the least precise fit, i.e., the greatest "basis risk." The curvature (negative convexity) predicted by the option analysis in Figure 1A is increasingly evident as one moves to the high-coupon GNMA 11s, 13s and 15s.

FIGURE 8A ▪ GNMA 9 Prices versus TBond Futures: Monthly, December 1977–December 1990

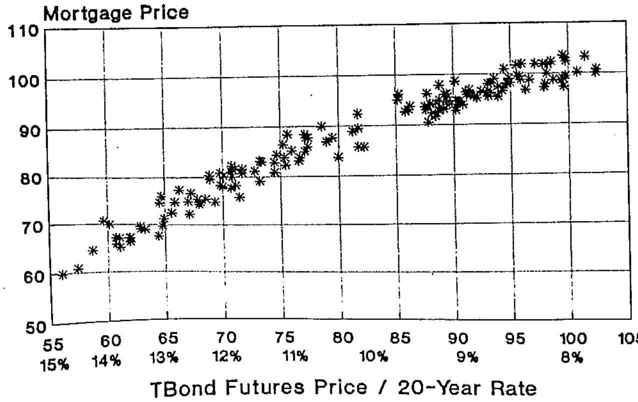


FIGURE 8B ▪ GNMA 11 Prices versus TBond Futures: Monthly, October 1979–December 1990

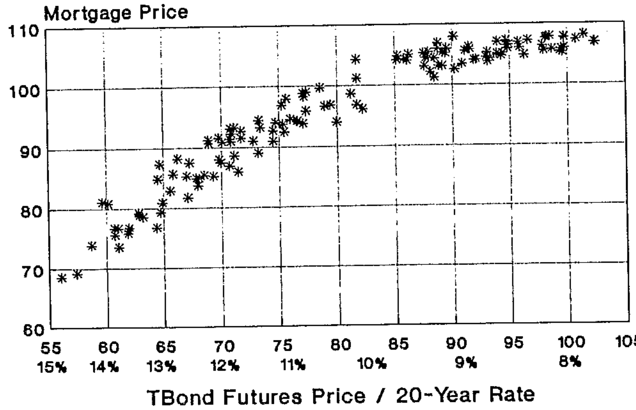


FIGURE 8C ▪ GNMA 13 Prices versus TBond Futures: Monthly, February 1981–December 1990

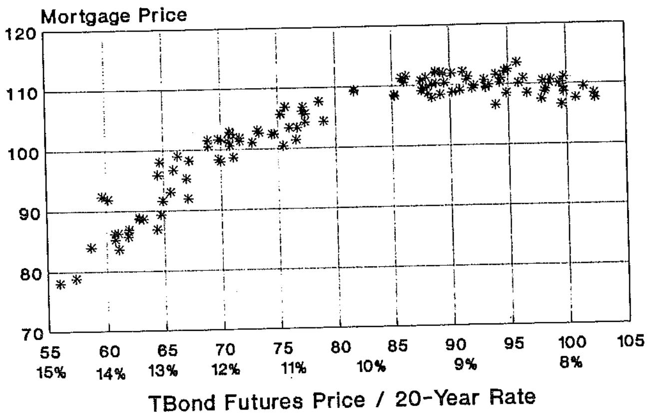
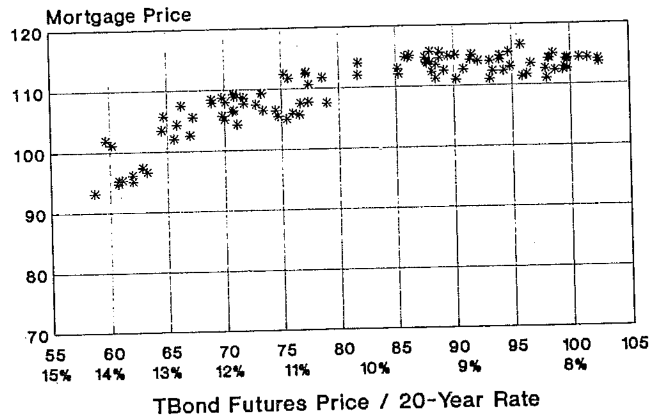


FIGURE 8D ▪ GNMA 15 Prices versus TBond Futures: Monthly, October 1981–December 1990



As Table 1 illustrates, these mortgages are effectively short call options on T-bond futures with different exercise prices. As these calls become in-the-money, the mortgage's price gains are limited substantially by significantly increased prepayments. The shapes of these curves are as anticipated in Section I, in that they resemble curves for riskless investments less put options with various exercise prices.

V. RISK ANALYSIS WITH IMPLIED MORTGAGE PRICE ELASTICITIES

The cross section of mortgage prices for different coupons can be used to find "implied price elasticities" or modified durations for mortgages. These implied price elasticities are useful as measures of risk and for the construction of hedges.

First, consider what an appropriate risk measure for a mortgage investment should be. For straight bonds, modified duration is extremely useful because of its close theoretical and practical relationship to price volatility. Unfortunately, for mortgages the standard calculation of modified duration is usually an incorrect and misleading measure of price sensitivity. This statement applies both to duration computed using scheduled cash flows without prepayments, as well as to calculations that base expected cash flows on a current forecast of future prepayment rates. The problem lies in the fact that the duration of cash flows changes systematically with interest rates as borrowers use the prepayment option to their benefit.

To see the potential error in using duration calculations for price volatility, consider the case of a high-coupon mortgage security before it has been burned out, such as GNMA 13s at the middle of 1985. The prepayment rate at that time was about 10% to 15% annually. As the par mortgage rate was near 11.5%, the 13s were premiums by 1.5%. Looking at Figure 8C's S-curve of the prepayment function at that time, one sees that mortgages in this premium range have the most sensitivity of prepayments to movements in interest rates.

Duration computed at that time, based upon cash flow forecasts, would be in the four- to five-year range. The standard risk management usage of duration would imply that this mortgage moves 4% to 5% for 100-basis point moves up or down in rates. During the subsequent year (June 1985 to June 1986), the par mortgage rate dropped by almost 200 basis points, so the duration-based estimate is that the GNMA 13 would increase in price by approximately 8% from 106 to 114. In actuality, the price increased only by 0.5% to 106.56.

Why did the GNMA 13s increase in price by such a small amount as rates fell sharply? As rates fell, prepayments accelerated sharply to the 40% level. Increased prepayments cause capital losses on paydowns of premiums, which largely offset the price benefit from discounting cash flows at the 200-basis point lower rates. Was this foreseeable, given the drop in rates by 200 basis points?

Although the extent of the prepayment increase was not easily predicted, rational models would all have shown some significant sensitivity of premium mortgage prepayments to rate moves. By using a prepayment *function* rather than a point estimate, one recognizes that for premium securities price moves upward are limited by prepayment increases. Similarly, price moves of premiums downward are also limited by reduced prepayments as rates increase. Thus, premiums have shorter "effective durations" and price elasticities due to the systematic prepayment effect. For discount mortgages, the same type of analysis gives the result that effective durations should be greater than standard computations, as price moves upward on discounts are enhanced by the capital gains of increased prepayments caused by lower rates.

Market participants are now well aware of the effects of changing prepayments on price volatility, although not many were in mid-1985. As the example demonstrates, correct understanding of the prepayment function and its price effect can be more important than interest rate moves in the prices of premium securities. One method for using market prices to capture that effect is to use a "roll-up, roll-down" approach (which can be justified as an approximation, given the homogeneity of option prices in the ratio of the underlying asset's price to the exercise price).

This approach simply estimates what the value of a GNMA 13 will be if rates decrease by 1% by using the current market price of the GNMA 14. As rates decrease by 1%, a GNMA 13 that has a coupon 1.5% over the old par mortgage rate will then have a 2.5% premium to the new rate. The GNMA 14s, however, have a premium of 2.5% over the old par rate. The price of the GNMA 14s presumably reflects the market's prepayment forecast for 2.5% premium securities, as well as its valuation of that option. Using the price of the 14s takes advantage of some of the market's knowledge. Correspondingly, for a 1% increase in rates, the GNMA 13 becomes an 0.5% premium, to which the price of GNMA 12s currently corresponds.

Using the roll-up, roll-down approach on June 30, 1985, gives a price elasticity of 2.7% for the GNMA 13s if rates move down by 1%, and a price elasticity of 3.9% if rates move up by 1%. This is consistent with the adversely asymmetric payoff pattern (negative convexity) anticipated due to the prepayment option.

In December 1985, after rates had dropped sharply, and prepayments had begun to accelerate, the GNMA 13 roll-up, roll-down elasticities were reduced to 1.2% and 0.8%, as the market began to feel the prepayment option's shortening of effective durations. These "implied price elasticities" from market prices would have provided much better volatility predictions in the subsequent year than would standard duration calculations.

Figures 9A through 9C give the monthly time series of implied elasticities based upon the price spreads of GNMA 9s and 8s, 11s and 10s, and 13s and 12s, respectively. Generally, these implied elasticities behave in sensible ways. In each graph, elasticities decrease as interest rates fall and bond prices increase, because prepayments increase and effective durations shorten. Furthermore, comparing the elasticities across graphs, one sees that implied elasticities for high coupons (premiums) are smaller than for low coupons (discounts). Certainly, that pattern of risk estimates is validated by subsequent price volatility, as we show later. Implied price elasticities are very useful in assessing subsequent price risk.*

The non-stationarities of mortgage price risks are striking in these implied price elasticity graphs. Seeing these, it just is not sensible to treat GNMA 13s or other coupons as if they have a well-defined, stationary risk profile that is valid for long periods of time. The risks in fact depend very much upon the level of interest rates, which affects whether the mortgage is a premium or a discount security, as well as the speed of prepayments.

To develop a more stationary risk profile for fixed-rate mortgage investments, the transformation is

FIGURE 9A ▪ Implied Elasticities: GNMA 9 and 8 Prices: Monthly, December 1977–December 1990

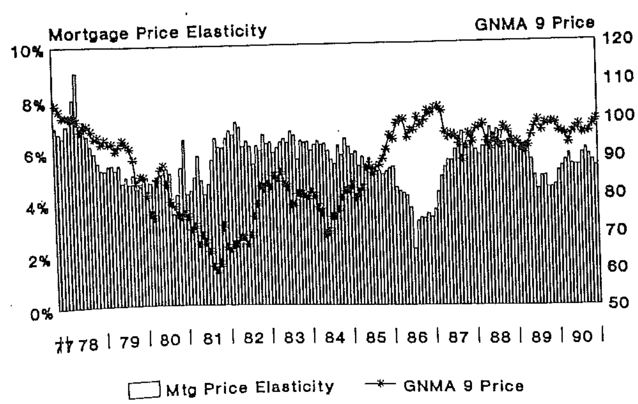


FIGURE 9B ▪ Implied Elasticities: GNMA 11 and 10 Prices: Monthly, October 1979–December 1990

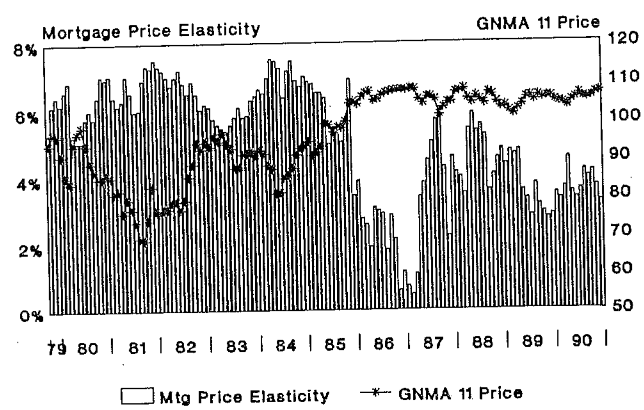
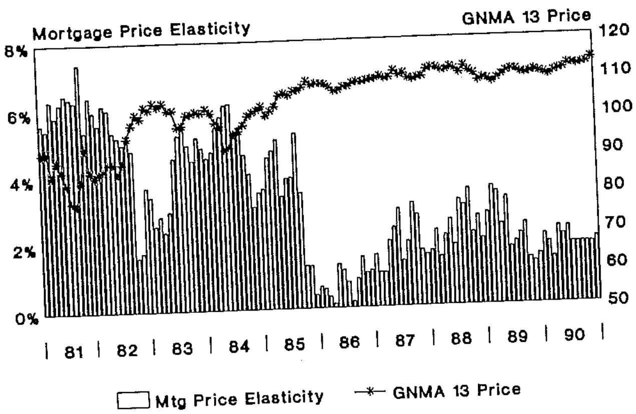


FIGURE 9C ■ Implied Elasticities: GNMA 13 and 12.5 Prices: Monthly, February 1981–December 1990



made to trading strategies that invest in mortgages with constant spreads to the par coupon. Thus, instead of considering a buy-and-hold strategy with GNMA 13s, consider a strategy of adjusting each month always to hold the mortgage coupon that is, say, 1.5% over the par coupon. As rates increase (decrease), higher- (lower-) coupon mortgages are bought. Table 5 shows the time series of elasticities from these trading strategies, and Figures 10A–10C present the results graphically.

Figures 10A through 10C show that these strategies give much more stationary risk profiles than investing in constant-coupon mortgages. Figure 10A shows that the implied elasticities for investing in GNMAs with discounts of 1% or more from the par coupon are quite stable at approximately 6% for the thirteen-year period.

TABLE 5 ■ Summary of Elasticities

		Monthly Cross-Sectional Simple Averages (%)						12-Month Moving Averages (%)					
	Par Mtg	Discount		Premium				Discount		Premium			
Date	Yield	<-1%	0% to -1%	0% to +1%	1% to 2%	2% to 3%	>3%	<-1%	-1% to 0%	0% to 1%	1% to 2%	2% to 3%	3% to 4%
123177	8.50	7.2	6.9	6.8									
33178	8.73	6.2	7.6	7.5									
63078	9.33	6.4	7.3										
93078	9.32	7.4	6.3										
123178	10.13	6.7						6.8	7.0	7.7			
33179	10.06	7.0						6.9	6.6	8.6			
63079	9.88	7.0	5.1					7.1	6.0				
93079	11.50	5.9						6.8	5.6				
123179	11.67	5.3	6.4					6.5	5.7				
33180	14.08	5.9						6.2	5.8				
63080	11.39	5.7	5.1					5.8	5.6				
93080	13.57	5.4						5.8	5.7				
123180	13.72	6.3						5.9	5.1				
33181	14.57	6.0						5.9	5.1				
63081	15.94	5.9						6.0					
93081	18.42	6.1						6.0					
123181	15.95	6.6	4.6	4.3				6.1	4.8	4.2	4.0		
33182	16.01	6.0	5.2					6.1	4.8	4.2	4.0		
63082	16.14	5.9	4.8					6.1	4.7	4.5	4.0		
93082	13.38	5.9	5.5	3.7	2.7	4.3		6.1	4.7	4.4	3.7	4.3	
123182	12.21	6.1	4.4	5.3	2.2	1.7	2.0	6.1	4.7	4.3	2.5	2.7	2.3
33183	12.13	5.5	4.8	5.3	2.0	0.4	0.2	5.9	4.7	4.5	2.5	2.1	1.5
63083	12.49	6.0	4.9	5.4	2.8	1.2	0.6	5.9	4.8	4.6	2.4	1.9	1.2
93083	12.79	5.8	5.5	4.9	3.6	2.2	0.1	5.9	5.0	4.7	2.3	1.6	1.1
123183	12.69	6.4	5.7	4.9	3.7	2.4	1.4	5.9	5.2	5.0	2.8	1.6	0.9
33184	13.20	6.5	5.6	5.2	3.2	3.8		6.1	5.4	5.0	3.1	1.9	1.2
63084	14.52	6.6	6.2	3.9	3.8			6.3	5.7	4.8	3.6	2.3	1.4

TABLE 5 ■ Continued

		Monthly Cross-Sectional Simple Averages (%)						12-Month Moving Averages (%)					
	Par Mtg	Discount			Premium			Discount			Premium		
Date	Yield	<-1%	0% to -1%	0% to +1%	1% to 2%	2% to 3%	>3%	<-1%	-1% to 0%	0% to 1%	1% to 2%	2% to 3%	3% to 4%
93084	13.46	6.4	5.5	4.1	3.1	4.0		6.4	5.6	4.8	3.6	3.0	1.7
123184	12.45	6.6	5.2	4.8	3.9	2.6	2.5	6.5	5.5	4.6	3.5	3.3	2.6
33185	12.65	5.9	5.6	4.7	4.6	2.3	3.2	6.4	5.4	4.5	3.6	3.2	2.9
63085	11.40	6.2	5.0	3.2	3.8	2.7	2.4	6.3	5.1	4.4	3.5	2.9	2.7
93085	11.41	5.4	5.1	5.2	4.0	2.6	1.9	6.1	5.1	4.3	3.9	2.5	2.6
123185	9.76		5.2	5.6	3.3	1.6	1.3	5.8	5.2	4.6	3.9	2.3	2.2
33186	9.00		4.5	4.9	3.1	1.3	1.4	5.7	5.0	4.7	3.6	2.1	1.9
63086	9.58		4.3	5.2	1.8	0.4	2.2	5.5	5.1	5.0	3.4	1.7	1.8
93086	9.24		2.8	5.4	3.9	0.6	1.3	5.1	4.6	5.1	3.3	1.3	1.5
123186	8.61		2.7	3.5	3.8	0.6	1.3		3.9	4.9	3.4	1.0	1.5
33187	8.53		5.2	5.0	3.1	1.0	1.6		3.7	4.7	3.5	0.8	1.5
63087	9.75	5.6	5.5	5.3	3.8	2.2	1.9	5.4	4.0	4.7	3.7	1.2	1.5
93087	10.80	6.6	6.2	6.1	4.1	2.8	2.0	5.8	5.1	4.9	3.6	1.6	1.7
123187	10.01	6.6	6.3	5.4	3.3	1.9	1.9	6.0	5.9	5.3	3.5	2.0	1.8
33188	9.24		5.9	6.0	3.9	2.0	1.6	6.0	6.0	5.6	3.6	2.3	1.9
63088	9.90	6.7	6.3	5.8	4.6	2.7	1.4	6.4	6.2	5.7	3.7	2.4	1.8
93088	9.39		6.3	5.8	3.7	2.3	1.7	6.5	6.2	5.6	3.7	2.5	1.7
123188	10.34	6.4	5.6	5.0	2.7	2.2	2.3	6.6	6.1	5.5	3.6	2.5	1.7
33189	10.69	5.9	5.4	4.7	2.9	3.5	1.0	6.4	5.9	5.3	3.3	2.7	1.7
63089	9.51	5.0	4.7	4.3	3.3	2.2	0.8	6.2	5.6	5.0	3.1	2.7	1.5
93089	9.30		5.2	4.4	3.2	2.6	1.1	5.9	5.3	4.6	3.1	2.7	1.4
123189	9.33		4.5	4.2	2.9	2.9	1.3	5.6	4.9	4.4	3.0	2.9	1.2
33190	9.92	5.6	5.3	4.7	3.6	2.7	1.4	5.4	4.9	4.4	3.2	2.7	1.2
63090	9.63	5.4	5.1	4.7	3.0	3.0	2.0	5.3	4.9	4.4	3.3	2.9	1.4
93090	9.77	5.9	5.6	5.3	3.8	3.6	1.8	5.5	5.1	4.7	3.4	3.0	1.6
123190	9.14		5.2	4.3	3.3	4.2	1.8	5.5	5.3	4.8	3.6	3.2	1.8
Averages		6.10	5.37	4.99	3.37	2.32	1.58	6.06	5.29	4.95	3.40	2.34	1.72

Figure 10B shows that buying coupons 1% to 2% over par gives a relatively stable elasticity of 3.5%, and Figure 10C shows that 3% or greater premium investments have relatively stable elasticities near 1.5% to 2%. Thus, a stable risk strategy in fixed-rate mortgages can be constructed, but it does require dynamic adjustment of the portfolio's coupon mix.

VI. DYNAMIC HEDGING STRATEGIES

Here we test simulated dynamic hedging strategies for mortgages using the implied elasticities from mortgage prices. For every month from January 1978 through December 1990, implied elasticities are computed for adjacent mortgage coupons. These elasticities are sorted by their spreads to the par mortgage coupon as in Table 5, typically with two coupons averaged in each bucket, e.g., 11.0s and 11.5s. To reduce the impact of price reporting errors on the results, the monthly elasticities for the last twelve months are averaged for each 1% coupon bucket and used in the construction of the hedges.

A separate dynamic hedging simulation is run for each mortgage coupon; every month its spread to the par coupon is computed, and its elasticity is estimated from the functions of Table 5. Based upon that elasticity and the current elasticity of the near twenty-year Treasury bond futures, the hedge position is computed, as well as its gain or loss during the next month. Adding the mort-

FIGURE 10A ▪ Implied Elasticities for GNMA Discounts: Coupon–Par: <-1%

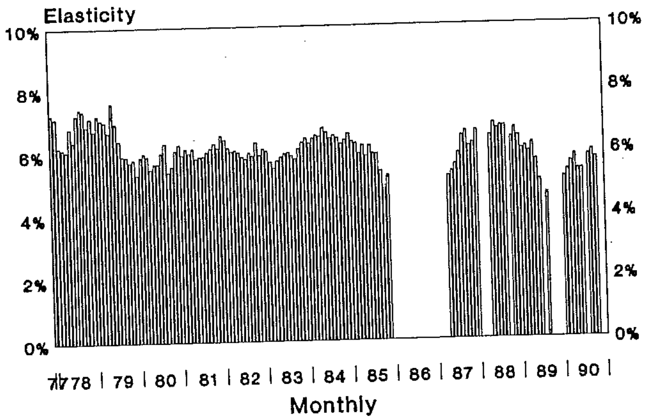


FIGURE 10B ▪ Implied Elasticities for GNMA Premiums: Coupon–Par: +1% to +2%

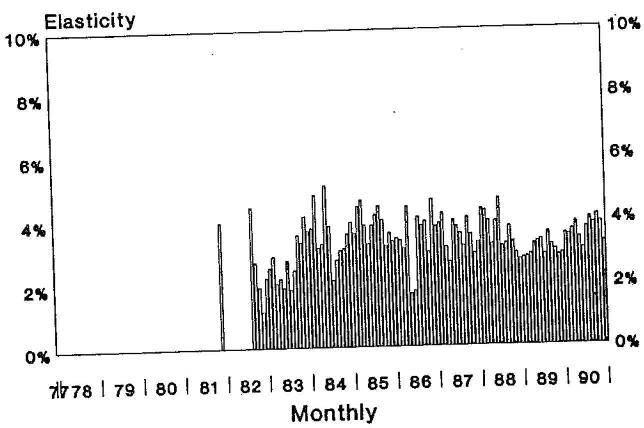
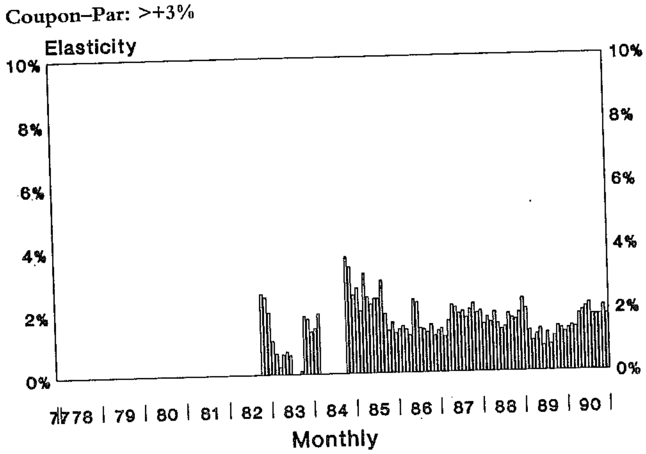


FIGURE 10C ▪ Implied Elasticities GNMA High Premiums: Coupon–Par: >+3%



gage's coupon income, its gain or loss on principal paid down, and any capital gain or loss due to price changes gives the total profit for the month, from which we compute the rate of return.

The performance of the hedges can be shown in several ways. First, the hedges are examined graphically according to three different perspectives, following which the statistical data are examined. Figure 11 graphs the cumulative total return on GNMA 9s, which were available for the entire period of 1979–1990 (losing one year to the moving average development of the elasticity function).

Graphs for the other coupons are very similar and are not shown. The pattern in all graphs shows that the hedging was successful, in that the return is more stable. At the same time, the hedge gives a lower return than the unhedged position, as interest rates generally fell during the simulation periods.

Hedge fluctuation due to basis risks is more apparent if the dominant uptrend is removed by examining excess returns. As most financing of mortgage securities is done through repurchase agreements ("repos") at rates near or below three-month LIBOR, excess returns over LIBOR were computed for each mortgage coupon. Figures 12A and 12B are representative of these results.

GNMA 8s and 9s, which were discounts in most of the period, generally earned a sub-LIBOR hedged return, while premiums such as GNMA 13s and GNMA 15s earned hedged returns in excess of LIBOR. In both Figures 12A and 12B, the mirroring of futures gains and losses with cash gains and losses is apparent. With a hedge, when the cash profit is up, the futures hedge profit is down, and vice versa.

FIGURE 11 ▪ Cumulative Investment Performance: GNMA 9s Value of $1 Unhedged and Hedged

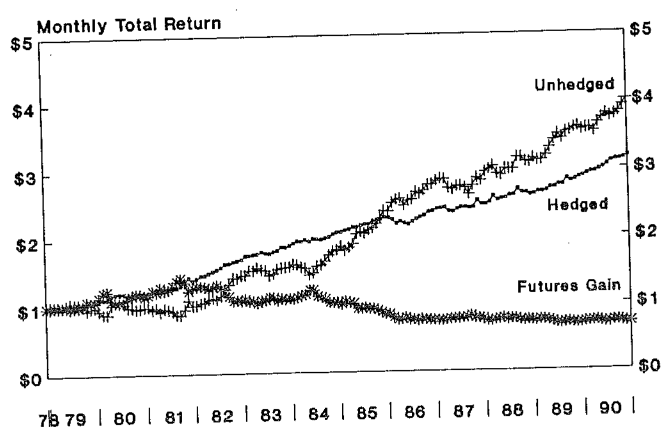


FIGURE 12A ▪ Cumulative Investment Performance: GNMA 9 Return in Excess of LIBOR

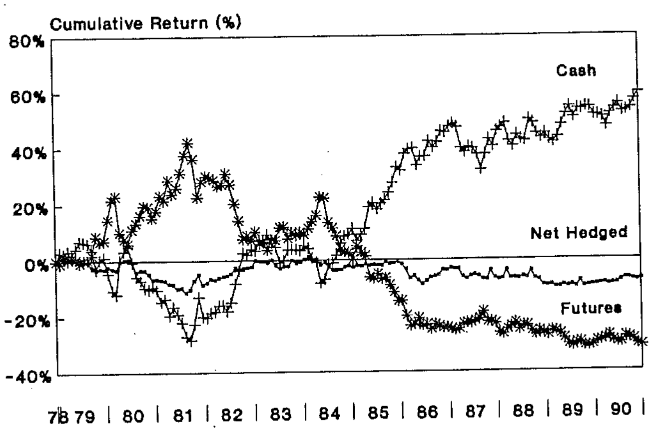
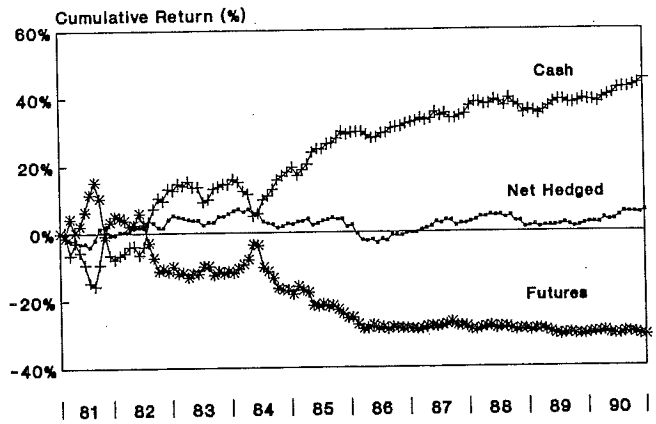


FIGURE 12B ▪ Cumulative Investment Performance: GNMA 13 Return in Excess of LIBOR



The "net hedged" return is much more stable than either futures or cash returns, and stays nearer the zero excess return line. As these returns are hedged to betas near zero, the equilibrium excess returns to the hedging strategies should on average be near zero.

Figures 13A and 13B clearly show the hedge effectiveness, as they give scatter plots of monthly and cumulative cash gains and losses versus corresponding futures gains and losses. The graphs for other coupons look quite similar to 13A and 13B. Futures gains and losses of the hedges are approximately "equal but opposite," as the slopes are near 1.00. Comparing Figure 13B to 13A, one sees the more precise fit over time as many monthly gain and loss fluctuations in the basis cancel out.

Another interesting way to view hedged returns is to graph their monthly gains and losses on the same scale as the unhedged returns. The hedged returns should show less volatility. Figures 14A and 14B show these graphs for GNMA 9s and 13s, respectively.

These figures show clearly the lower volatility of hedged monthly returns during the volatile interest rate period of 1979-1986. In the more recent period of relatively low interest rate volatility, the more volatile GNMA 9s show reduced volatility from hedging, but the GNMA 13s do not. During this recent period of lower rates, the GNMA 13s have sold for high premiums and have relatively little price volatility. As a result, the basis risk of the hedge is so great that the hedge effects little reduction in volatility. Thus, Figures 14A and 14B presage the different statistical results for the subperiods analyzed in the next section.

The most interesting features of these mortgage hedges are the dynamic option creation aspects. As shown in Section I, mortgages should have negative con-

FIGURE 13A ▪ Monthly Hedge Performance: GNMA 13s Hedged with TBond Futures

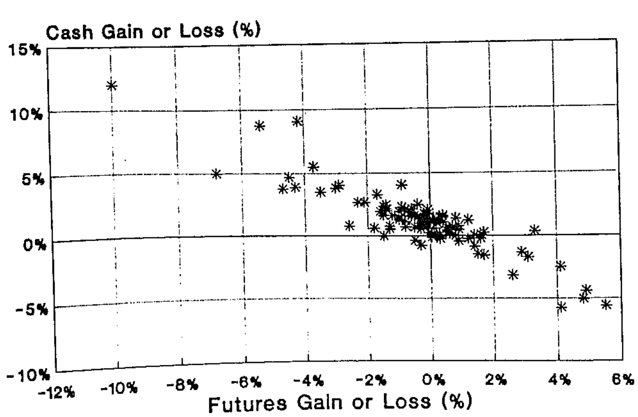


FIGURE 13B ▪ Cumulative Hedge Performance: GNMA 13s Hedged with TBond Futures

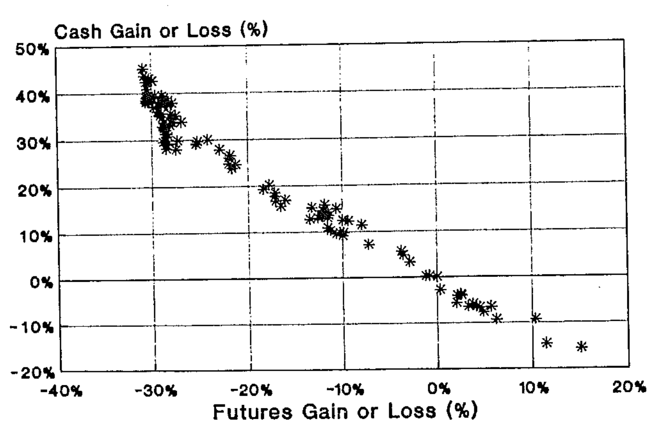


FIGURE 14A ▪ Futures Hedge Performance: GNMA 9 Total Return Hedged versus Unhedged

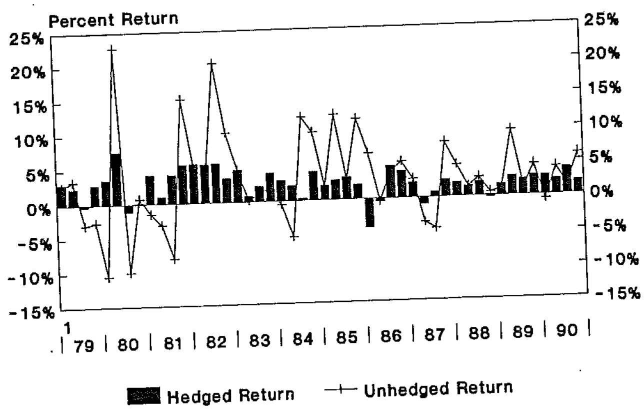
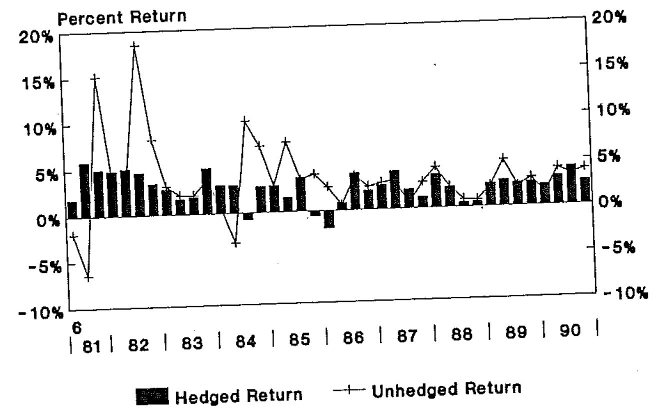


FIGURE 14B ▪ Futures Hedge Performance: GNMA 13 Total Return Hedged versus Unhedged



vexity due to the written prepayment options. Section III showed price graphs that verify the presence of the expected curvature. Hedges for mortgages essentially create put options, as mortgage risks are analogous to those of written puts.

Figure 15 graphs the hedge elasticities for GNMA 13s versus Treasury bond prices. These elasticities summarize the dynamic trading strategy for GNMA 13s. When bond prices are high, elasticities (risks) are low, and hedge positions are low. When bond prices are low, prepayments slow, and the bond has a high elasticity and requires a large hedge position. Thus, the hedge sells short large amounts of T-bond futures at low prices and buys some of them back as prices increase (and rates fall).

The similarities to theoretical delta hedging curves of the option pricing literature are striking. This is the same type of trading strategy for creating a put option, as it should be. Graphs for other coupons trace out portions of that curve, with the portion depending upon whether the security was primarily a premium or a discount. GNMA 13s were both above and below par so they trace out the entire curve.

Another expected option-like feature is demonstrated in Figure 16. This plots the actual cumulative hedge profit versus Treasury bond futures prices. As the hedge creates a put option, the payoffs should be similar to the payoffs on a put option. Again, the dynamic hedge gives the expected payoff pattern, as the payoffs created have positive convexity that is a hedge for the negative convexity of the prepayment option. Graphs (not shown) of these cumulative payoffs for other coupons also resemble put options, with the strike prices varying with the mortgage's coupon, as expected.

Another important aspect of mortgage hedging and pricing is illustrated in Figure 16, as the curve drifts to the left as time passes. This normal pattern is

FIGURE 15 ▪ Dynamic Mortgage Hedges: GNMA 13 Elasticities versus TBond Futures

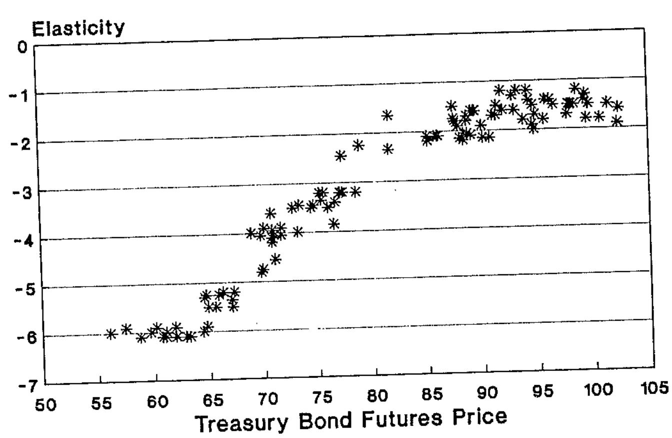
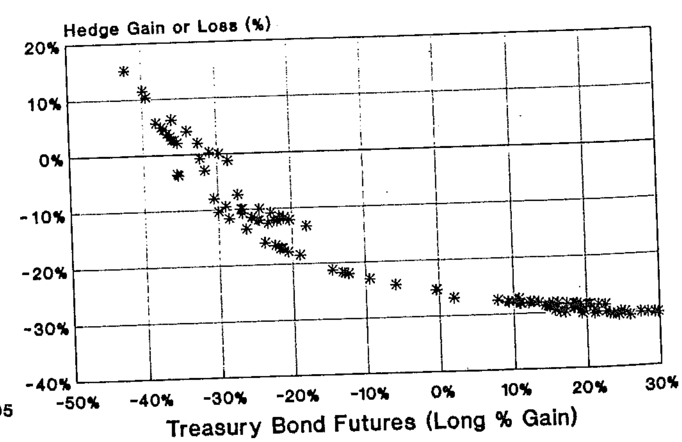


FIGURE 16 ▪ Dynamic Mortgage Hedges: GNMA 13 Cumulative Hedge Gains versus TBonds



attributable to the "whipsaw loss" or option creation cost present in any dynamic option replication. Whipsaw losses are incurred when the option creation strategy involves buying more contracts at higher prices and selling at lower prices, as put and call option creation strategies do. With that dynamic trading strategy, an increase in price followed by a return to the previous price results in a net loss from buying high and selling low, which is a whipsaw loss. As time passes, whipsaw losses accumulate. They are the inevitable costs of option creation.

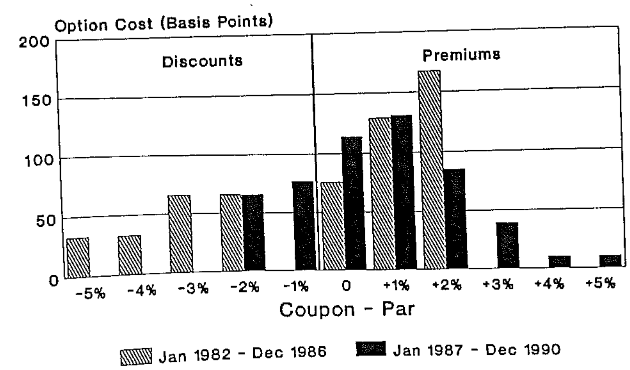
Estimates of option creation costs are important in mortgage pricing, as they differ by coupon, are economically significant, and are the most complex part of a modern "option-adjusted spread" analysis. The extent of the loss is related more to the volatility of the trading position than to the general scale of it (which relates to the mortgage's elasticity). Thus, the whipsaw cost for a coupon is found by comparing its dynamic hedging returns to those of a constant hedge with the same average elasticity.

As that average elasticity is known only ex post, it could be viewed as one that assumes "perfect foresight." As the perfect foresight elasticity has the same overall interest rate exposure as the actual dynamic hedge, the difference in average return will be the loss due to the dynamic hedge adjustments about the average elasticity.

We have estimated whipsaw costs for each mortgage coupon for two major subperiods for which there were a meaningful amount of data by coupon, 1982-1986 and 1987-1990. Figure 17 shows the annualized excess returns of the perfect foresight hedges over those of the dynamic hedging strategies.

The amounts range from lows of ten to twenty-five basis points for deep discounts and very high premiums to 130 to 170 basis points for coupons between par and 3% premiums to par. As the amount of dynamic hedge adjustment can be seen from Table 5 (and Figure 15) to be greatest for coupons in that low premium range, the general shape of the whipsaw curve is sensible.

FIGURE 17 ▪ GNMA Mortgage Option Creation Costs: Sorted By Coupon–Par



0 Includes options costs for coupons between 0 and 1% over par mtg coupon.

In terms of the usual option pricing nomenclature, whipsaw is caused by changes in the "delta hedge ratio." Changes in delta are sometimes called "gammas," which Cox and Rubinstein [1985, Chapter 5] show to be greatest for options that are at the money. Low premium mortgages are mortgages whose prepayment options are at the money for many borrowers, as there are some costs to refinancing.

Note that whipsaw costs are lower in the more recent period of low volatility of interest rates, as expected. The general levels of whipsaw costs by coupon, in Figure 17, are of the same general magnitudes (but somewhat higher for the low premiums) as those assumed by many research firms today.

VII. STATISTICAL ANALYSIS OF MORTGAGE RISKS, RETURNS, AND HEDGES

This section compares hedged and unhedged returns on mortgages from 1979 to 1990 to Treasury returns of comparable duration, as well as to returns of major bond and stock indexes. Betas are presented for mortgages relative to the Salomon Brothers Mortgage Index, to twenty-year Treasury bond futures, and to the Standard & Poor's 500 stock price index.

Unfortunately, data are not available for all mortgage coupons for the entire period (see Table 4). GNMA 10s and 11s were introduced only in late 1979, after the surge in interest rates generated originations of those higher-rate mortgages. GNMA 13s, 14s, and 15s were introduced at various times during 1981; prior to that time mortgage interest rates had never been that high. An oddity is the fact that rates jumped so quickly in 1981 and 1982 that GNMA 12s were not issued in large enough amounts to be quoted in the *Wall Street Journal* until the end of 1983. Finally, GNMA 14s and 15s had prepaid so much that their prices became of extremely poor quality, and the *Wall Street Journal* stopped quoting their prices in March 1990.

Given this ragged data set, the analysis is performed for three subperiods: calendar years 1979-1981, 1982-1986, and 1987-1990. The first and second subperiods had generally very high and volatile rates; the more recent subperiod had lower rates and lower rate volatility.

Table 6 gives for each of the subperiods the means and standard deviations of monthly returns both hedged and unhedged on all GNMA coupons from 8% to 15%. For comparison purposes, we analyze Ibbotson Associates' series of "Intermediate-Term Government Bonds" and "Long-Term Government Bonds," as they represent total returns on benchmark five-year and twenty-year government bonds. Returns for the Salomon Brothers Mortgage Index, Standard & Poor's 500 stock price index, the Shearson Lehman Government/Corporate Index, three-month Treasury bills, and three-month LIBOR are also shown.

During the 1979-1981 period, average mortgage returns unhedged and returns on other long-term bonds were lower than those on Treasury bills, as rates increased significantly during the period. Hedged returns on mortgages were much higher than unhedged returns, as the hedges provided protection for the fall in rates.

Comparing standard deviations across subperiods, we see that the 1979-1981 period had the highest volatility of returns and rates, with the ranking of returns reversed as rates fell in the 1982-1986 and 1987-1990 periods. In both of those periods, hedge losses reduced the returns on mortgages. In both 1982-1986 and 1987-1990, however, the hedged returns on mortgages exceeded those of Treasury bills. Unhedged returns in mortgages were similar to those of comparable duration Treasuries in the 1982-1986 period and exceeded them in the 1987-1990 period.

Next, let us examine Table 6 for the effectiveness of the dynamic hedging strategies in reducing risk. For GNMA 8s, 9s, 10s, and 11s, in all subperiods, the dynamically hedged returns are of substantially lower volatility than the unhedged returns. For the high coupons, hedges were very effective in reducing risk in the 1982-1986 period when rates were high and there was great rate volatility. In the 1987-1990 period, the

TABLE 6 ■ Mortgage and Bond Returns: Means and Standard Deviations (Monthly)

Long-Term Bonds and Stocks	1979–1981		1982–1986		1987–1990	
	Mean(%)	Std Dev (%)	Mean(%)	Std Dev (%)	Mean(%)	Std Dev(%)
5-Year Treasury	0.52	3.32	1.33	1.77	0.65	1.39
20-Year Treasury	0.04	5.28	1.71	3.70	0.64	2.71
Shearson Lehman Gov't/Corp	0.42	3.39	1.40	1.85	0.67	1.50
Standard & Poor's 500	1.21	4.39	1.61	4.23	1.09	5.52
Salomon Mortgage Index	0.16	4.80	1.62	2.36	0.78	1.52
GNMA 8 Unhedged	0.12	5.16	1.77	3.07	0.76	2.48
GNMA 9 Unhedged	0.15	5.05	1.74	2.96	0.79	2.25
GNMA 10 Unhedged	N/A	N/A	1.71	2.83	0.77	1.93
GNMA 11 Unhedged	N/A	N/A	1.61	2.67	0.80	1.29
GNMA 12 Unhedged	N/A	N/A	N/A	N/A	0.84	1.00
GNMA 13 Unhedged	N/A	N/A	1.42	2.03	0.88	0.71
GNMA 14 Unhedged	N/A	N/A	1.30	1.69	0.83*	0.54*
GNMA 15 Unhedged	N/A	N/A	1.26	1.48	0.79*	0.55*
Short-Term Bills and Hedges	**1979–1981**		**1982–1986**		**1987–1990**	
	Mean(%)	Std Dev (%)	Mean(%)	Std Dev (%)	Mean(%)	Std Dev(%)
Treasury Bills (3 month)	0.95	0.24	0.69	0.16	0.57	0.10
LIBOR (3 month)	1.17	0.25	0.81	0.20	0.68	0.08
GNMA 8 Unhedged	0.88	1.83	0.92	1.24	0.59	1.43
GNMA 9 Unhedged	0.92	1.62	0.91	1.18	0.61	1.36
GNMA 10 Unhedged	N/A	N/A	0.89	1.23	0.58	1.29
GNMA 11 Unhedged	N/A	N/A	0.87	1.14	0.62	0.86
GNMA 12 Unhedged	N/A	N/A	N/A	N/A	0.71	0.75
GNMA 13 Unhedged	N/A	N/A	0.83	1.07	0.81	0.67
GNMA 14 Unhedged	N/A	N/A	0.76	1.12	0.80*	0.65*
GNMA 15 Unhedged	N/A	N/A	0.75	1.10	0.76*	0.57*

Data for GNMA 14s and 15s end in March 1990.

TABLE 7 ▪ Dynamic Hedge Effectiveness

	A: Dynamic Futures Hedges								B: Treasury Bond Futures							
	1979–1981		1982–1986		1987–1990		Nov 81–Mar 90		1979–1981		1982–1986		1987–1990		Nov 81–Mar 90	
	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2
GNMA 8	-1.19	0.92	-0.94	0.84	-1.11	0.88	-1.03	0.87	0.92	0.91	0.64	0.78	0.68	0.90	0.71	0.83
GNMA 9	-1.17	0.93	-0.93	0.84	-1.03	0.86	-1.00	0.87	0.91	0.92	0.62	0.78	0.60	0.89	0.67	0.82
GNMA 10	NA	NA	-0.90	0.82	-0.95	0.80	-0.97	0.85	NA	NA	0.56	0.70	0.49	0.85	0.60	0.75
GNMA 11	NA	NA	-0.92	0.82	-0.82	0.64	-0.92	0.84	NA	NA	0.50	0.62	0.32	0.66	0.52	0.65
GNMA 12	NA	NA	NA	NA	-0.69	0.44	-0.78**	0.74**	NA	NA	NA	NA	0.18	0.45	0.26**	0.44**
GNMA 13	NA	NA	-0.82	0.75	-0.56	0.22	-0.89	0.80	NA	NA	0.32	0.44	0.10	0.21	0.33	0.45
GNMA 14	NA	NA	-0.75	0.63	-0.29*	0.07*	-0.83	0.73	NA	NA	0.24	0.34	0.05*	0.06*	0.26	0.36
GNMA 15	NA	NA	-0.70	0.52	-0.46*	0.20*	-0.83	0.67	NA	NA	0.22	0.38	0.08*	0.20*	0.24	0.40
Avg. Slope t-statistic =	20.5		14.3		9.6		17.0		19.2		9.9		10.0		12.4	

Notes: A: GNMA 8 Unhedged Return = a + b [GNMA 8 Dynamic Futures Hedge Gain %]
B: GNMA 8 Unhedged Return = a + b [Near Treasury Bond Futures Gain %]

* Data for GNMA 14s and 15s end in March 1990.

** Data for GNMA 12s cover only January 1984–December 1990.

dynamic hedges were not helpful in reducing the risks of high coupons. Their primary risks then were prepayment basis risks that are independent of (or very non-linear in) interest rates. During 1987–1990, elasticities for the high coupons were very small, so mortgage investors were aware of the limited usefulness of interest rate hedges for them.

In an alternative analysis of dynamic hedging, Table 7 shows the results of regressions of unhedged mortgage returns on the dynamic futures hedge returns. These results are compared to those for similar regressions on Treasury bond futures (excess) returns for a long position. Both of these regressions benefit from being able to fit the slope ex post. This is an advantage over the actual dynamic hedges, as they used only data available at the times the hedges were placed.

There are two interesting things to note about Table 7. First, the slopes in the dynamic futures hedge regressions should be approximately 1.00. They are for the discounts, but they are less than that for the high coupons, particularly during the 1987–1990 period. As futures hedge positions are chosen in advance, negative convexities of the mortgages will make the hedges seem too large ex post when rates decline. This is at least a partial explanation for slopes being less than 1.00 in the last two subperiods and greater than 1.00 in the first subperiod.

The slopes for the Treasury bond futures regressions give the estimated interest rate elasticities of the mortgages divided by the elasticity of the twenty-year Treasury bond contract. In each period, those estimates logically decline in each period for the higher coupons.

The second point may be more subtle, but it is of significant interest, as comparison displays the hedging advantage of dynamic adjustments. The period of greatest rate volatility for which there were many coupons is the second subperiod, 1982–1986. For mortgages hedged with a fixed slope (estimated ex post) in T-bond futures, the R-squareds range from 0.78 to 0.38.

Note that the dynamic mortgages constructed from the same Treasury bond futures returns, but with dynamically changing hedge ratios, show R-squareds improved to a range of 0.84 to 0.52. For the high-coupon mortgages, which went through significant changes in elasticities during that period, the improvement in hedge correlation is especially large.

Thus, in times of great rate volatility, dynamic hedging strategies significantly improve hedge correlation. In times of low rate volatility (e.g., 1987–1990), the dynamic hedging strategy does not improve upon a static hedging strategy.

The results of the same regression equations are also shown in Table 7 for the longest time period for which coupons from 8% to 15% were continuously available, November 1981 to March 1990. They show even more clearly the higher correlations of the dynamic hedging strategies than those of the static hedging strategies with the same hedge instrument.

Finally, Table 8 shows the betas, or return sensitivities, of mortgage returns on different coupons to movements in the Salomon Brothers Mortgage Index and to the Standard & Poor's 500 stock price index. As the Salomon index includes these mortgages, it is not sur-

TABLE 8 ▪ GNMA Mortgage Betas

	A: Salomon Mortgage Index						B: Standard & Poor's 500					
	1979–1981		1982–1986		1987–1990		1979–1981		1982–1986		1987–1990	
Unhedged	Beta	t(b)	Beta	t(b)	Beta	t(b)	Beta	t(b)	Beta	t(b)	Beta	t(b)
GNMA 8	1.06	38.3	1.26	29.0	1.44	12.7	0.40	2.1	0.31	3.6	0.04	0.6
GNMA 9	1.05	49.7	1.21	27.5	1.28	11.7	0.41	2.2	0.31	3.7	0.03	0.6
GNMA 10	NA	NA	1.16	30.7	1.06	10.1	NA	NA	0.28	3.6	0.01	0.2
GNMA 11	NA	NA	1.08	25.6	0.72	10.6	NA	NA	0.26	3.4	0.03	0.9
GNMA 12	NA	NA	NA	NA	0.50	7.1	NA	NA	NA	NA	0.03	1.4
GNMA 13	NA	NA	0.78	17.0	0.27	4.9	NA	NA	0.16	2.7	0.03	1.9
GNMA 14	NA	NA	0.61	12.6	0.16*	3.2*	NA	NA	0.12	2.4	0.03*	1.8*
GNMA 15	NA	NA	0.49	9.7	0.21*	4.7*	NA	NA	0.15	3.6	0.02*	1.3*

* GNMA 14s & 15s only from 1/87 to 3/90

prising to see very strong relationships there. Furthermore, the higher betas for discounts than for premiums make intuitive sense and are consistent with the elasticity analyses.

What is more interesting is that the GNMA 8s and 9s have had recent increases in betas. This is partly because the index also includes premiums, which have had reduced elasticities as rates have fallen, and partly due to the growing weight on FNMA and FHLMC mortgage securities, which have lower elasticities.

The mortgage index regressions of Table 8 and the Treasury bond futures regressions of Table 7 give bond market betas for mortgages. Betas relative to the stock market are in the second panel of Table 8.

Stock market betas for mortgages move much like other bonds' betas move across the subperiods. During the 1979–1981 period and the 1982–1986 period, these betas are significantly positive in the 0.20 to 0.40 range, consistent with other studies. This is so because interest rates rose when the stock market dropped in the recessions of 1980 and 1981–1982, and rates dropped when the stock market recovered during the growth years from late 1982 through 1986. In the 1987–1990 period, the stock market did well, while rates were relatively stable, leading to betas that were insignificantly different from zero.

VIII. SUMMARY

This article has presented the theoretical reasons for negative convexity of mortgages and showed that the data support the theory. A method that uses the market's implied price elasticities for risk analysis and hedging is found to be successful in constructing hedges of these very complicated securities. The dynamic nature of the hedging strategies inevitably generates whipsaw losses attributable to replication of the prepayment option. These losses, however, are reasonable in size and in pattern compared to those estimated by current mortgage researchers.

Returns on mortgages were examined for both hedged and unhedged positions, with average levels of returns found to exceed Treasury returns of comparable duration. Hedging simulations indicate that some hedged returns exceeded LIBOR financing costs, and some did not. The evidence presented here appears to be consistent with returns that are commensurate with mortgage risks. The methods for estimating whipsaw option replication costs and implied price elasticities for mortgages with changing risks, however, should help investors in their attempts to identify relative value in mortgage markets.

ENDNOTE

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*This roll-up, roll-down approach for implied elasticities needs to be modified if the maturities of adjacent coupons differ substantially from that of the mortgage coupon considered. If rates jump 1%, a GNMA 13 with twenty years to maturity is treated as a GNMA 12, which has twenty-three years to maturity. The different years to maturity also reflect different seasoning and prepayments, which might be significant. For the coupons examined here (from 8% to 15%), a compensating weighted-average maturity (WAM) adjustment did not alter implied elasticities significantly. For GNMA 7$^{1}/_{2}$s, however, the differences in WAMs and seasoning (versus the GNMA 8s) were significant. A use of implied elasticities involving GNMA 7$^{1}/_{2}$s would be reasonable only after adjusting for the WAM differential.

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