



Central Bank Policy Impacts on the Distribution of Future Interest Rates

Douglas T. Breeden and Robert H. Litzenberger***

Reference notes for talks at central banks in New York and at the
International Monetary Fund in Washington, August 2016

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***We thank Robert Merton and Robert Litterman for helpful comments and Rebekah Ackerman, Song Xiao, Lu Liu and their predecessors for research assistance.

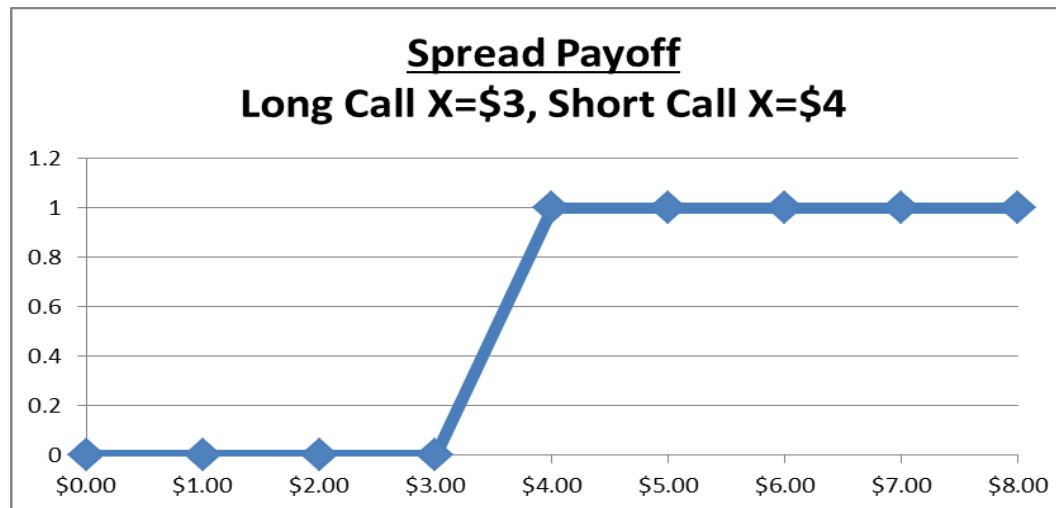
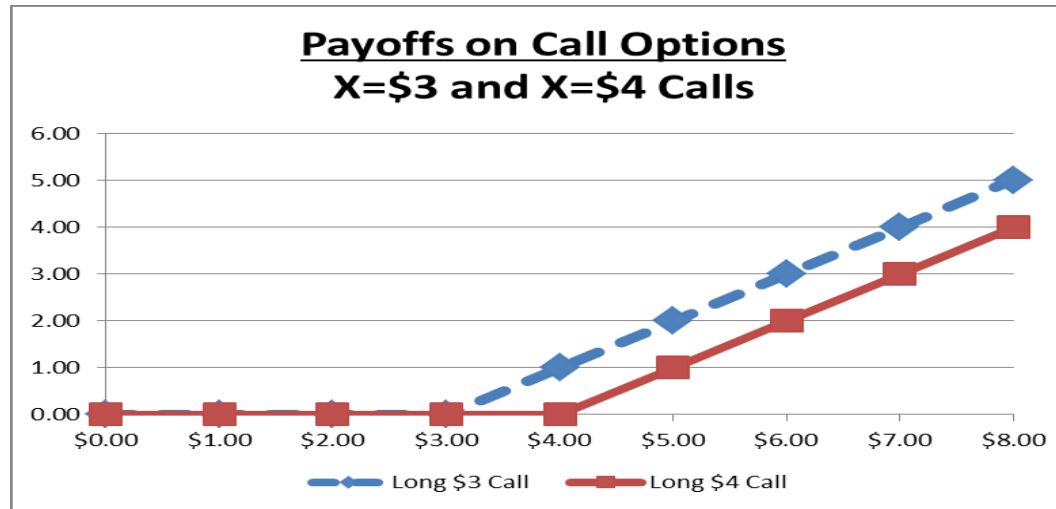
I. Overview of the Presentation

1. Before and after analysis shows that the U.S. Federal Reserve and the European Central Bank's policies often have significant impacts on the distributions for future interest rates. Fed, ECB and Bank of England research papers show central bank awareness.
2. Interest Rate Caps and Floors have been used for the last 30 years to hedge interest rate risks of financial institutions. They are portfolios of interest rate put and call options. We show how to use their prices to estimate the market's implied "insurance prices" for what LIBOR will be 3 to 5 years in the future.
3. Empirically, interest rate insurance prices 2003-2016 have shifted from bell-shaped curves to positively skewed ones. Some key market prices show "bipolar" views on future rates that reflect either (1) normalization or (2) fears of recession or deflation.

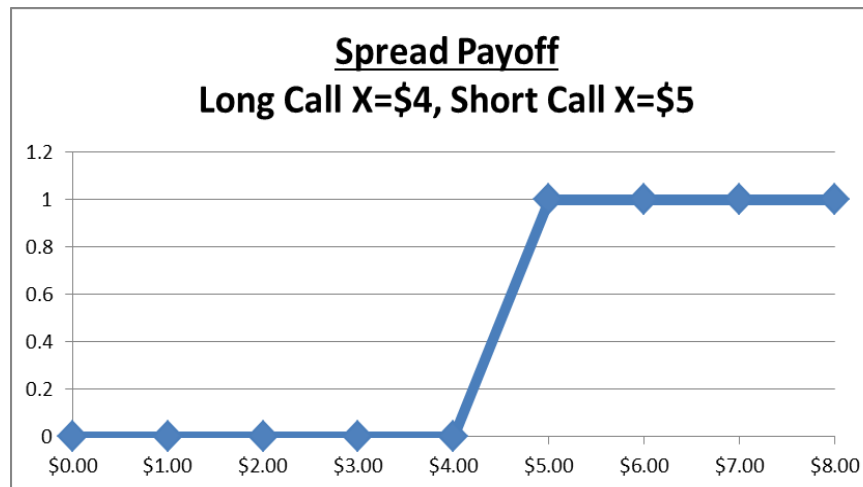
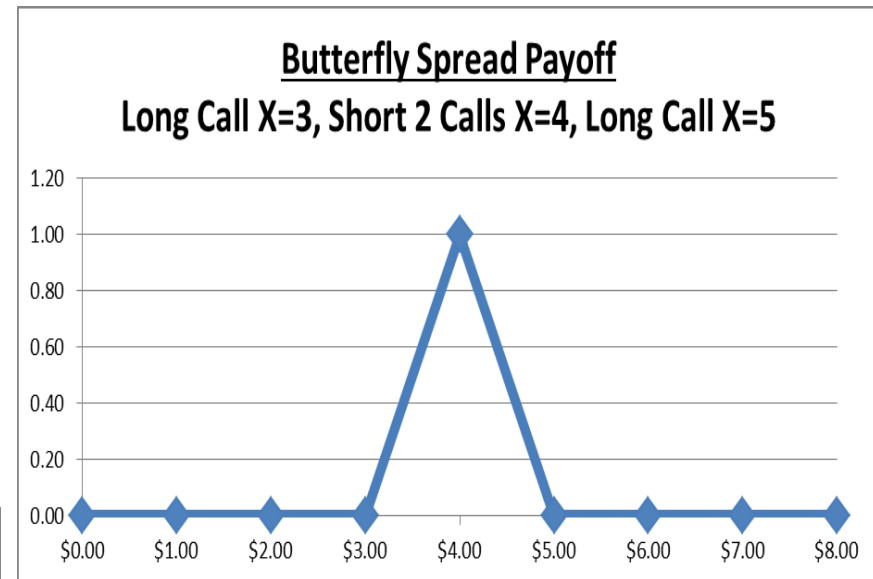
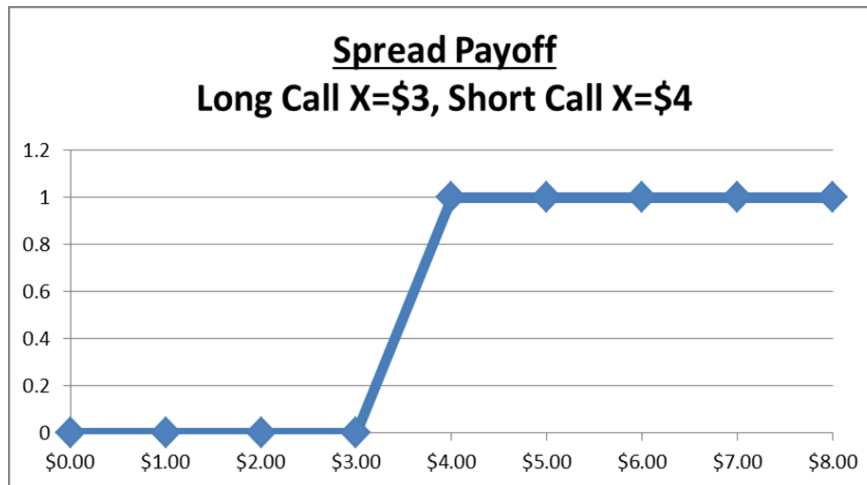
II. How to Find Interest Rate Insurance Prices From Option Prices:

*See Ross (1976), Quarterly Journal of Economics article “Options and Efficiency” and Breeden and Litzenberger (1978) Journal of Business article, “Prices of State-Contingent Claims Implicit in Option Prices.” B-L’s MIT working paper in 2013 on “Central Bank Policy Impacts on the Distribution of Future Interest Rates” gives the method for calculations in this talk. .

Option payoffs with continuous movements in the underlying asset prices...



Butterfly spread of options is a spread of spreads:
Payoffs are a pure bet on a specific range, zero elsewhere



Butterfly spread with strikes of
\$3, \$4, \$5 pays off only if
Underlying asset falls in that range.

Notes for Nerds: Theorem: If Risk Neutral Density is Linear in the Rate Range, then Digital Option (Arrow) Value Equals Butterfly Cost

Proposition: The relationship between butterfly spread values and digital option values:

If the risk-neutral density (RND) is a linear function of the interest rate within the range of the butterfly strikes, then the value of a digital option that pays off \$1.00 over the middle half of the range is equal to the value of the butterfly.

Proof: Let x be the interest rate, such that $x = c$ at the lower strike of the butterfly, $x = c + 1$ at the mid-point strike of the butterfly, and $x = c + 2$ at the high strike of the butterfly.

Assume that between c and $c+2$ the risk-neutral density = $RND = a + b(x - c)$

The forward value of a digital option that pays off \$1.00 between $x = c + 0.5$ and $x = c + 1.5$ is:

$$\int_{c+0.5}^{c+1.5} [a + b(x - c)] \cdot 1 dx = a + b$$

The forward value of a butterfly is $\int_c^{c+1} \{[a + b(x - c)](x - c)\} dx + \int_{c+1}^{c+2} \{[a + b(x - c)](c + 2 - x)\} dx$

$$= \frac{1}{3}bx^3 + \frac{1}{2}(a - 2bc)x^2 + (bc^2 - ac)x \Big|_c^{c+1} - \frac{1}{3}bx^3 + (bc + b - \frac{1}{2}a)x^2 + (2a - 2bc - bc^2 + ac)x \Big|_{c+1}^{c+2} = a$$

Of course, since forward values are equal at the same date, present values are also equal.

Q.E.D.

¹ Do note that there is a macro inconsistency in applying this approach with RNDs linear in rates where the $\{a,b\}$ coefficients change from rate range to rate range, as would be realistic. With overlapping triangles, this would give an RND for the 4% to 5% range that is different for the 3/4/5 butterfly than for the 4/5/6 butterfly. Thus, this Proposition's result is just an approximation that is for useful intuition about butterflies and digital options.

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**Prices of State-contingent
Claims Implicit in Option Prices***

(Journal of Business, 1978, vol. 51, no. 4)

B-L derived that the price of \$1.00 received if underlying price ends between Y_1 and Y_2 and the Black-Scholes formula holds is:

$$\Delta(Y_1, Y_2, T) = B(T)\{N[d_2(X = Y_1)] - N[d_2(X = Y_2)]\}. \quad (7)$$

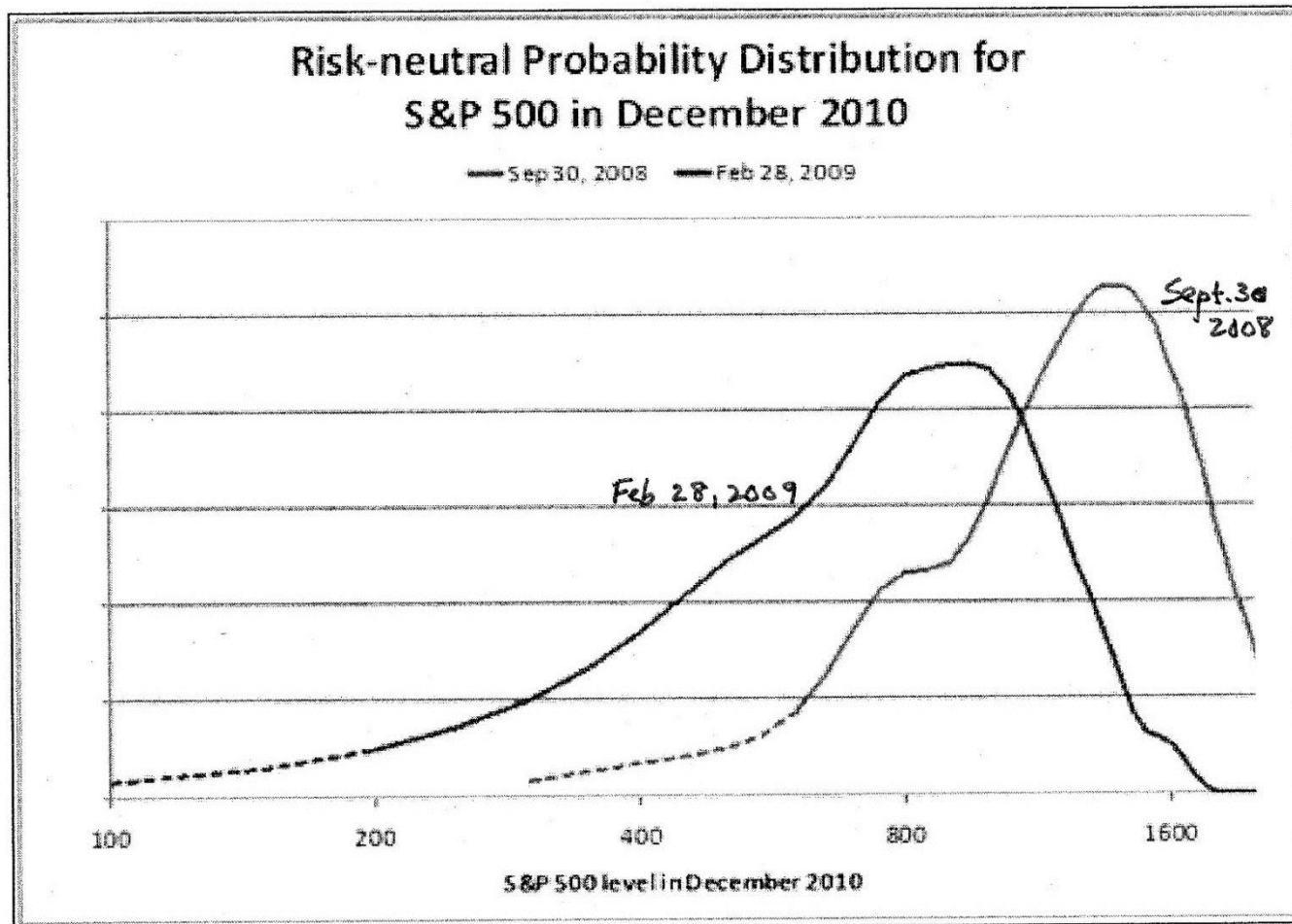
TABLE 2 **Values of the Cumulative Pricing Function and the Prices of Delta Securities: An Example***

Market Change in 1 Year (%)	1-Year Price Relatives (Y_1/M_0)	Cumulative Pricing Function [$G(Y_1/M_0)$]	Prices of Delta Securities [$\Delta(\dots; t = 1)$]
-40	.6	93.7¢	3.0¢
-30	.7	90.7¢	9.0¢
-20	.8	81.7¢	15.7¢
-10	.9	66.0¢	18.9¢
0	1.0	47.1¢	17.3¢
+10	1.1	29.8¢	12.8¢
+20	1.2	17.1¢	8.1¢
+30	1.3	8.9¢	4.6¢
+40	1.4	4.3¢	2.3¢
+50	1.5	2.0¢	

* Parameters for this example are: $\sigma = .20$, $\delta = .04$, $r_1 = .06$, and $t = 1$ year.

Freakonomics article: “Quantifying the Nightmare Scenarios”

Eric Zitzewitz (Dartmouth) Uses Breeden-Litzenberger 1978 Technique
In *Freakonomics* Blog by Justin Wolfers, March 2, 2009



9/30/2008:
S&P500= 1166
VIX = 39.4%

2/28/2009
S&P500 = 735
VIX = 46.4%

Breeden-Litzenberger Method (1978, 2013) used by Central Banks to find price distributions from option prices.

Probability distributions of future asset prices implied by option prices

1996 Bank of England Quarterly

By Bhupinder Bahra of the Bank's Monetary Instruments and Markets Division.

Introduction

Many monetary authorities routinely use the forward-looking information that is embedded in financial asset prices to help in formulating and implementing monetary policy. For example, they typically look at changes in the forward rate curve implied by government bond prices to assess changes in market perceptions of future short-term interest rates.⁽¹⁾ But, although implied forward rates are informative about the market's mean expectation for future interest rates, they tell us nothing about the range of expected outcomes around such estimates. For this, we can turn to options markets.

exercising it only if the price of the underlying asset lay above the strike price at that time.

Consider a set of European options on the same underlying asset, with the same time-to-maturity, but with different exercise prices. The prices of such options are related to the probabilities attached by the market to the possible values of the underlying security on the maturity date of the options. Intuitively, this can be seen by noting that the difference in the price of two options with adjacent exercise prices will reflect the value attached to the ability to exercise the options when the price of the underlying asset lies between the two exercise prices. This price difference in turn depends on the probability of the underlying asset price

The Breeden and Litzenberger approach

Breeden and Litzenberger (1978) derived a relationship linking the curvature of the call pricing function to the terminal RND function of the price of the underlying asset. In particular, they showed that the second partial derivative of the call pricing function with respect to the exercise price is directly proportional to the terminal RND function. Details about the derivation of the Breeden and Litzenberger result are given in Bahra (1996). The rest of this article focuses on how this result can be applied in order to estimate market RND functions for short-term interest rates in the future and how such RND functions can be used for policy analysis.

FEDERAL RESERVE BANK OF MINNEAPOLIS

BANKING AND POLICY STUDIES

Methodology for Estimating Risk Neutral Probability Density Functions

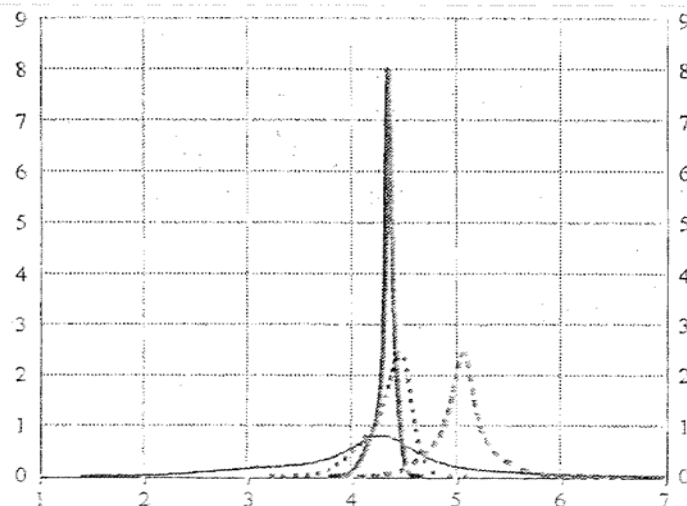
We estimate risk neutral probability density functions (RNPDs) for a variety of different asset classes using a variation of the technique developed by Shimko (1993). This procedure involves fitting a curve to the implied volatilities of a series of options and expressing the volatility as a function of the strike price. The implied volatilities are then translated into continuous call option prices, and the risk neutral distribution of the underlying asset is obtained through the Breeden-Litzenberger (1978) method.

European Central Bank's Monthly Bulletin, February 2011, uses the Breeden-Litzenberger 1978 method to estimate interest rate distributions for what Euribor will be in 3 Months:

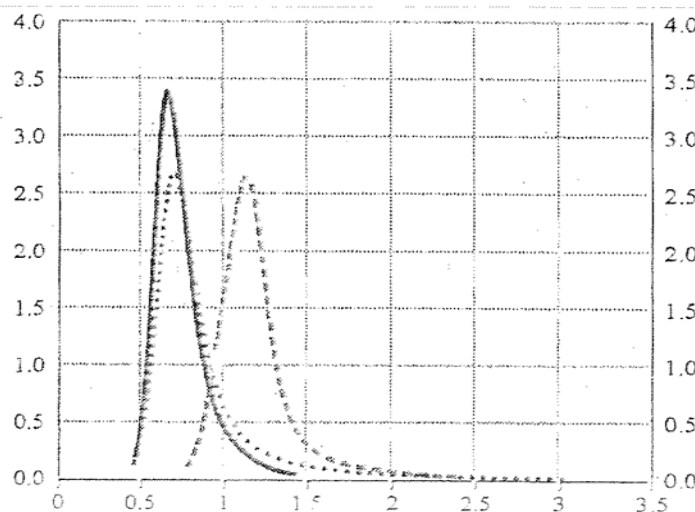
THE INFORMATION CONTENT OF OPTION PRICES DURING THE FINANCIAL CRISIS

x-axis: interest rate
y-axis: density

— 4 June 2007
..... 10 August 2007
- - - - 1 September 2008
—— 8 October 2008



— 1 April 2010
..... 20 May 2010
- - - - 14 January 2011



Sources: NYSE Liffe and ECB calculations.

Disadvantages of Many Prior Approaches

■ 1. *Short-term option prices used.*

Most options mature in 3 months to 18 months, as many markets only have active markets for those maturities. Often there are not options actively traded for a large number of standardized strike prices. We use interest rate caps and floors that have longer term maturities from 2 to 10 years.

■ 2. *Parametric vs. nonparametric approach.*

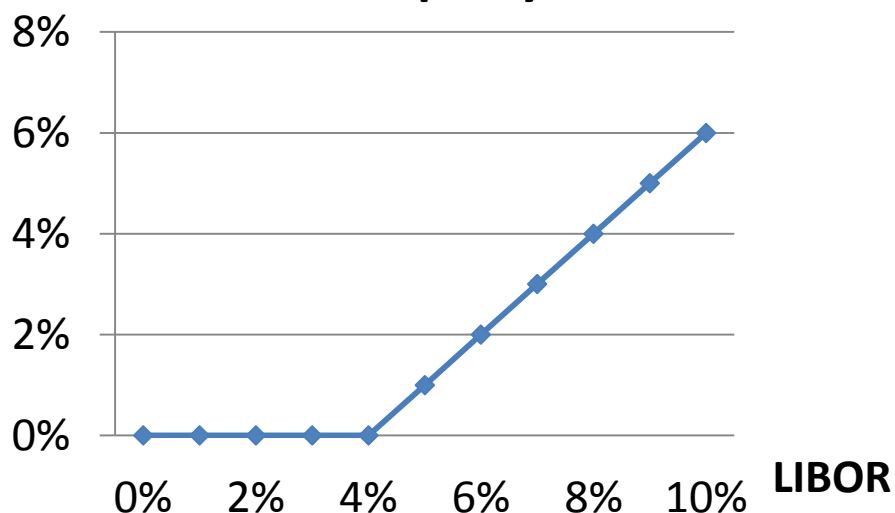
Applications often parameterize option prices with 3 or 4 parameters (mean, variance, skewness, kurtosis) and estimate implied volatility surfaces and entire risk-neutral densities. It is well-known among practitioners that these methods can be off significantly in estimating tail risks.

Payoffs on Interest Rate Caps and Floors

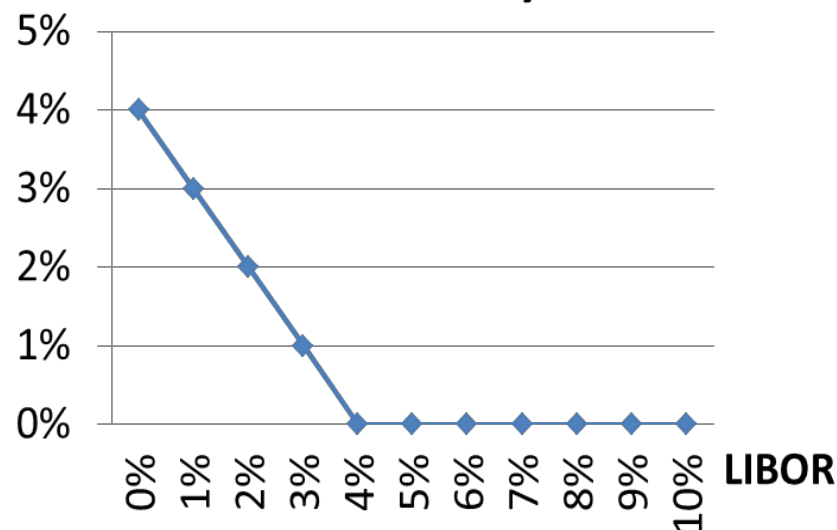
Purchaser of a 5-year **interest rate cap** on 3-month LIBOR, with a “strike rate” (exercise price) of 4% receives quarterly for 5 years the difference between then-current LIBOR and 4%. (0 if <4%).

Caps hedge against higher interest rates. As rates increase, the cap’s cash flows increase and pay increased funding costs. Caps win when rates increase, like portfolios of put options on bond prices.

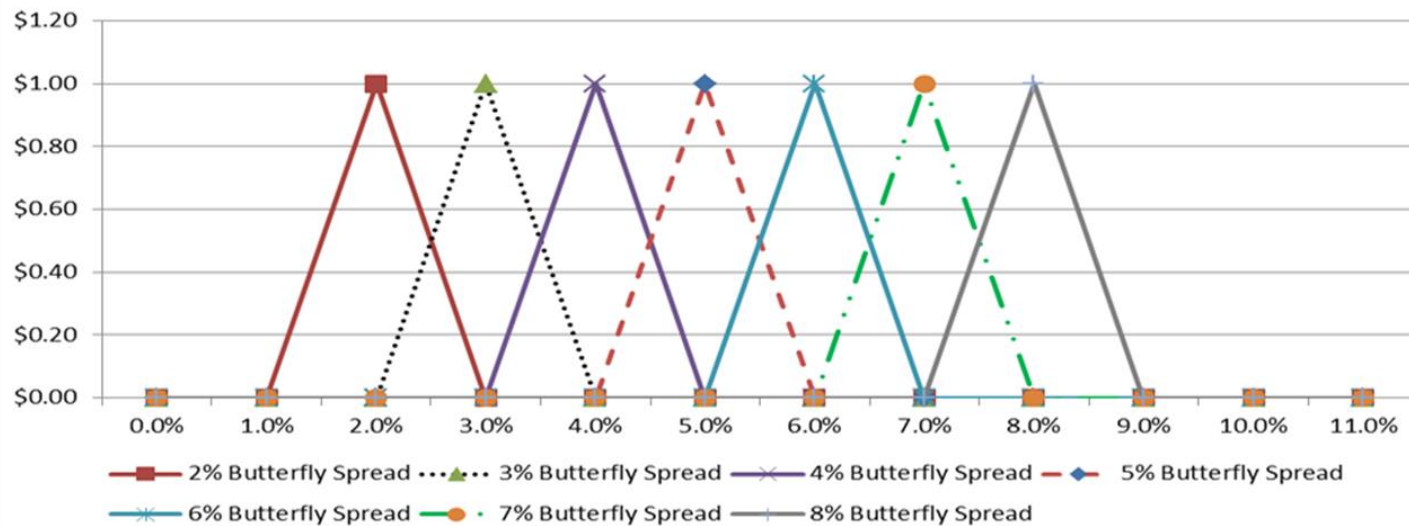
Cap Payoff



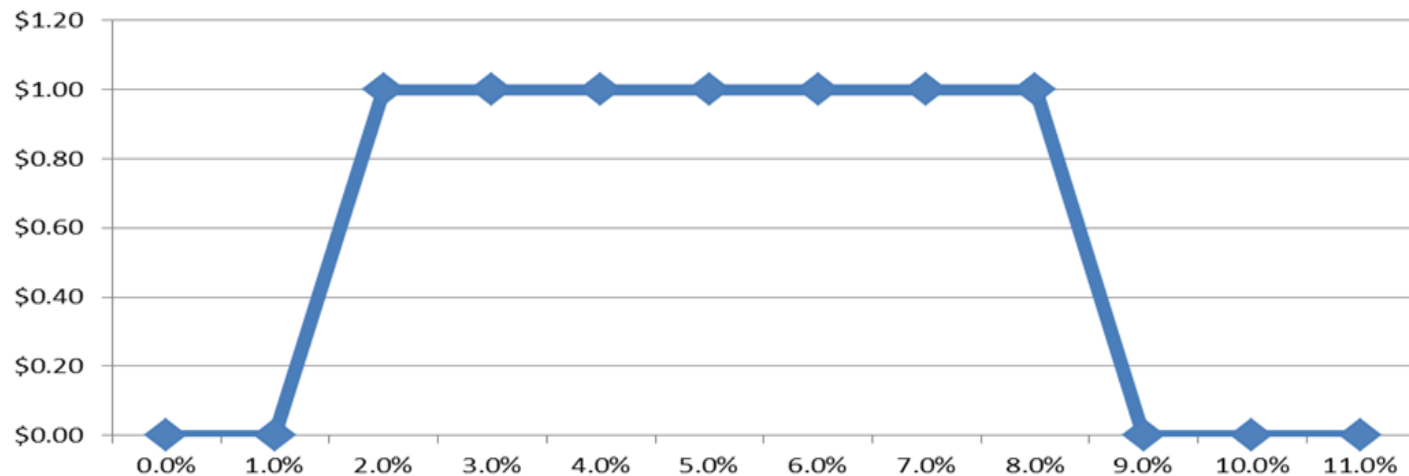
Floor Payoff



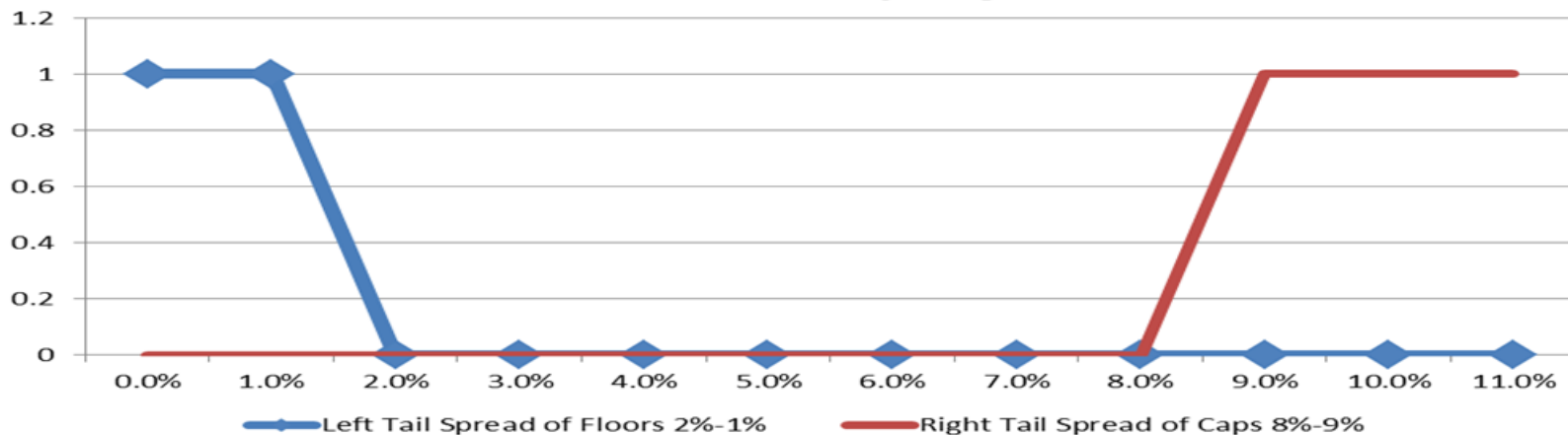
Payoffs on Butterfly Spreads: 2% to 8% Centers



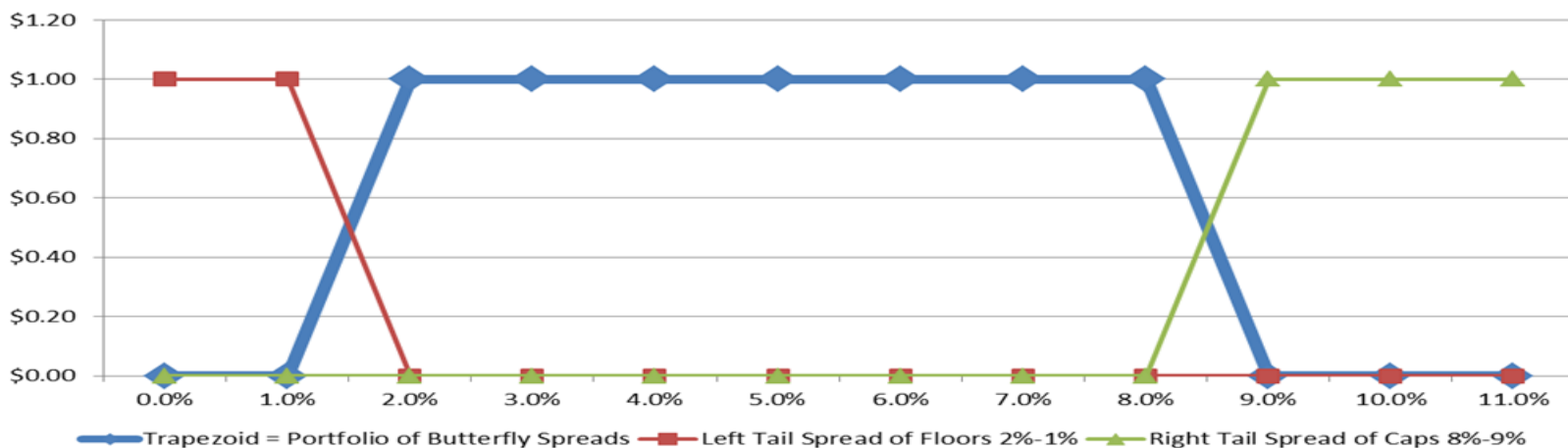
Trapezoid = Portfolio of Butterfly Spreads



Payoffs on Tail Spreads of Floors and Caps **Floor Left Tail: 2%-1%; Cap Right Tail 8%-9%**



Trapezoid = Portfolio of Butterfly Spreads + Left and Right Tail Spreads = Riskless Zero Coupon Bond



Butterfly Spread and Tail Spread Costs and Risk Neutral Probabilities (Insurance Prices)

	<u>Spread Cost</u>	<u>“Risk-Neutral Probability”</u>
“0%” = Left tail spread: Long 1%, Short 0% floorlet	\$0.290	0.297
1% Butterfly spread (Long 0%, Short 2 1%, Long 2%)	\$0.320	0.328
2% Butterfly spread (Long 1%, Short 2 2%, Long 3%)	\$0.180	0.184
3% Butterfly spread	\$0.080	0.082
4% Butterfly spread	\$0.037	0.038
5% Butterfly spread	\$0.028	0.028
6% Butterfly spread	\$0.014	0.014
7% Butterfly spread	\$0.007	0.007
8% Butterfly spread	\$0.007	0.007
9%+ = Right tail spread: Long 8%, Short 9% caplet	<u>\$0.015</u>	<u>0.015</u>
Totals	\$0.977	1.000

III. True Probabilities vs. Insurance Prices or “Risk Neutral Probabilities”

True Probabilities vs. **Insurance Prices or “Risk Neutral Probabilities”**

Insurance prices or “risk neutral probabilities” differ from true, objective probabilities, because investors price assets higher for those that pay off most when times are bad (negative beta). Thus, their insurance prices (risk neutral probabilities) exceed their true probabilities.

States that correspond to good economies will have lower insurance prices, and their insurance prices will underestimate the true probabilities.

Notes for Nerds: In a general state preference model:

Inserting eq. 6 for the zero coupon bond gives:

$$\frac{\phi_{tr_j}^*}{\pi_{tr_j}} = \frac{E[\tilde{u}'_{ts} | r_j]}{E[\tilde{u}'_t]} \quad (12)$$

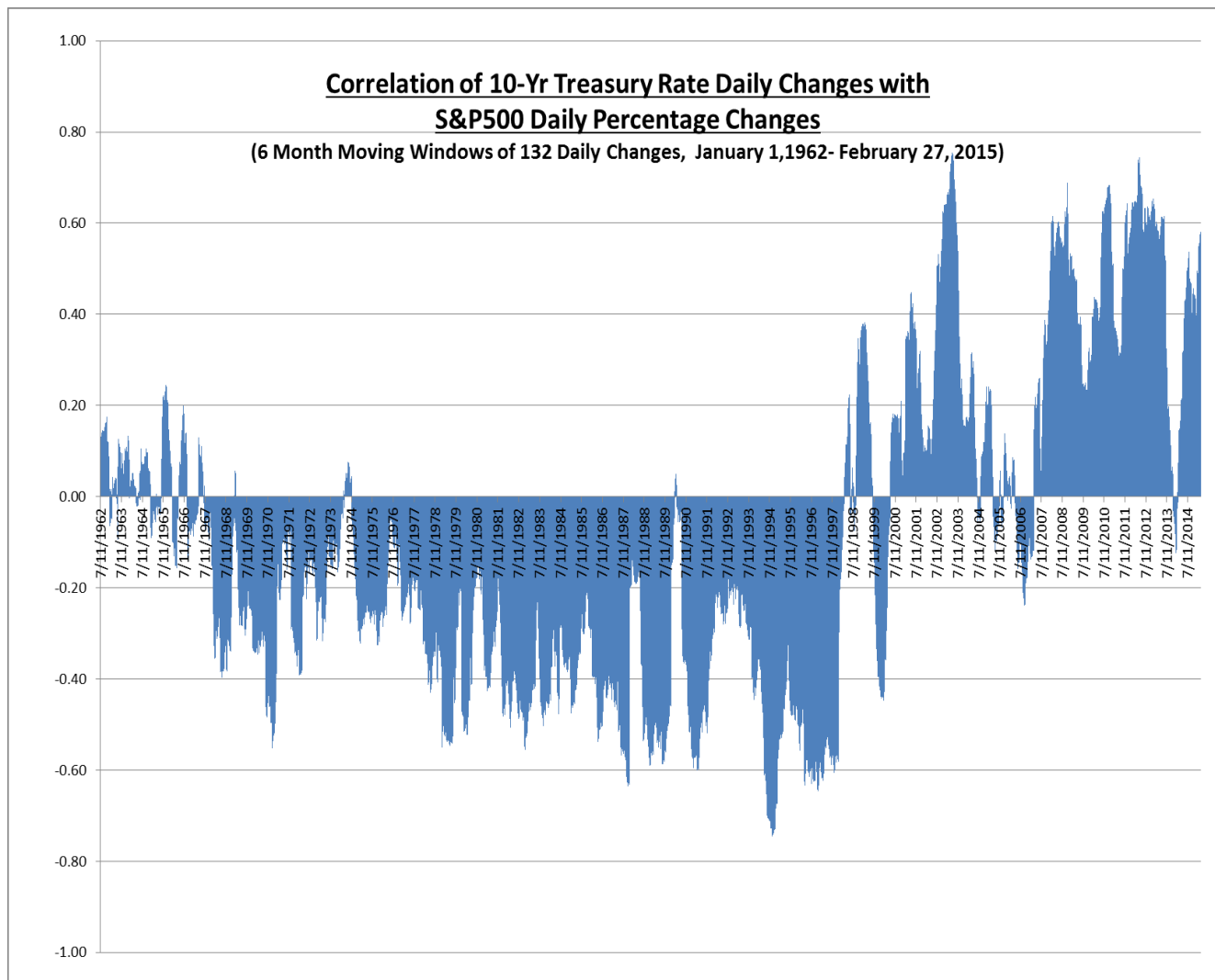
Thus, we see that the risk-neutral probability to true probability ratio at the optimum for r_j is equal to the expected marginal utility of consumption, conditional upon the interest rate being at the specified level, divided by the unconditional expected marginal utility of consumption at time t . So if we are looking at butterfly spreads or digital options centered upon LIBOR = 2%, we need to compute the conditionally expected marginal utility of consumption, given that 2% rate.

If assume power utility (CRRA) and lognormally distributed consumption, we get a simple formula for state price to probability ratios:

$$\log\left(\frac{\phi_{ts}^*}{\pi_{ts}}\right) = \gamma \left[\mu_t - g_{ts} - \frac{1}{2} \gamma \sigma_c^2 \right] t \quad (19)$$

As expected, higher growth states for consumption have lower $\left(\frac{\phi_{ts}^*}{\pi_{ts}}\right)$ ratios. One could input different estimates of relative risk aversion and different states' growth rates and consumption volatility into the eq. 19 and compute the estimated log of the risk neutral probability to the true probability.

When rates are high, is marginal utility high or low? Depends on the time period.



This graph shows the dramatic switch from negative to positive in 1999/2000 in the correlation between changes in the 10-year interest rate and moves in the S&P 500.

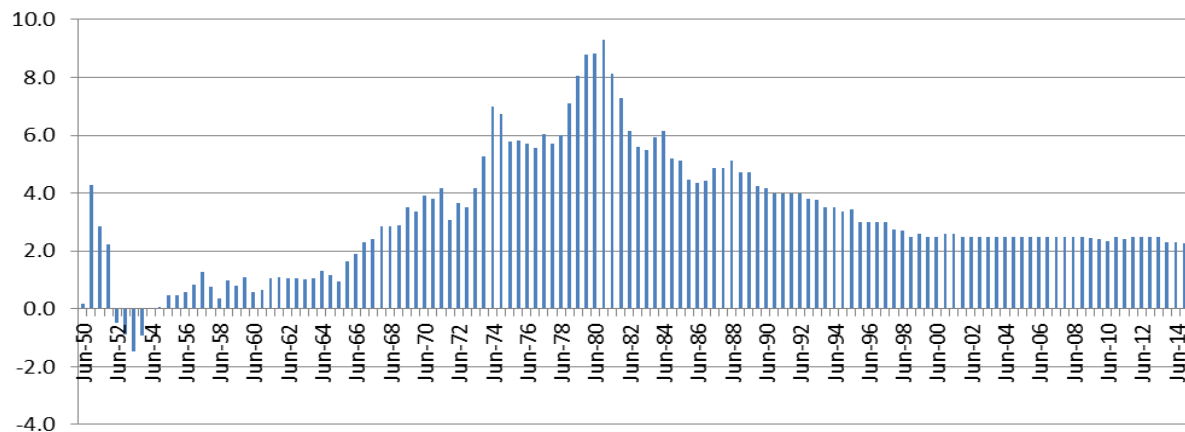
This switch in correlation reflects a shift from supply-oriented inflation concerns in the 1970s and 1980s to inflation concerns dominated more by demand issues.

The beta of long-term bond returns versus stock returns and the economy thus shifted from positive to negative. The fair risk premium on long-term bonds should have shifted from positive to negative, as long-term bonds became excellent hedges for risks of a bad economy.

What caused correlations of interest rates and stocks to change sign in 2000-2015?

Livingston Survey of Long Term Inflation Forecasts

10 Year Forecasts from 1990, 2 Yr Prior.
Semiannual data, June 1950-December 2014

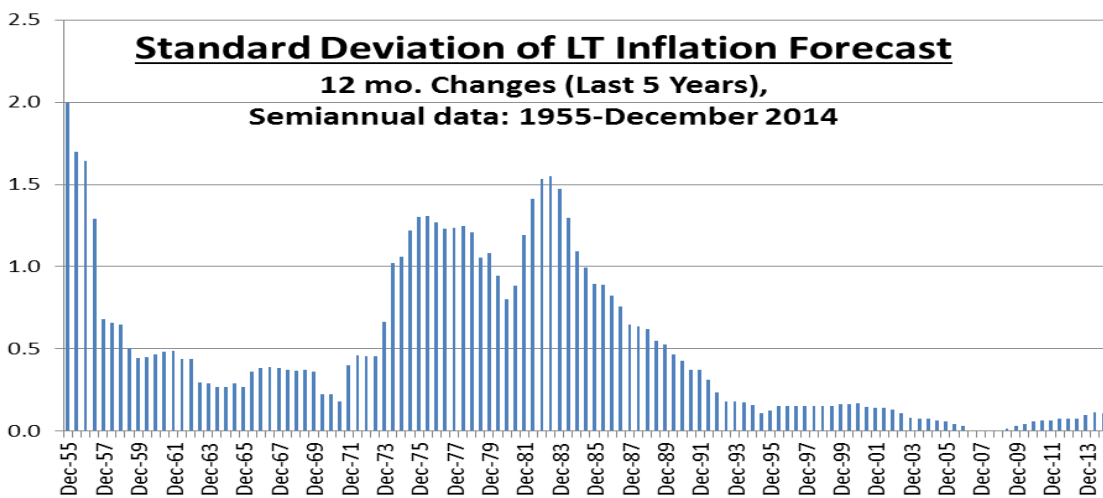


The Livingston/Philly Fed semiannual survey of inflation expectations shows the dramatically higher inflation rate in the 1970s and early 1980s, with notable surges in 1974/75 due to oil price and grain price shocks, as well as in 1981/82 when the second major round of OPEC oil price shocks occurred.

After the Volcker Fed in 1979-1981 let interest rates increase dramatically while focusing on controlling money supply growth, inflation was sharply reduced. Inflation expectations peaked in 1981 at 9% and dropped to less than 5% by the end of the 1980s. With continued monetary discipline, the 10-year inflation forecast dropped through the 1990s until it hit 2.5% in 1999/2000. The inflation rate forecast has remained anchored between 2.0% and 2.5% from 2000 to 2014, with very low volatility of the inflation forecast.

Standard Deviation of LT Inflation Forecast

12 mo. Changes (Last 5 Years),
Semiannual data: 1955-December 2014

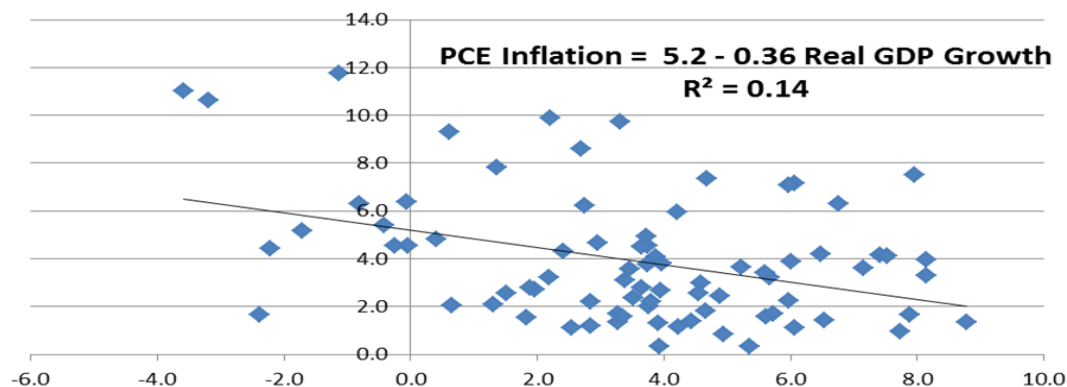


What caused correlations of interest rates and stocks to change sign in 2000-2015?

PCE Inflation vs. Real GDP Growth 1960-1999.

Supply Oriented Inflation:

Low Supply, Low GDP, High Inflation



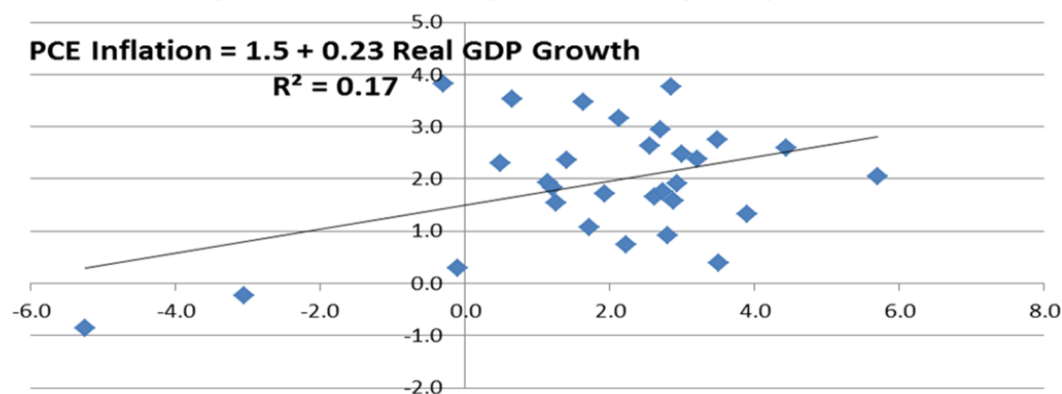
In the 40-year period from 1960 to 1999, higher real GDP growth occurred in conjunction with lower inflation, and recessions generally happened with high inflation, led by the big 1974/75 and 1981/82 recessions.

Constricted oil and grain supplies caused high inflation at times of these significant recessions. High inflation led to high interest rates, so the USA had high rates in recessions. Bond returns had positive stock correlations with supply risks: returns were negative when the economy was down, positive when the economy was strong.

PCE Inflation vs. Real GDP Growth 2000-2014.

Demand Oriented Inflation:

High Demand, High GDP, High Inflation



In the 15-year period from 2000 to 2014, higher real GDP growth occurred in conjunction with higher inflation, and recessions generally happened with low inflation, led by the Great Recession of 2008/09.

Weak demand in the Great Recession led to very low interest rates. Supply uncertainties were dominated by demand uncertainties in this period.

Bond returns had negative stock correlations when demand risks dominated, as their returns were very positive (due to the very low rates) when the economy was down sharply in 2008/9.

Illustration of True Probabilities Related to Risk Neutral Probabilities

True probability = $K \times \text{Risk Neutral} \times \exp(\text{Gamma} \times (\text{gts} - \mu))$

Assumes: CRRA-Lognormal real growth model

Real Growth on Nominal Rate: 1998 to 2011 Data

Intercept -3.71 (t= -2.2)

Slope 1.42 (t= 3.8)

MuCgrow 3

Relative Risk Aversion (Gamma)

Nominal Real 2 4 8

Rate Growth Ratio of True Probability to Risk Neutral*

1	-2.29	0.90	0.81	0.65
2	-0.87	0.93	0.86	0.73
3	0.55	0.95	0.91	0.82
4	1.97	0.98	0.96	0.92
5	3.39	1.01	1.02	1.03
6	4.81	1.04	1.08	1.16
7	6.23	1.07	1.14	1.29
8	7.65	1.10	1.20	1.45
9	9.07	1.13	1.27	1.63
10	10.49	1.16	1.35	1.82

*=Up to a scalar multiple

Real Growth on Nominal Rate: 1977 to 1997 Data

Intercept 4.11 (t= 3.2)

Slope -0.12 (t= -0.8)

MuCgrow 3

Relative Risk Aversion (Gamma)

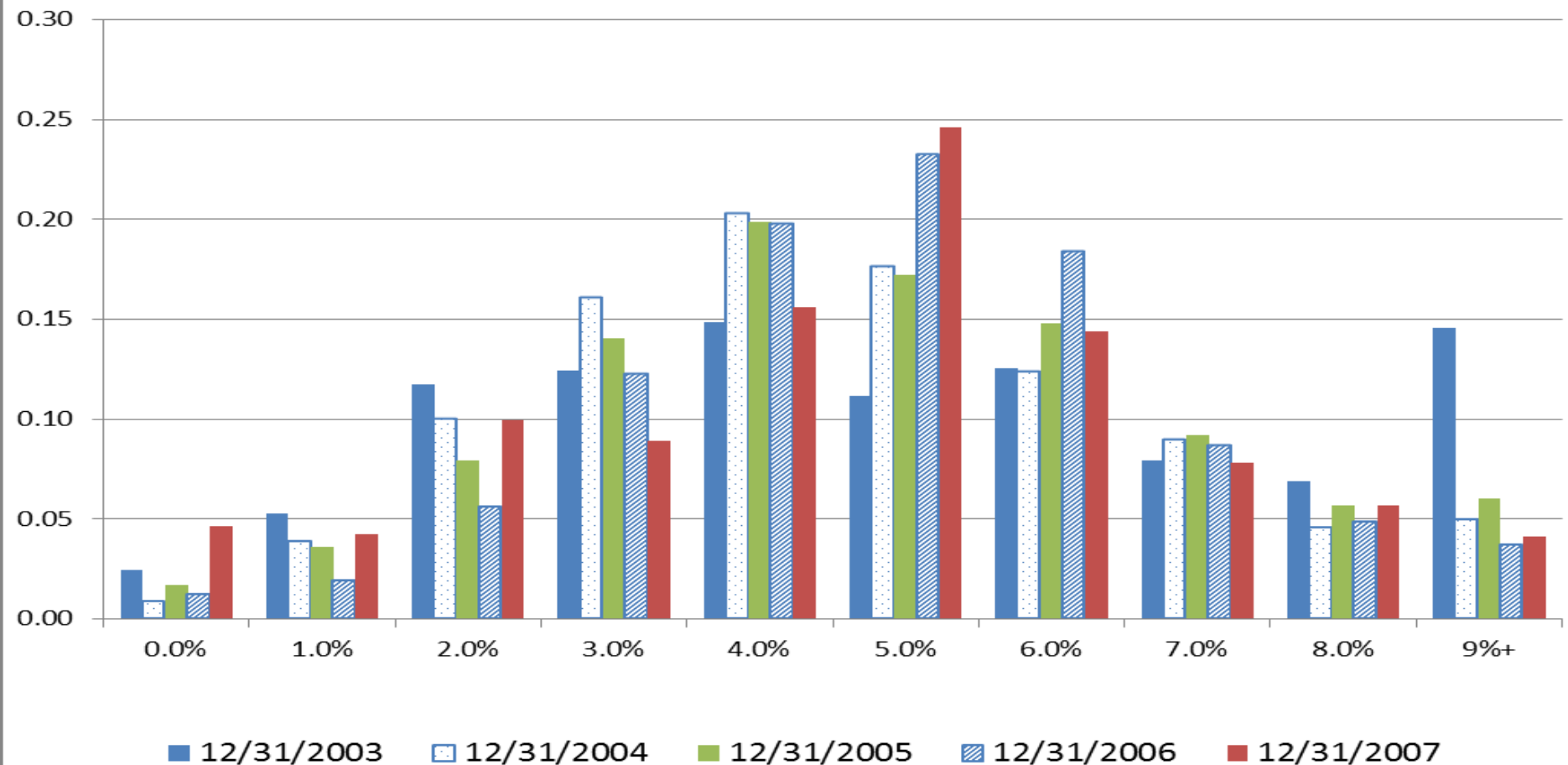
Nominal Real 2 4 8

Rate Growth Ratio of True Probability to Risk Neutral*

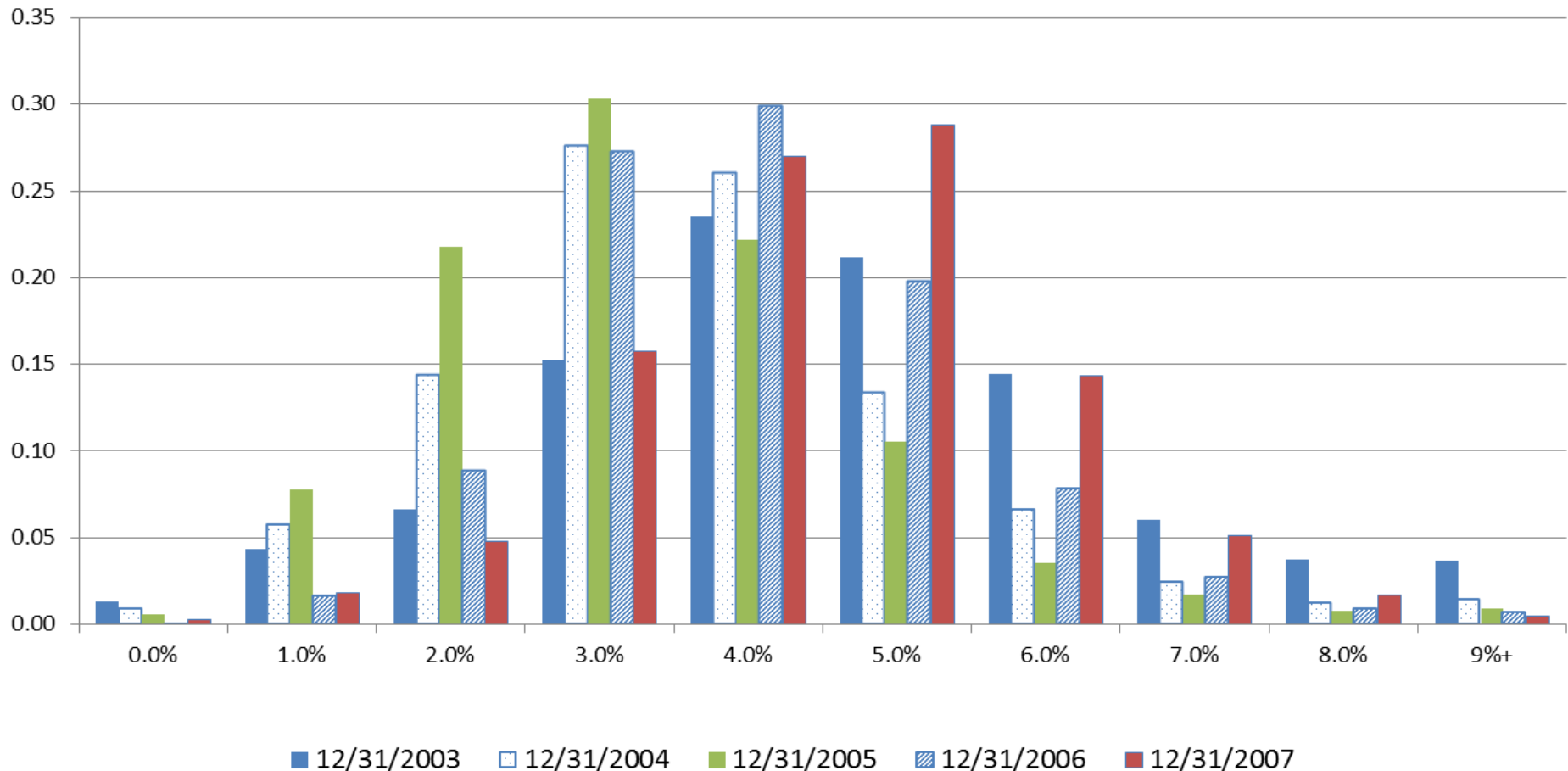
1	3.99	1.02	1.04	1.08
2	3.87	1.02	1.04	1.07
3	3.75	1.02	1.03	1.06
4	3.63	1.01	1.03	1.05
5	3.51	1.01	1.02	1.04
6	3.39	1.01	1.02	1.03
7	3.27	1.01	1.01	1.02
8	3.15	1.00	1.01	1.01
9	3.03	1.00	1.00	1.00
10	2.91	1.00	1.00	0.99
11	2.79	1.00	0.99	0.98
12	2.67	0.99	0.99	0.97
13	2.55	0.99	0.98	0.96
14	2.43	0.99	0.98	0.96
15	2.31	0.99	0.97	0.95
16	2.19	0.98	0.97	0.94

IV. Estimates of Interest Rate Insurance Prices
Implicit in Prices of Interest Rate Caps and Floors
2003-2007.

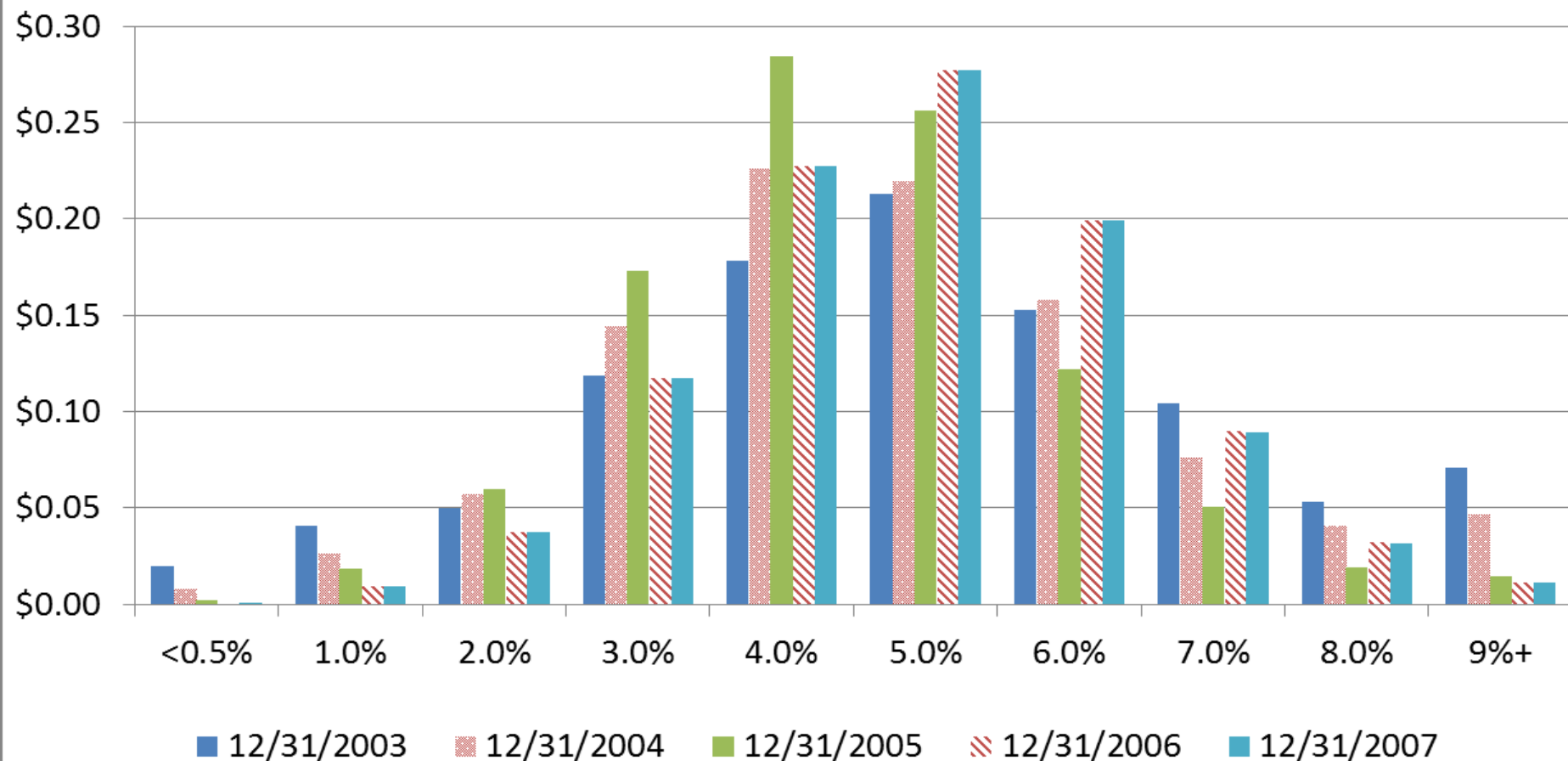
USA Insurance Prices for 3-Month LIBOR in 5 Years,
as of December 31, 2003, 2004, 2005, 2006, 2007:
Relatively Symmetric Distributions



Euro Area Insurance Prices for 6-Month Euribor in 5 Years,
as of December 31, 2003, 2004, 2005, 2006, 2007
Relatively Symmetric Distributions



British Pound Insurance Prices for 3-Month Interbank Rate in 5 Years
as of Dec 31 2003, 2004, 2005, 2006, 2007:
Relatively Symmetric Distributions

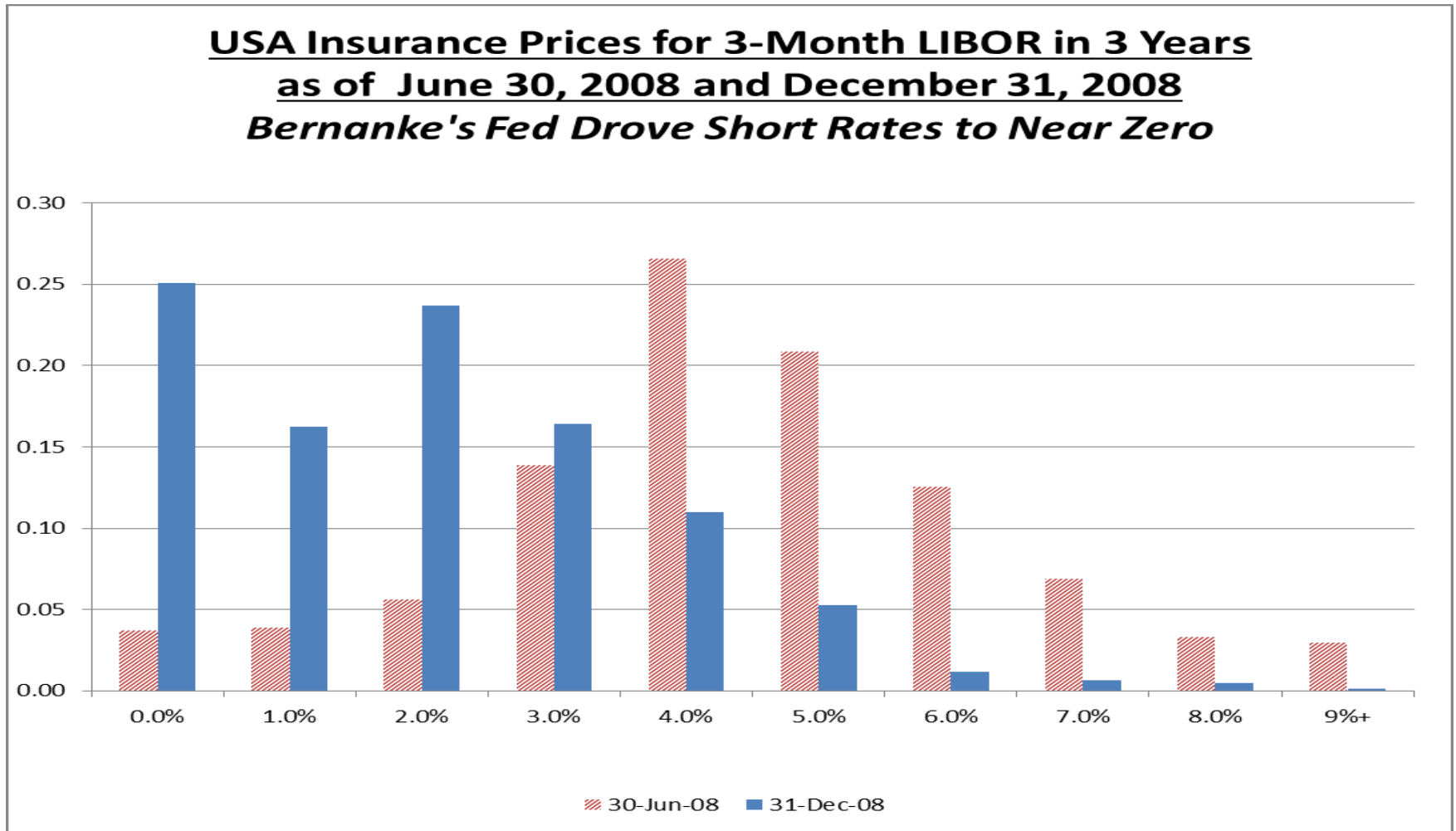


V. Impact of U.S. Federal Reserve Policy Announcements on Interest Rate Insurance Prices for 3-Month LIBOR: 2008-2016

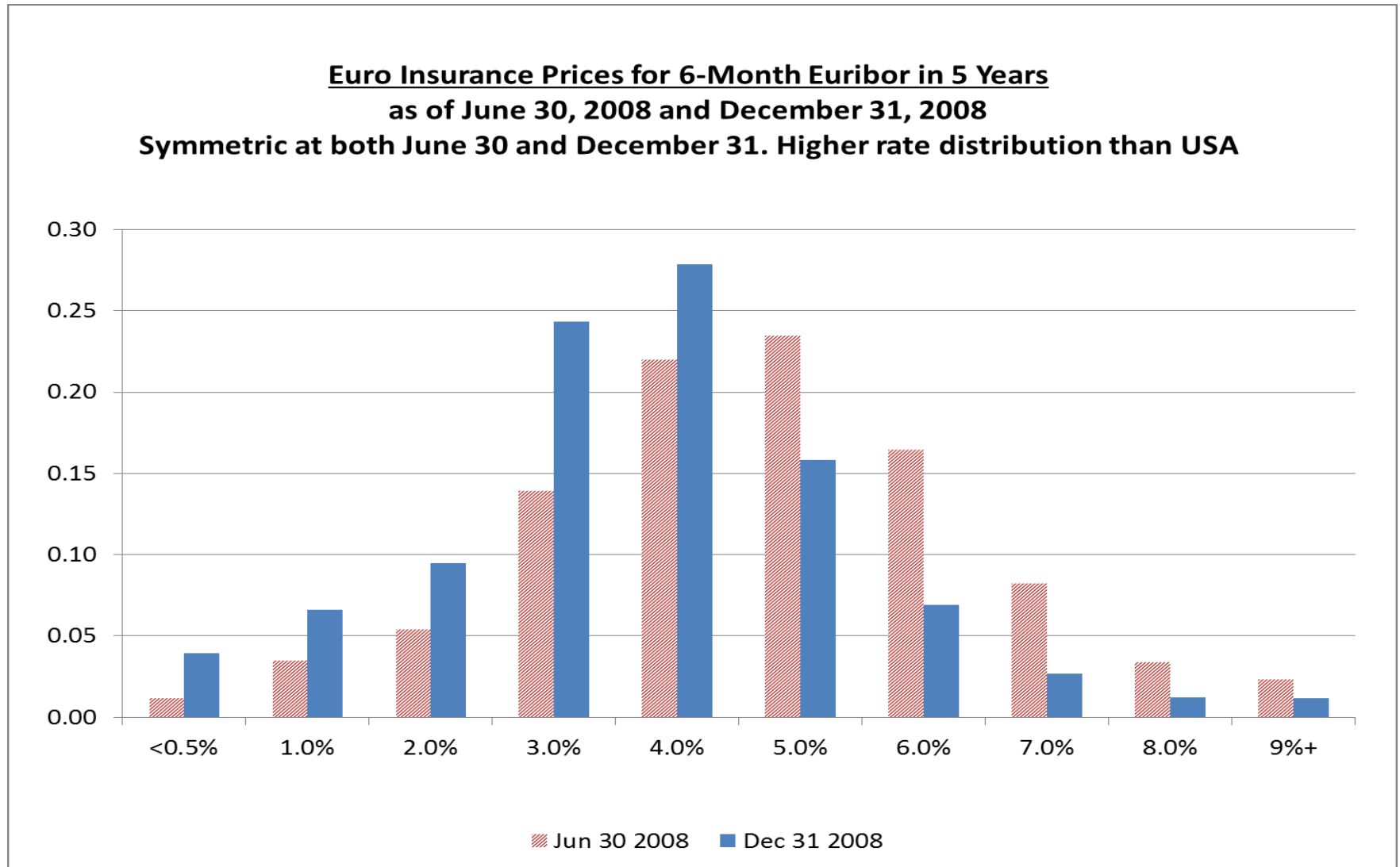
Major Federal Reserve Announcements 2008-2016

- **December 2008.** Cut rates to record lows in financial panic.
- **March 2009:** Will keep rates close to zero for “extended period.” Stock market bottoms March 9th. Unemployment rate increases to peak of 10.0% in October 2009.
- **August 2011:** Budget impasse. Fed “will keep rates extremely low “at least until 2013.”
- **September 2012:** Low “at least until 2015”
- **December 2012:** Will tie low rates to range in Unemployment (>6.5%), Inflation(<2%).
- **May/June 2013:** **May 22:** Given economic strength, Fed is seriously considering “tapering” asset purchases (QE3). **June 19:** Housing market is strong and supportive; tapering QE3 likely in 2nd half 2013.
- **Sept 18, 2013:** Fed announces “No tapering yet” and surprises markets.
- **Dec 18, 2013.** Bernanke Fed announces beginning of tapering, \$10 billion/month.
- **March 19, 2014.** Yellen Fed indicates short rates may rise in 6 months after end of tapering, perhaps by mid-2015, earlier than markets expected.
- **April 30, 2014.** Job growth strong. Unemployment rate drops sharply: 6.7% to 6.3%.
- **October, 2014.** Unemployment at 5.9%. Yellen Fed ending asset purchases (QE).
- **March, 2015.** Unemployment at 5.5%, rapid job growth. Fed drops “patience” talk. “Dots” show that Fed members expect a slower ramping up of rates after liftoff.
- **December, 2015.** Fed “lifts off” and raises its policy rate 0.25%, first since the Great Recession.

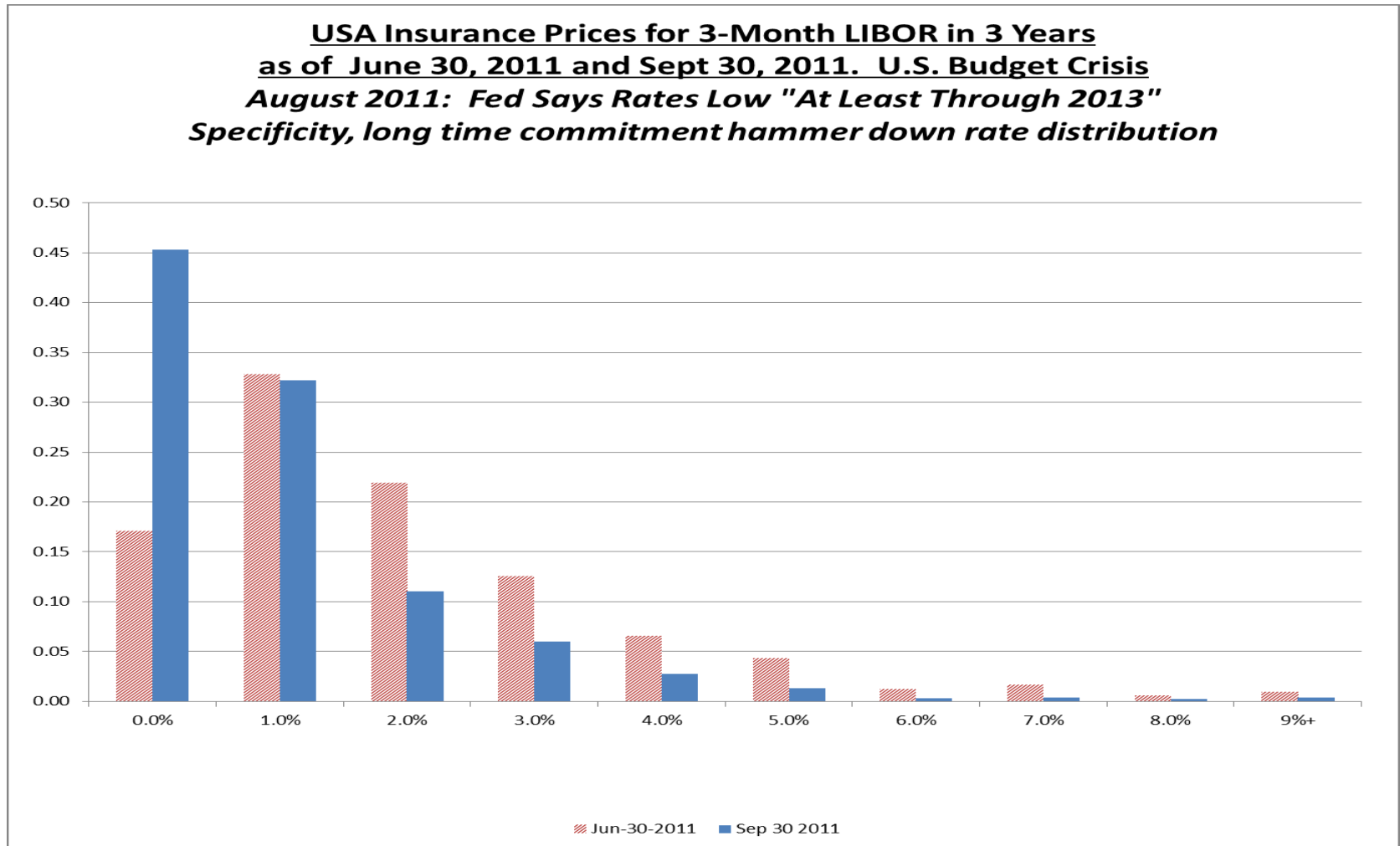
2008: U.S. Rate Distribution Transformed from Symmetric to Positive Skewness (Concentrated near zero, but long right tail)



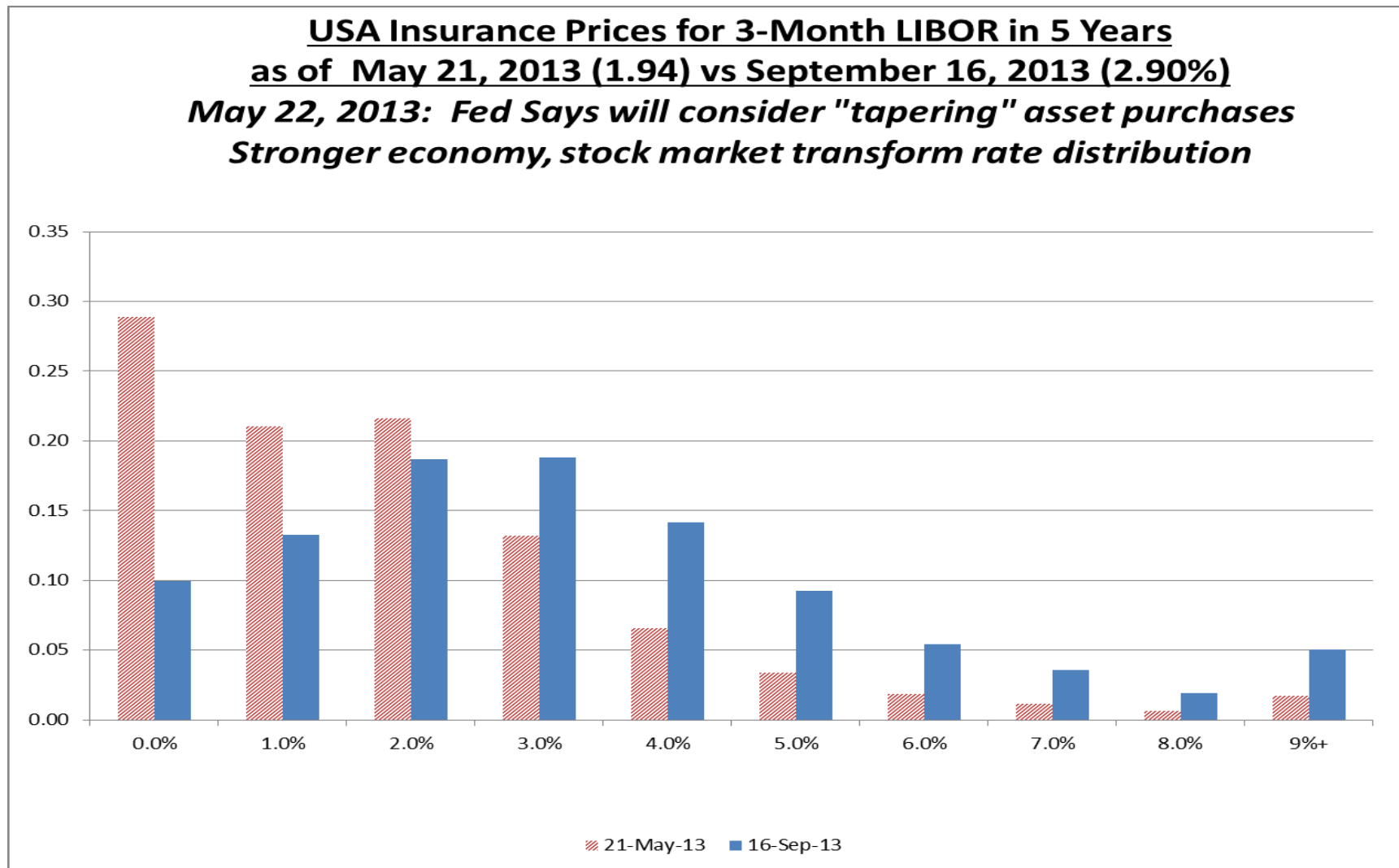
Dec 2008: Euro Area Rate Distribution Unaffected by USA problems



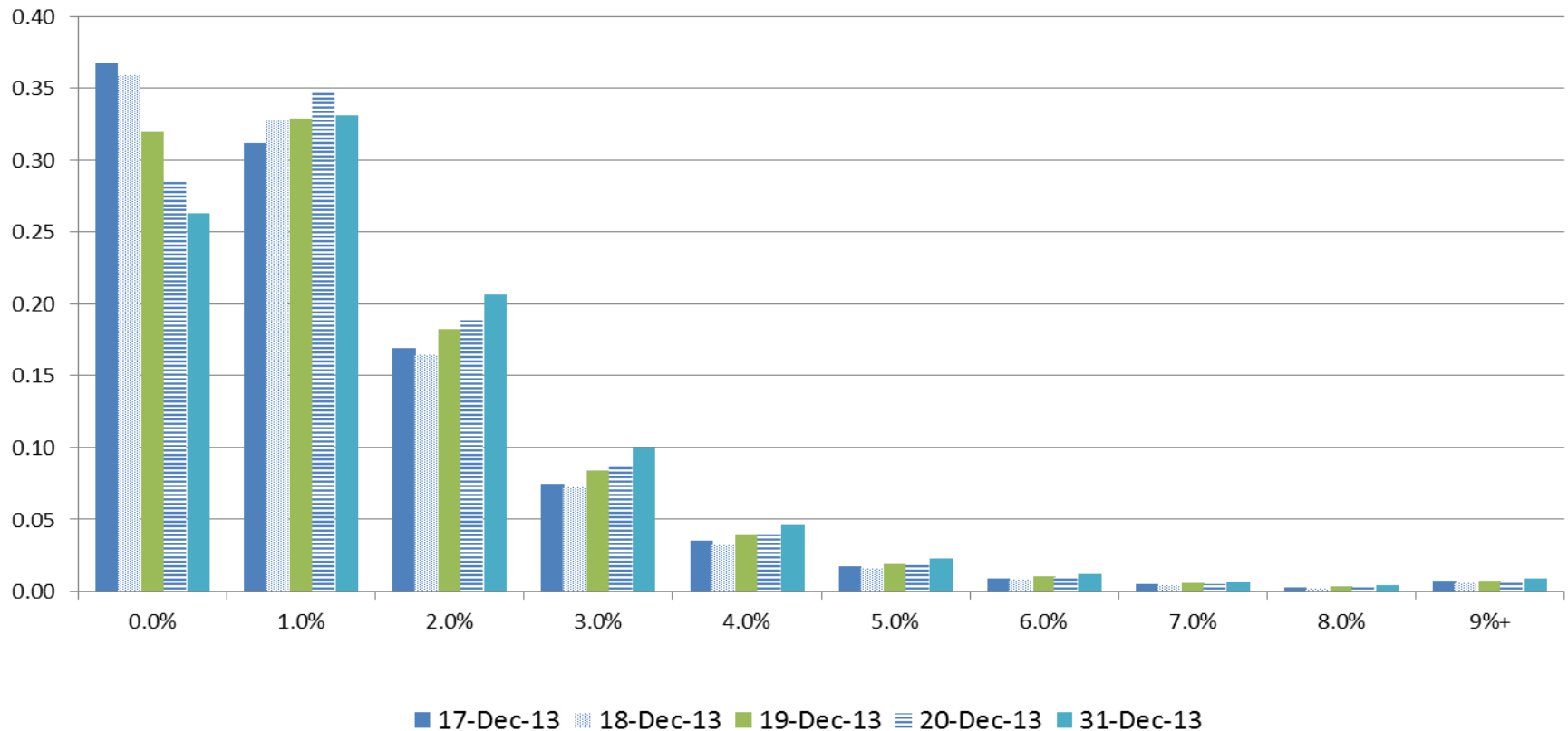
Panic during budget impasse causes Fed to commit low rates 2.5 years.



Summer 2013: Stronger economy shifts distribution towards symmetry



USA Insurance Prices for 3-Month LIBOR in 3 Years
Daily: December 17 (2.85% 10 Yr) to Dec 31, 2013 (3.04%)
Dec 18: Bernanke Fed announces start of tapering asset purchases
Rate distribution shifts to higher rates

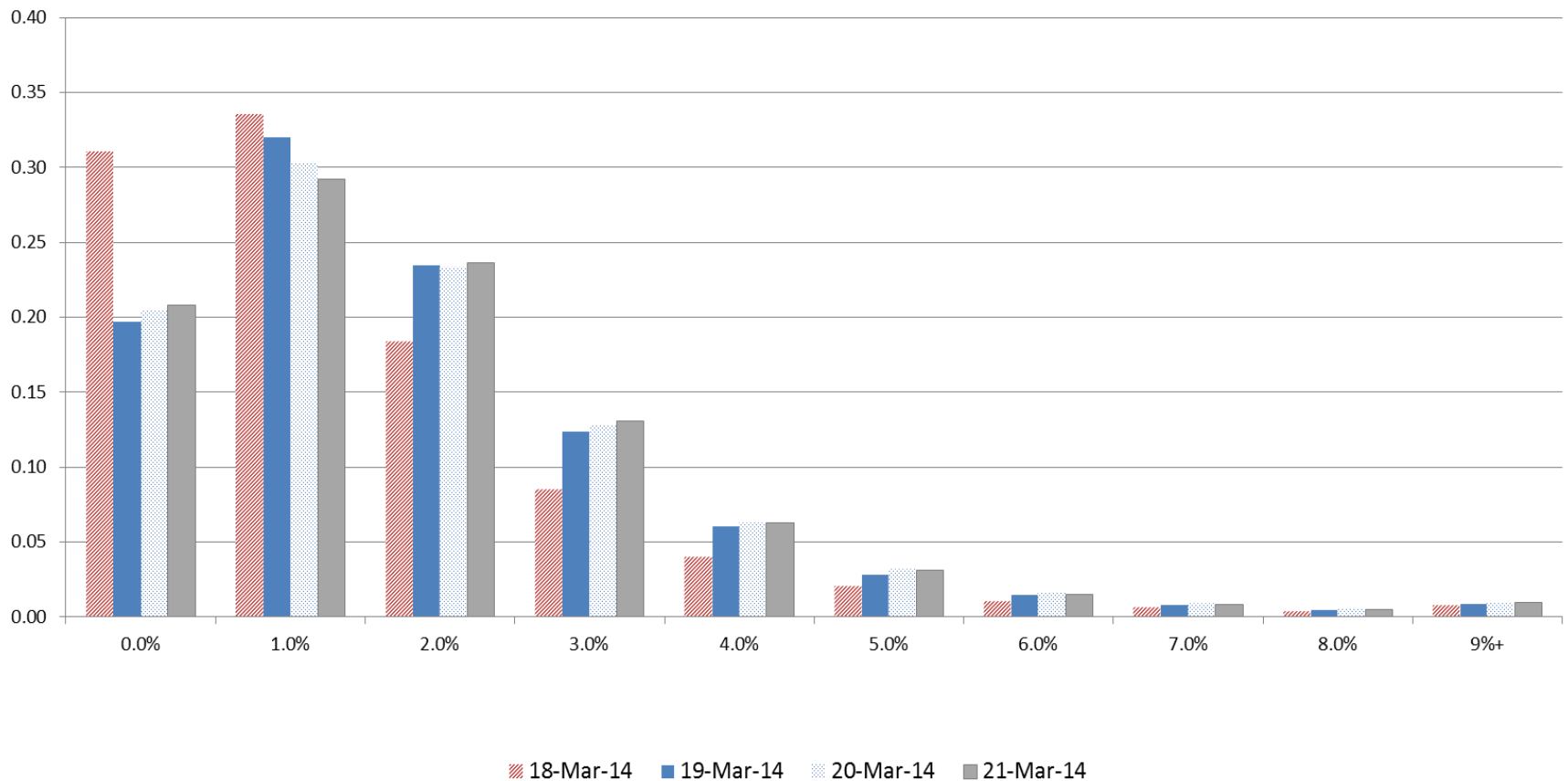


USA Insurance Prices for 3-Month LIBOR in 3 Years

Daily: March 18 2014 (2.68% 10 Yr) to March 21 2014 (2.75%)

March 19: Fed Chair Yellen says rates could increase in mid-2015 (6 mos after taper).

Rate distribution shifts higher for shorter term.



VI. Interest Rate Insurance Prices
for Euribor During the Sovereign Debt Crisis
2010-2016

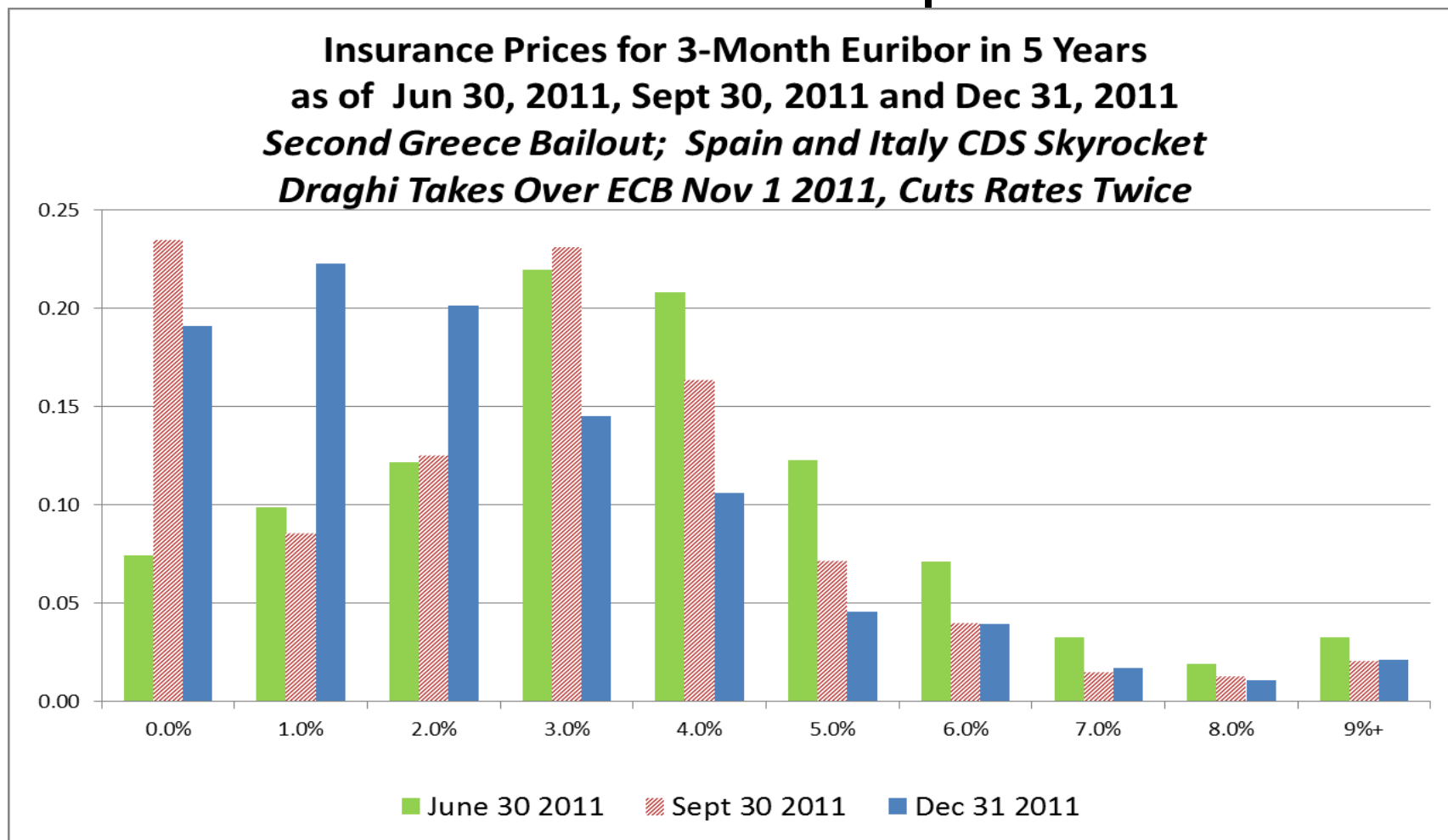
Key Events in the European Sovereign Debt Crisis

European Central Bank 2010-2016

Sources: BBC, Reuters

- January-May 2010: Greek deficit revised upward from 3.7% to 12.7%. “Severe irregularities” in accounting. EU agrees to \$30 billion, then \$110 billion bailout of Greece. Ireland bailed out in November 2010.
- July-August 2011: Talk of Greek exit from Euro. Second bailout agreed. EC President Barroso: sovereign debt crisis spreading. Spain, Italy yields surge.
- November 1, 2011: Mario Draghi takes over European Central Bank from Jean-Claude Trichet. Draghi cuts rates twice quickly.
- September, 2012: ECB ready to buy “unlimited amounts” of bonds of weaker member countries. Draghi says ECB will do “whatever it takes to preserve the Euro.” “...and believe me, it will be enough.”
- May/June 2013: U.S.Fed considers “tapering” asset purchases, as economy strengthens. Long term interest rates move up sharply.
- June-October, 2014: European economies weak, inflation expectations lower. Draghi cuts rates twice to 0.05%. Announces QE, buying ABS, possibly even from Italy and Spain, up to 1 trillion Euro.
- January-March 2015: Draghi of ECB announces on January 22nd “Quantitative Easing” by massive asset purchases. Began QE March 9, 2015.

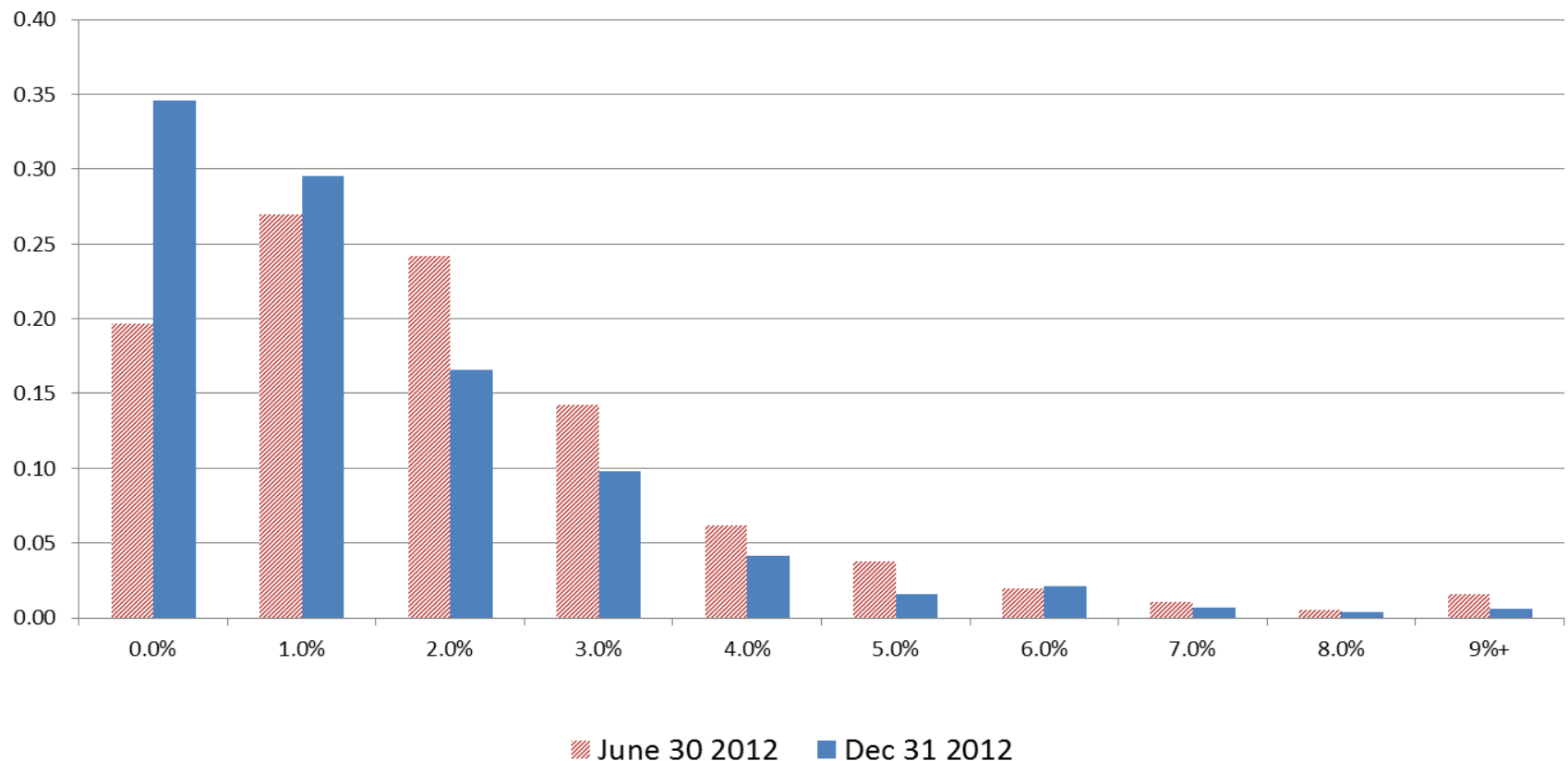
2011 Sovereign Debt Crisis: Draghi ECB cuts rates sharply. Massive shift in Euribor interest rate distribution to positive skewness like U.S.



Draghi Rescues the Euro in 2012 with “Whatever it takes...”

**Insurance Prices for 3-Month Euro LIBOR in 5 Years
as of Jun 30, 2012 and Dec 31, 2012.**

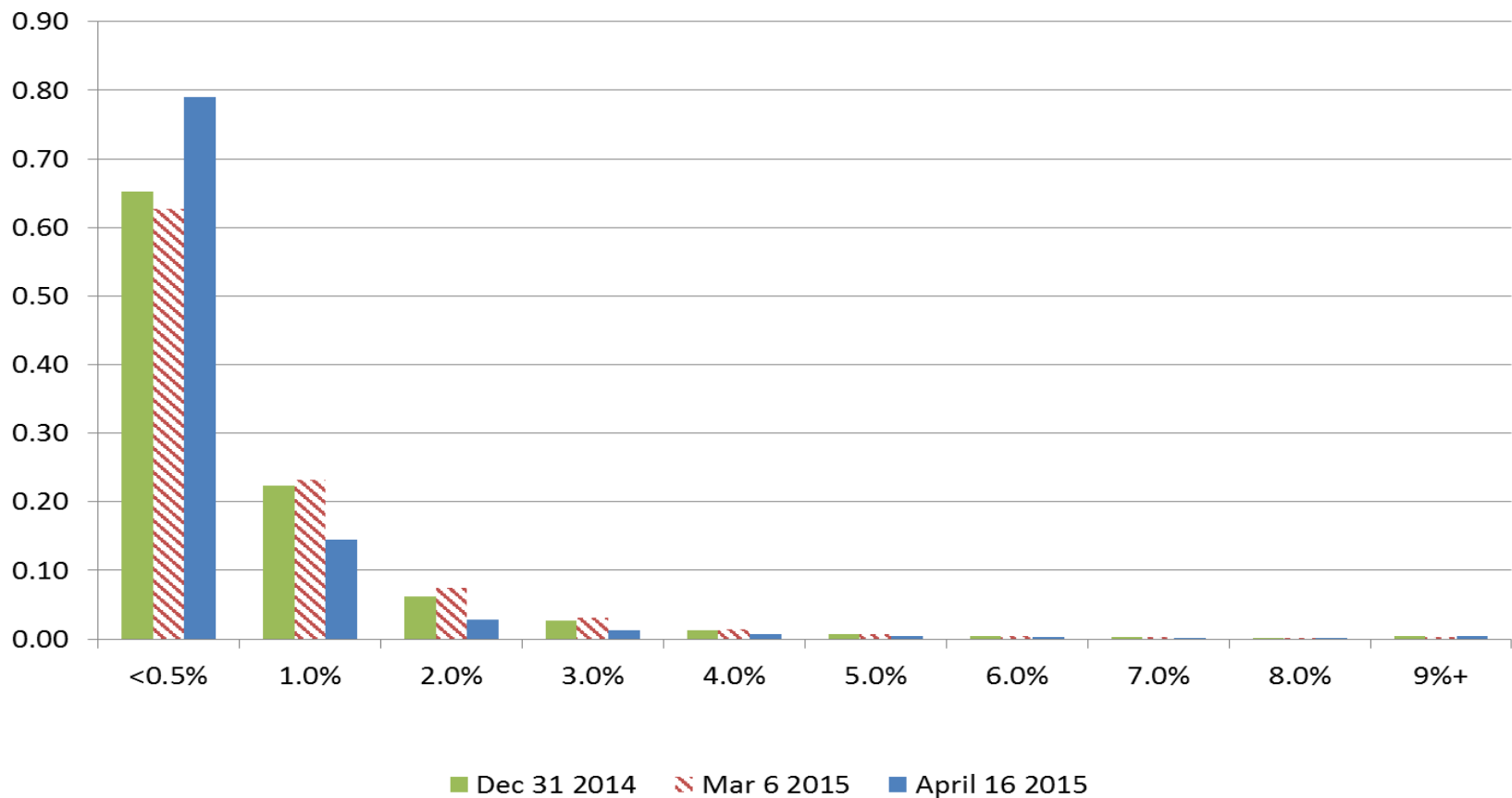
***Draghi says ECB ready to buy "Unlimited amounts" of bonds of weaker
members. Will do "Whatever it takes to preserve the Euro"***



VII. 2015: Draghi's European Central Bank Massive "Quantitative Easing" Program:

***ECB announced QE January 22, 2015,
implemented it starting March 9 2015,
buying massive amounts of Eurozone bonds.
Long rates drop sharply in Eurozone, UK, USA.***

Euro Insurance Prices for 3-Month Euribor in 5 Years
Dec 31 2014 (Bund 10Yr=0.54%), Mar 6 (0.39%), April 16 2015 (0.08%):
ECB QE announced Jan 22 2015, started March 9 2015.
Euribor distribution in 5 years hammered down to less than 1.0%

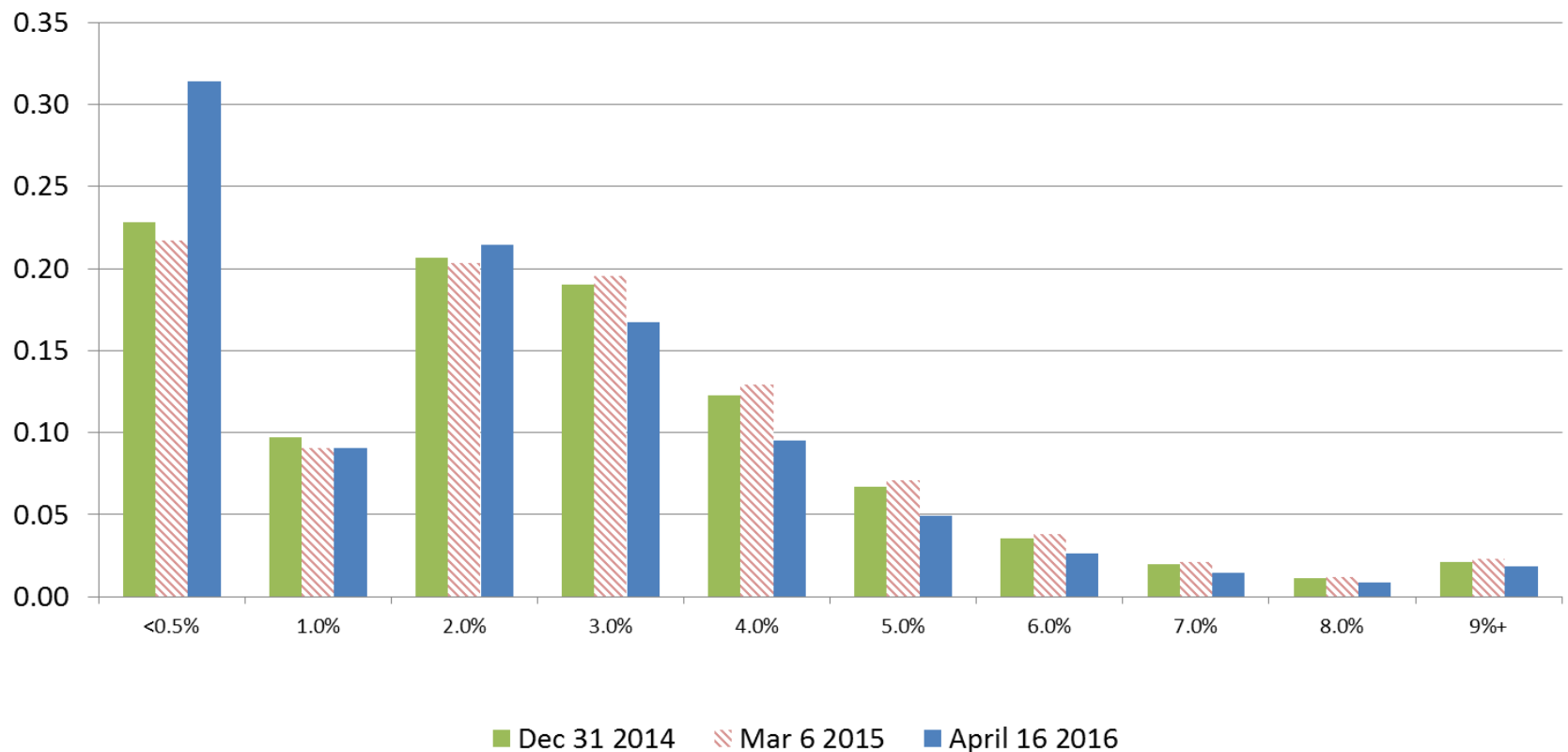


USA Insurance Prices for 3-Month LIBOR in 5 Years

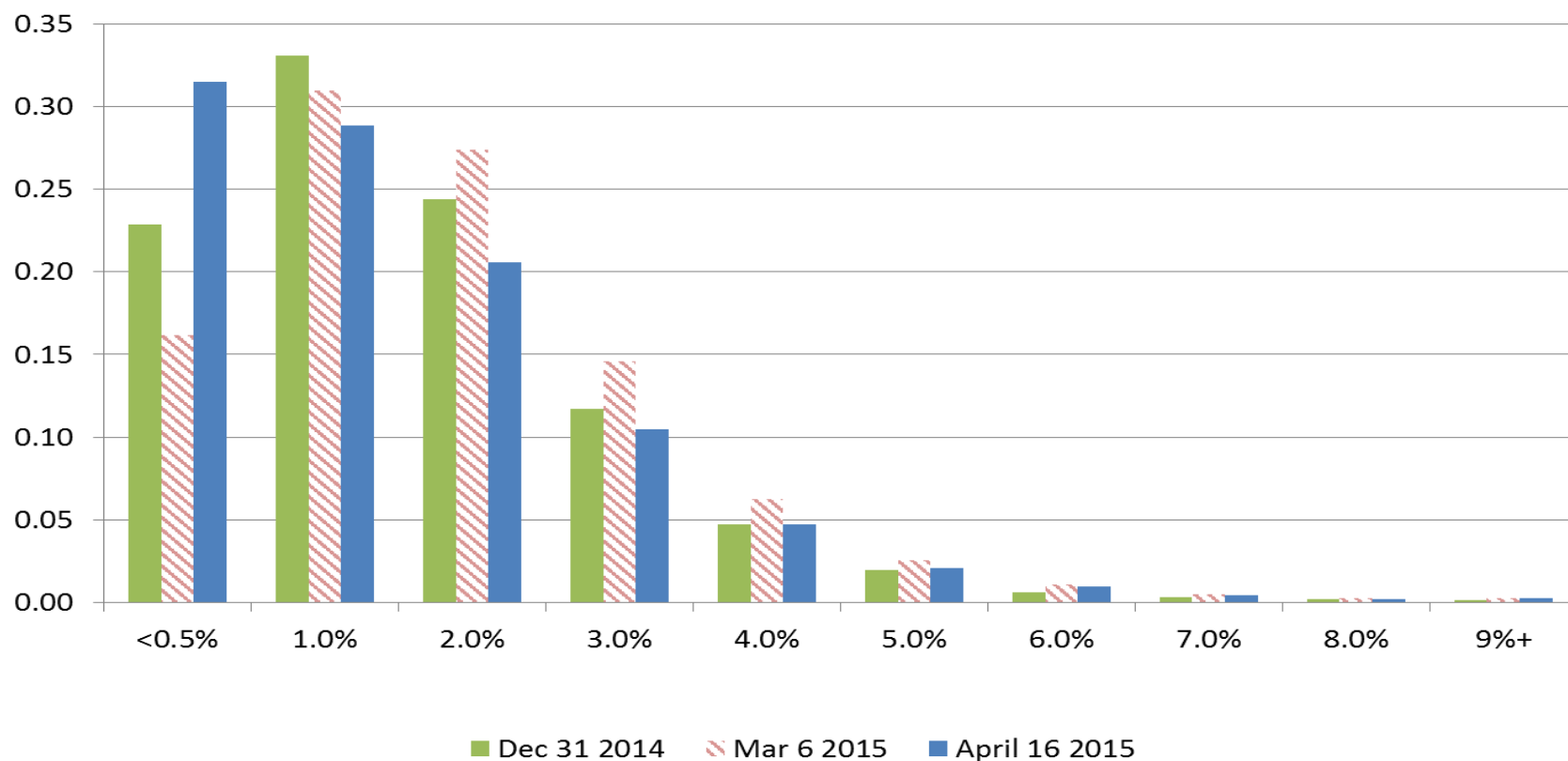
Dec 31 2014 (10 Yr=2.17%), March 6 (2.24%), April 16 2015 (1.88%):

Rates drop with ECB QE March 9th. Dots show flatter ramp after liftoff.

Bimodal dist'n for LIBOR 5 yrs. Economy weak or strong then?



British Pound Insurance Prices for 3-Month Interbank Rate in 3 Years
Dec 31 2014 (10 Yr Gilt=1.76%), March 6 (1.95%), April 16 2015 (1.61%):
ECB QE announced Jan 22, implemented Mar 9 2015. Rates drop.
Mode shifts down <0.5% for 3 years out.



VIII. What are markets saying now in the USA, the Eurozone and the UK?

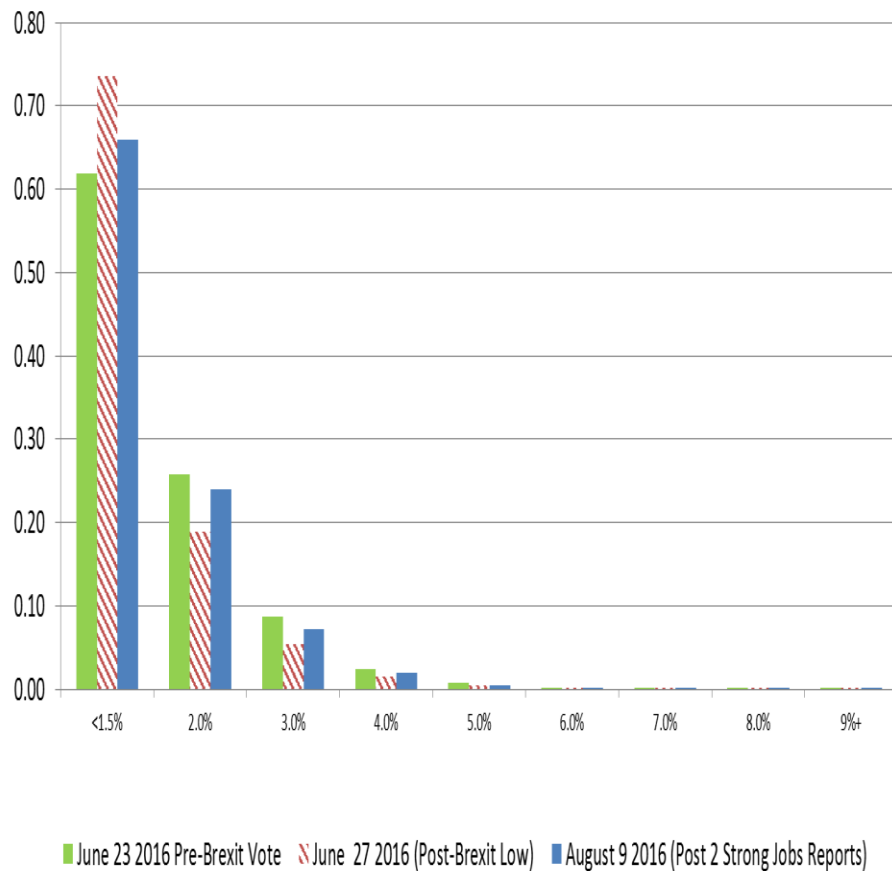
June 23, 2016's surprise vote for Brexit roiled markets and caused rates to drop sharply. Strong actions and statements by the Bank of England's Governor, Mark Carney, continued the shift towards low rates in the UK distribution. Strong jobs reports in the USA on July 8 and on August 5, 2016, caused some bounceback in rate distributions, especially in the USA.

USA Insurance Prices for 3-Month LIBOR in 3 Years

Jun 23 2016 (1.74% 10 Yr), Jun 27 (1.46%), August 9 2016 (1.55%)

June 23 - August 9 2016. USA rate dist'n drops sharply after Brexit vote.

Strong USA jobs reports July 8, Aug 5 aid bounceback in stocks,

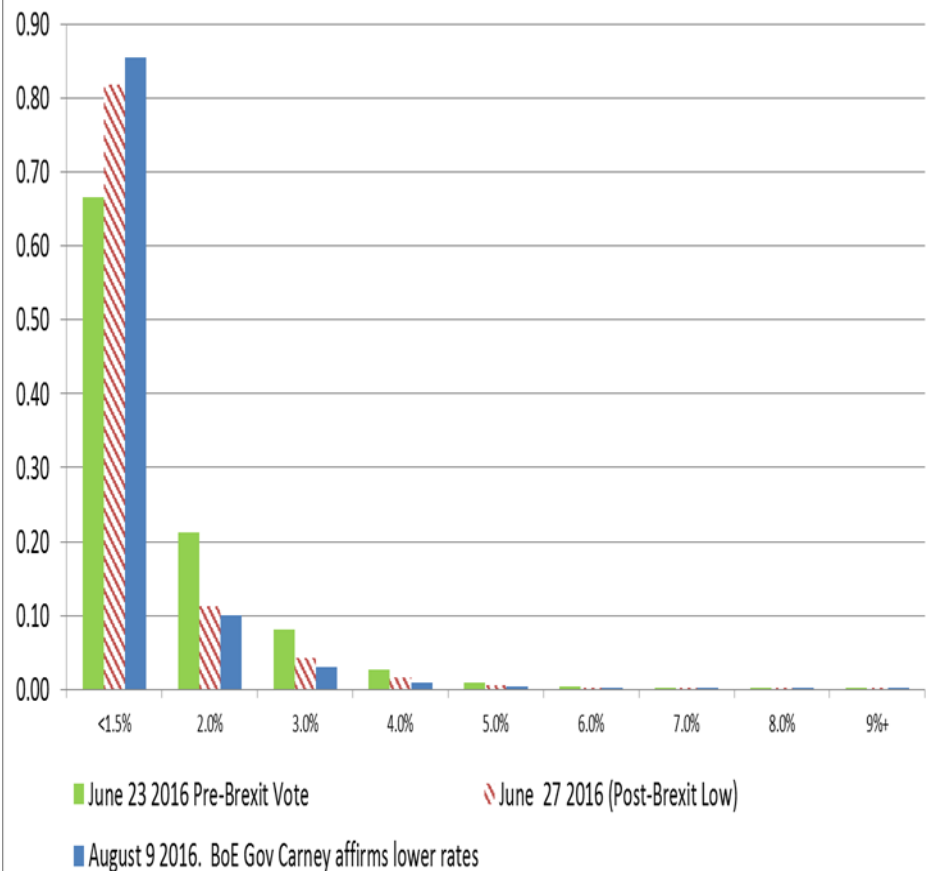


UK Insurance Prices for 3-Month Interbank Rate in 3 Years

Jun 23 2016 (1.37% 10 Yr Gilt), Jun 27 (0.93%), August 9 2016 (0.58%)

June 23 - August 9 2016. UK rate dist'n drops sharply after Brexit vote.

Strong easing by BoE Gov Carney increases shift.

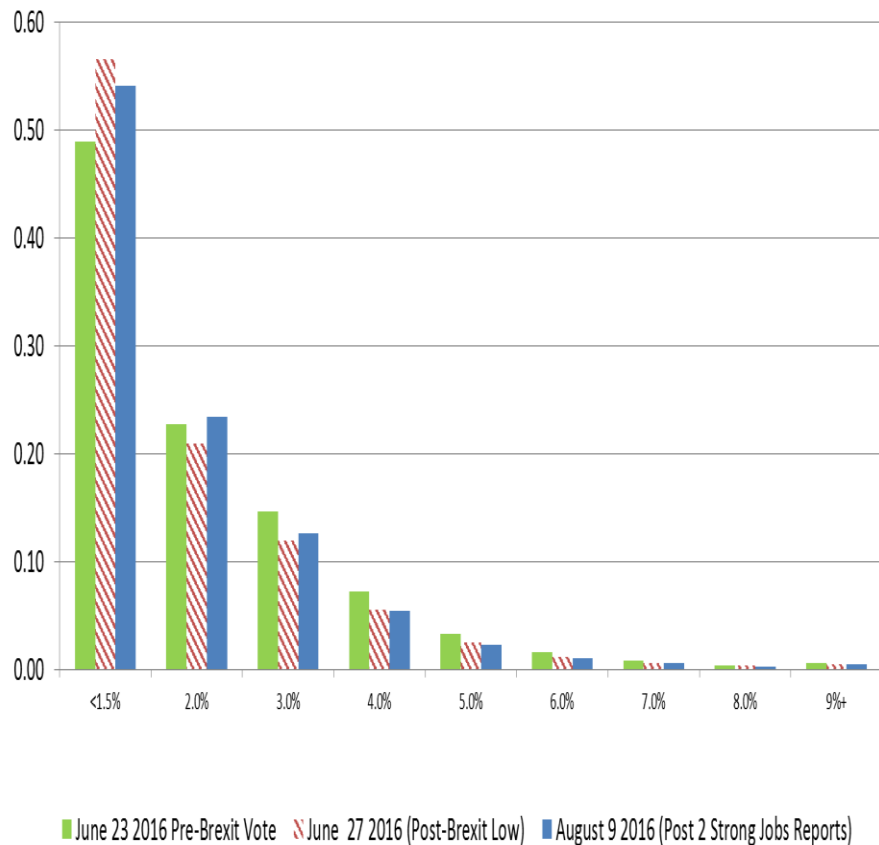


USA Insurance Prices for 3-Month LIBOR in 5 Years

Jun 23 2016 (1.74% 10 Yr), Jun 27 (1.46%), August 9 2016 (1.55%)

June 23 - August 9 2016. USA rate dist'n drops sharply after Brexit vote.

Strong USA jobs reports July 8, Aug 5 aid partial bounceback di

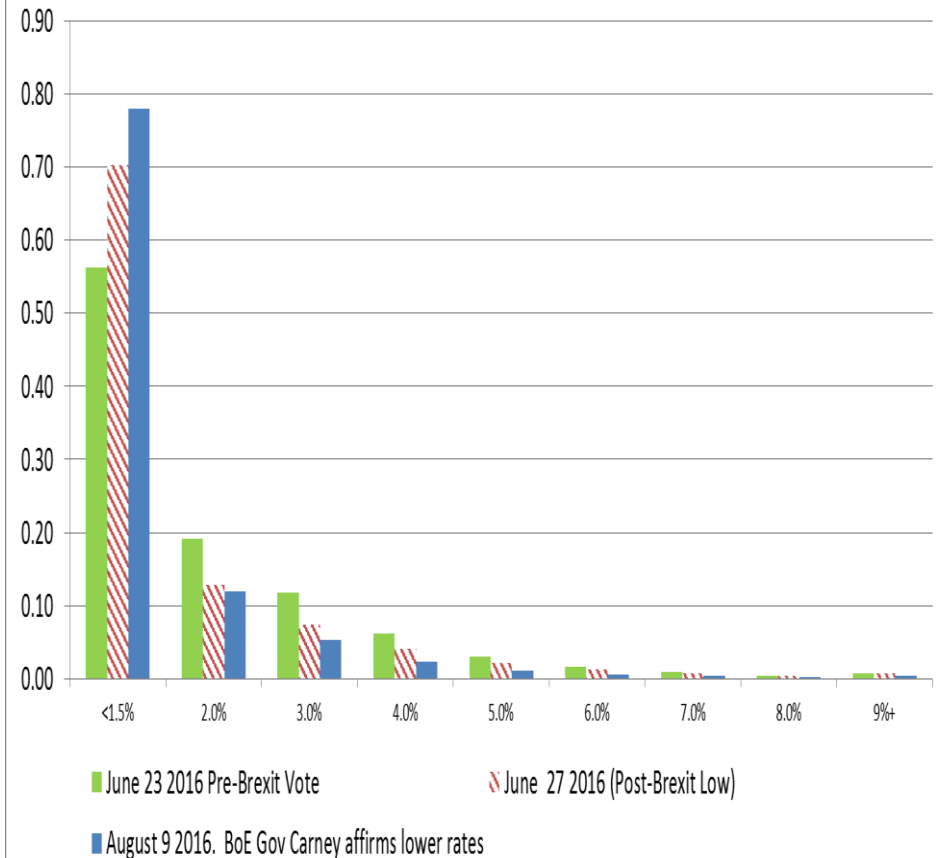


UK Insurance Prices for 3-Month Interbank Rate in 5 Years

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Strong easing by BoE Gov Carney increase rate shift

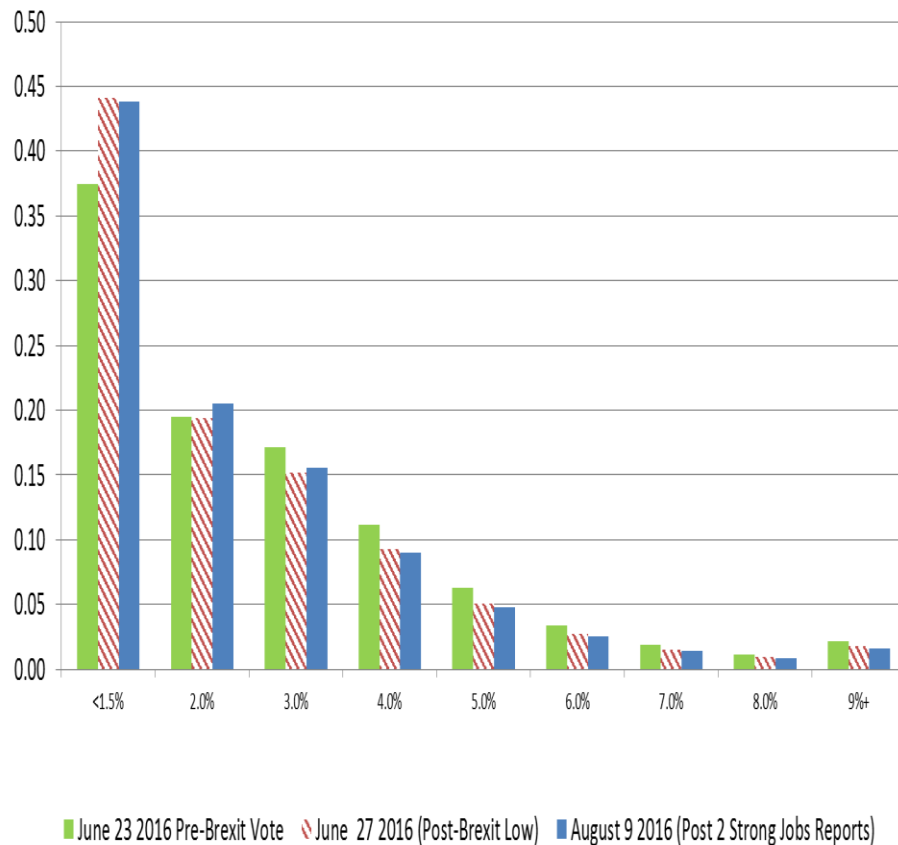


USA Insurance Prices for 3-Month LIBOR in 8-10 Years

Jun 23 2016 (1.74% 10 Yr), Jun 27 (1.46%), August 9 2016 (1.55%)

June 23 - August 9 2016. USA rate dist'n drops after Brexit vote.

Long-term dist'n has long tail. Normalization or despressed rates?

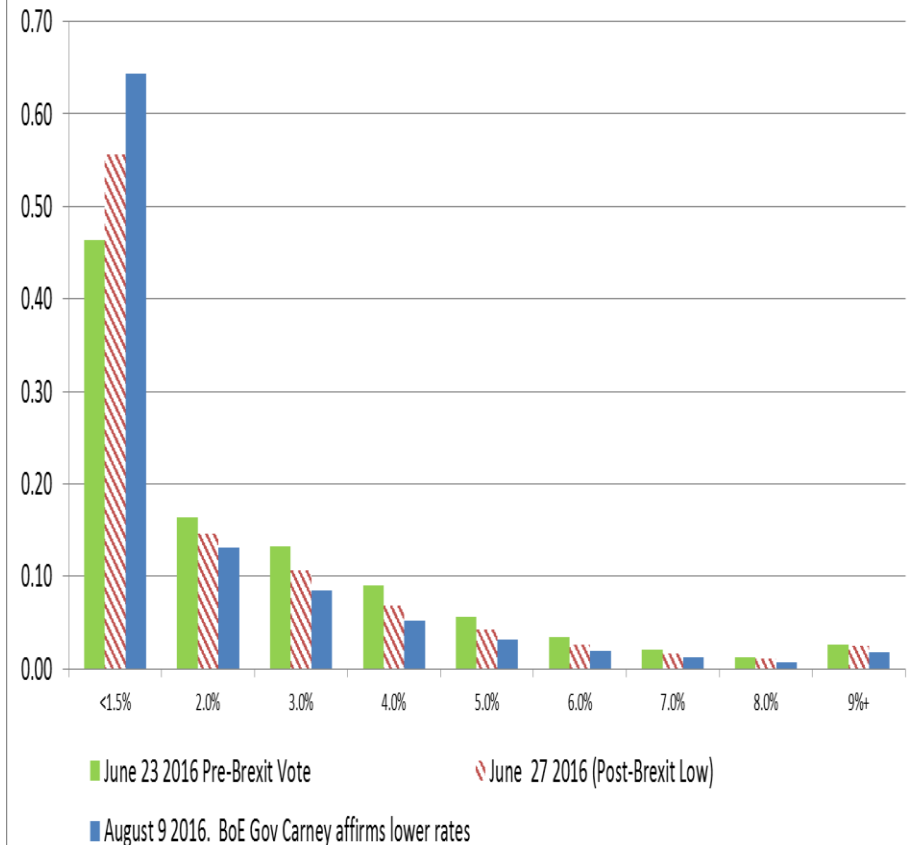


UK Insurance Prices for 3-Month Interbank Rate in 8-10 Years

Jun 23 2016 (1.37% 10 Yr), Jun 27 (0.93%), August 9 2016 (0.58%)

June 23 - August 9 2016. UK rate dist'n drops sharply after Brexit vote.

Strong easing by BoE Gov. Carney increases rate shift



VIII. Conclusions

1. Interest rate cap and floor prices show the market's implied pricing of insurance payoffs for interest rates from 1% to 9%+. A virtue of caps and floors is forecasts covering much longer periods (3, 5, 10 years), rather than 3-18 months from options.
2. The "Great Recession" in the USA and the "Sovereign Debt Crisis" in Europe show dramatic moves in distributions for rates after Central Bank interventions. Symmetric distributions shifted to highly positively skewed ones. Our technique is non-parametric, not relying on lognormality.
3. USA "lifted off" from zero rates in December 2015, given the relatively strong USA economy. Despite liftoff, US stock market is at all-time highs. Strong job market brought the unemployment rate down to 4.9%. However, markets are worried about economy in 5 years, as the "bipolar" rate distribution shows.
4. January-March 2015 Weakness in Europe caused European Central Bank President Mario Draghi to announce a massive "Quantitative Easing" program of asset purchases. Rates dropped sharply after that implementation, with German Bund 10-year yield down from 0.38% to 0.09% and to negative levels in 2016.
5. The U.K. vote for "Brexit" on June 23, 2016, roiled markets for a bit, with interest rates and the British Pound falling sharply. Stock prices dropped sharply, especially in the Eurozone, but have bounced back in most countries, partially due to strong jobs reports in the U.S. for June-July 2016. The Bank of England reduced its policy rate 25 basis points and prepared strong stimulus measures to combat weakness expected with Brexit and the uncertainties of negotiations with the EU. The insurance price distributions reflect lower rates anticipated in the UK for years.