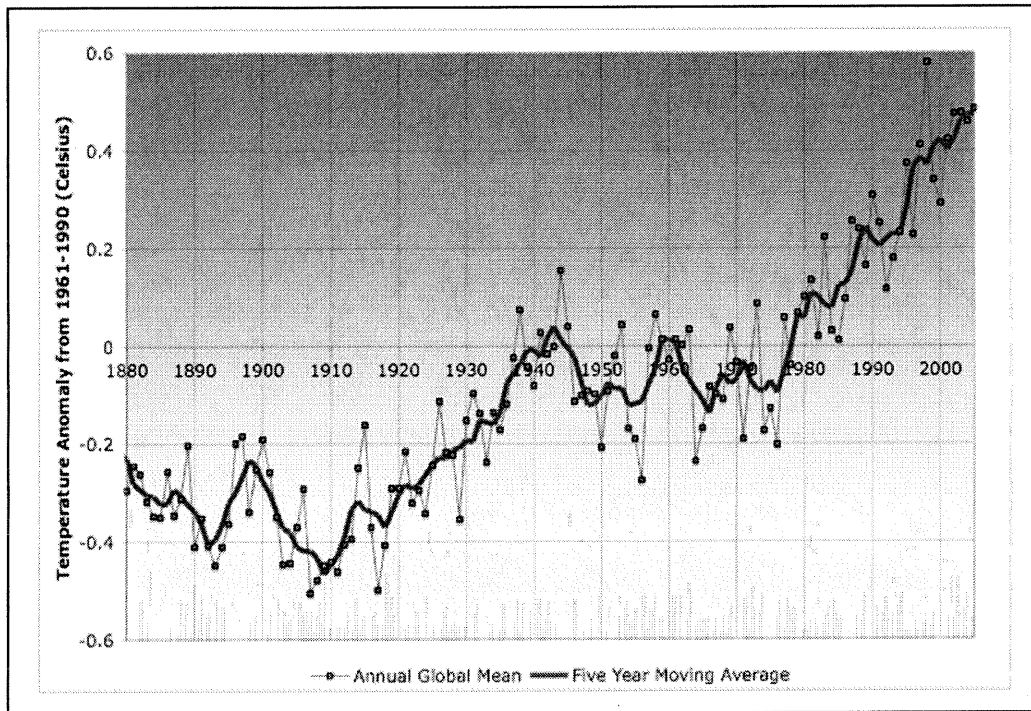


Global Warming Trends

1



A combination of hundreds of climate studies have traced changes in the average temperature of Earth since before the Industrial Revolution in the 1800's. The temperature data after 1960 can be approximated by the function

$$T(X) = +0.10X + 0.05$$

The function $T(X)$ gives the change in world temperature in degrees Celsius, and X is the number of decades since 1960.

For example, 1960-1969 is the decade $X=0$, and at this time, $T(0) = +0.05$, which means that the world was $+0.05$ C warmer than its average temperature before 1960.

Problem 1 - Graph this function over the period from 1960 to 2050.

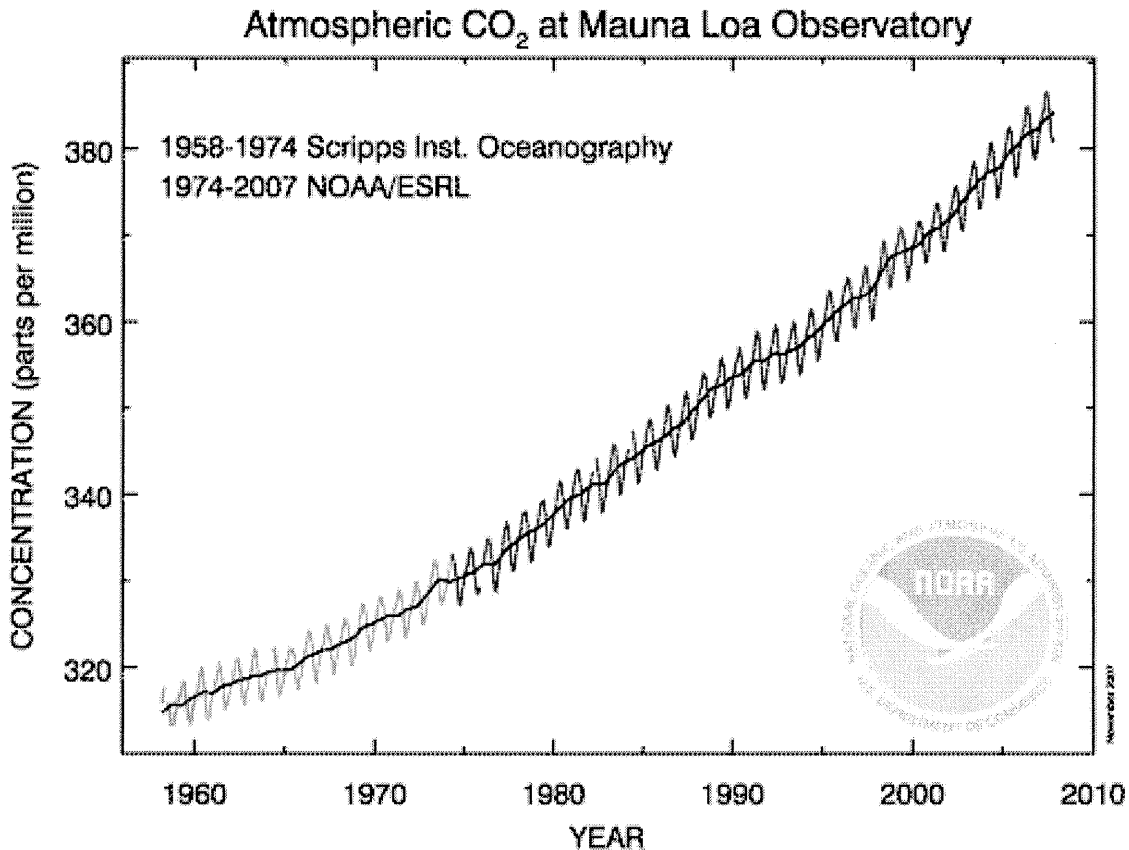
Problem 2 - What does it predict for the temperature change in 2000 and in 2050?

Problem 1 –

Problem 2 - For 2000, $X = 4$ decades, so $T(4) = +0.45$ Celsius.

For 2050, $x=9$ decades $T(9) = + 0.95$ Celsius.

Graph - Global temperature trend expressed as both the annual and five-year departure (anomaly) from the long-term mean temperatures, measured over 120 years (Hansen et al., 1999).



This is the Keeling Curve, derived by researchers at the Mauna Kea observatory from atmospheric carbon dioxide measurements made between 1958 - 2005. The accompanying data in Excel spreadsheet form for the period between 1982 and 2008 is provided at

<http://spacemath.gsfc.nasa.gov/data/KeelingData.xls>

Problem 1 - Based on the tabulated data, create a single mathematical model that accounts for, both the periodic seasonal changes, and the long-term trend.

Problem 2 - Convert your function, which describes the carbon dioxide volume concentration in parts per million (ppm), into an equivalent function that predicts the mass of atmospheric carbon dioxide if 383 ppm (by volume) of carbon dioxide corresponds to 3,000 gigatons.

Problem 3 - What would you predict as the carbon dioxide concentration (ppm) and mass for the years: A) 2020? B)2050, C)2100?

Answer Key

1.5.2

Data from: C. D. Keeling, S. C. Piper, R. B. Bacastow, M. Wahlen, T. P. Whorf, M. Heimann, and H. A. Meijer, Exchanges of atmospheric CO₂ and ¹³CO₂ with the terrestrial biosphere and oceans from 1978 to 2000. I. Global aspects, SIO Reference Series, No. 01-06, Scripps Institution of Oceanography, San Diego, 88 pages, 2001. Excel data obtained from the Scripps CO₂ Program website at http://scrippsco2.ucsd.edu/data/atmospheric_co2.html

Problem 1 - Answer: The general shape of the curve suggests a polynomial function of low-order, whose amplitude is modulated by the addition of a sinusoid. The two simplest functions that satisfy this constraint are a 'quadratic' and a 'cubic'... where 't' is the elapsed time in years since 1982

$$F1 = A \sin(Bt + C) + (Dt^2 + Et + F) \text{ and } F2 = A \sin(Bt + C) + (Dt^3 + Et^2 + Ft + G)$$

We have to solve for the two sets of constants A, B, C, D, E, F and for A, B, C, D, E, F, G. Using *Excel* and some iterations, as an example, the constants that produce the best fits appear to be: F1: (3.5, 6.24, -0.5, +0.0158, +1.27, 342.0) and F2: (3.5, 6.24, -0.5, +0.0012, -0.031, +1.75, +341.0). Hint: Compute the yearly averages and fit these, then subtract this polynomial from the actual data and fit what is left over (the residual) with a sin function.) The plots of these two fits are virtually identical. We will choose Fppm = F1 as the best candidate model because it is of lowest-order. The comparison with the data is shown in the graph below: red=model, black=monthly data. Students should be encouraged to obtain better fits.

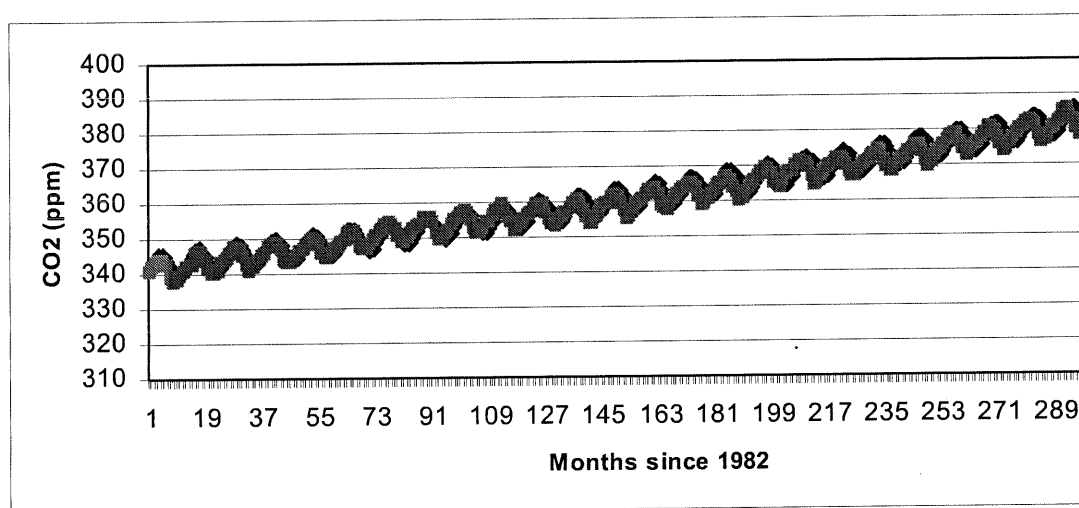
Problem 2 - Answer: The model function gives the atmospheric carbon dioxide in ppm by volume. Since 3000 gigatons = 383 ppm, take Fppm and multiply it by the conversion factor (3,000/383) = 7.83 gigatons/ppm to get the desired function, Fco2 for the carbon mass.

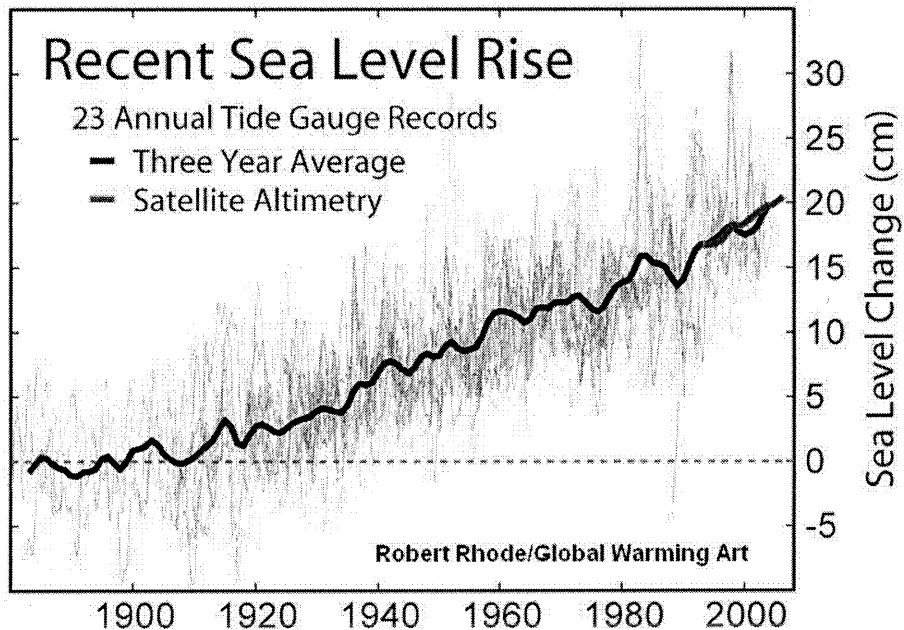
Problem 3 - What would you predict as the carbon dioxide concentration (ppm), and mass for the years: A) 2020? B)2050, C)2100? Answer:

A) $t = 2020 - 1982 = 38$, so $F_{co2}(38) = 7.83 \times 410 \text{ ppm} = 3,200 \text{ gigatons}$

B) $t = 2050 - 1982 = 68$, so $F_{co2}(68) = 7.83 \times 502 \text{ ppm} = 3,900 \text{ gigatons}$

C) $t = 2100 - 1982 = 118$, so $F_{co2}(118) = 7.83 \times 718 \text{ ppm} = 5,600 \text{ gigatons}$





As global temperatures increase, water expands. This means that the volume of the world's oceans will steadily increase in time. This causes the sea level to increase.

Also, some areas of the world are still changing after the enormous weight of the ice sheets from the last Ice Age have gone away. This causes land areas to rise, and so in those coastal areas, such as the East Coast of North America, sea levels are falling. A simple function that models the average sea level around the world is given by

$$H(X) = 0.21X - 401.1$$

where X is the year (currently $X=2012$), and H is the sea level change since 1910 in centimeters.

Problem 1 - What was the change in sea level by 2012?

Problem 2 - Compared to 2012, how much higher will it be in the year 2100 according to this model?

Problem 1 - In 2012, $H(2012) = 21.3$ centimeters.

Problem 2 - $H(2100) = 40.0$ centimeters. The increase will be about
 $40 \text{ cm} - 21.3 \text{ cm} = 18.7$ centimeters (about 7 inches).

To see the sea level rise where you live, check out:

http://tidesandcurrents.noaa.gov/sltrends/sltrends_global.shtml

Select your location in North America from the left menu, or your location around the world from the right menu. The display shows data taken by tide gauges at these locations since 1900.