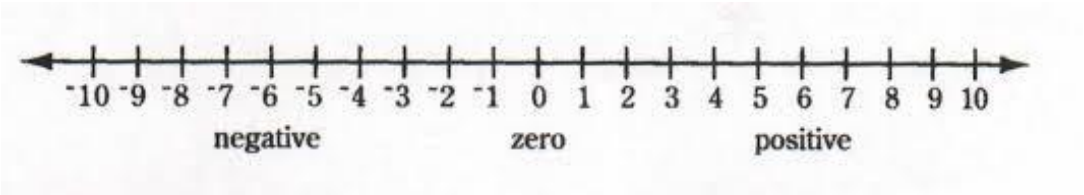
Order in the Real Numbers

Suppose that a and b represent two real numbers. If their graphs on the number line are the same point, then a **is equal to** b. If the graph of a lies to the left of b, then a **is less than** b, and if the graph of a lies to the right of b, then a **is greater than** b. The **law of trichotomy** says that for two numbers a and b, one and only one of the following is true.

$$a = b, \quad a < b, \quad \text{or} \quad a > b$$

When read from left to right, the inequality $a < b$ is read "a is less than b."



Additive Inverses and Absolute Value

For any nonzero real number x, there is exactly one number on the number line the same distance from 0 as x but on the opposite side of 0. In **Figure 6,** the numbers 3 and -3 are the same distance from 0 but are on opposite sides of 0. Thus, 3 and -3 are called **additive inverses, negatives,** or **opposites** of each other.

Figure 6

The **absolute value** of a real number can be defined as the undirected distance between 0 and the number on the number line.

The symbol for the absolute value of the number x is $|x|$, read **"the absolute value of x."** For example, the distance between 0 and 2 on the number line is 2 units, so

$$|2| = 2.$$

Because the distance between 0 and -2 on the number line is also 2 units,

$$|-2| = 2.$$

Absolute value is a measure of undirected distance, so ***the absolute value of a number is never negative.*** Because 0 is a distance of 0 units from 0, $\mathbf{|0| = 0.}$

DOUBLE NEGATIVE RULE

For any real number x, the following is true.

$$-(-x) = x$$

Section 6.1 and 6.2 Notes

Examples:
Select the lesser of the two numbers.

42. $-9, -14$

44. $-15, -16$

46. $5, |-2|$

48. $|-8|, |-9|$

Examples:
Decide whether each statement is true or false.

53. $6 > -(-2)$

55. $-4 \leq -(-5)$

57. $|-6| < |-9|$

59. $-|8| > |-9|$

61. $-|-5| \geq -|-9|$

63. $|6-5| \geq |6-2|$

Section 6.1 and 6.2 Notes

Sign Rules for Addition and Subtraction:

If the signs are the same, ADD and keep the sign.
If the signs are different, SUBTRACT and keep the sign of the larger number.

Ex. -3 – 6 = - 9

Ex. -6 + 2 = - 4

If you have trouble understanding this concept or remembering the rules, think in terms of MONEY or BANKING.

If you OWE someone money, that would be represented by a negative number.

Ex. If you owe someone 3 dollars, you borrow 6 more dollars, now you OWE them 9 dollars.

Ex. Your bank account is over drawn by 6 dollars, you deposit 2 dollars, and you still OWE the bank 4 dollars.

Sign Rules for Multiplication and Division:

I find that most students consider this to be the easier of the two sign rules.

In words:
If the signs are different, your answer will be negative.
If the signs are the same, your answer will be positive.

In Symbols:

+	*	+	=	+
-	*	+	=	-
+	*	-	=	-
-	*	-	=	+

Ex. (5)(3) = 15
Ex. (-5)(3) = -15
Ex. (5)(-3) = -15
Ex. (-5)(-3) = 15

Most people don't understand why a negative multiplied by a negative makes a positive. But remember, it's the same in the English language.

Ex. If I say "DO NOT, NOT go there" that means "GO THERE" …so even in the English language, two negatives make a positive.

DIVISION INVOLVING ZERO

$\frac{a}{0}$ **is undefined for all** a**.** $\frac{0}{a} = 0$ **for all nonzero** a**.**

Order of Operations

The order of operations is a set of well-defined rules, which you will use to simplify algebraic expressions.

1. Perform all operations within grouping symbols first, starting with the inner most set.
2. Evaluate exponential expressions.
3. Multiply or divide in order from left to right.
4. Add or Subtract in order from left to right.

Fortunately, a mnemonic was created to help us remember these rules.

PEMDAS

Parentheses	**P**lease
Exponents	**E**xcuse
Multiplication & **D**ivision	**M**y **D**ear
Addition & **S**ubtraction	**A**unt **S**ally

Section 6.1 and 6.2 Notes

Try These!

12. $-5 + (-2)$

14. $-6 + 17$

16. $-3 - (-8)$

18. $-9 + 16 + 5$

20. $15 - (-6) - (-8)$

22. $(-3)(-5)$

24. $-5(-17)(2)(-2)(4)$

26. $\dfrac{-100}{-50}$

28. $\dfrac{52}{-13}$

30. $\dfrac{0}{-7}$

32. $-8[4 + (7 - 8)]$

34. $-6 - 5(-8) + 3^2$

36. $\dfrac{3(-4) + (-5)(-2)}{2^3 - 2 + (-6)}$

38. $\dfrac{(-10 + 4) \cdot (-3)}{-7 - 2}$

Heights of Mountains and Depths of Trenches *The table shows the heights of some selected mountains and the depths of some selected ocean trenches.*

Mountain	Height (in feet)	Trench	Depth (in feet, as a negative number)
Foraker	17,400	Philippine	−32,995
Wilson	14,246	Cayman	−24,721
Pikes Peak	14,110	Java	−23,376

Source: World Almanac and Book of Facts.

Use the information given to answer Exercises 77–82.

77. What is the difference between the height of Mt. Foraker and the depth of the Philippine Trench?

78. What is the difference between the height of Pikes Peak and the depth of the Java Trench?

79. How much deeper is the Cayman Trench than the Java Trench?

80. How much deeper is the Philippine Trench than the Cayman Trench?

81. How much higher is Mt. Wilson than Pikes Peak?

82. If Mt. Wilson and Pikes Peak were stacked one on top of the other, how much higher would they be than Mt. Foraker?