Solving differential equations:

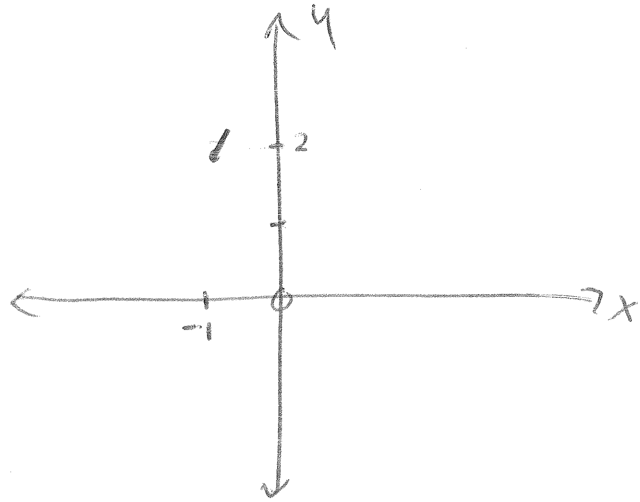
We can observe the general behavior of the solutions without actually solving the equation. Rather, we will construct a picture that is related to the solutions of the equation.

How? Consider $dy/dx = x + 2y$ $\boxed{*}$

First, suppose y is the particular solution of $\boxed{*}$ that satisfies $y(x_0) = y_0$. Then the graph of y passes through (x_0, y_0). At that point the slope (of y) is $x_0 + 2y_0$ (why?)

For example, the slope of the solution that passes through $(-1,2)$ is $-1+2(2)=3$. Indicate this by drawing a small line segment that passes through $(-1,2)$ and has slope $=3$. We call this a SLOPE SEGMENT.

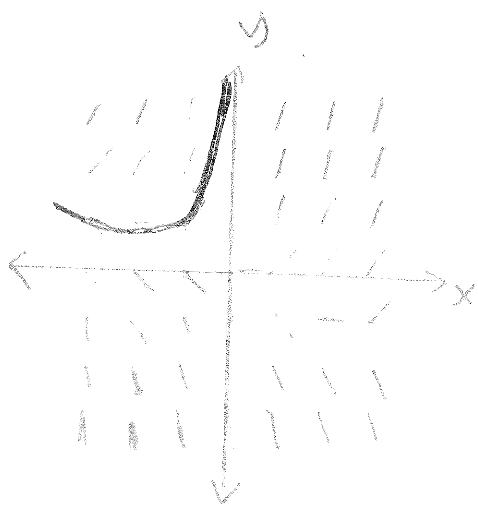
$$\frac{dy}{dx} = x+2y$$



SLOPE SEGMENT - A linear approximation to the solution satisfying $y(x_0)=y_0$.

Local Linearity!

Taking a significant number of well-scattered points in the plane and drawing slope segments we create a picture - <u>SLOPE FIELD</u> of the differential equation.



Connecting associating segments gives a smooth curve.
We call this the <u>Solution curve</u>.

Approximate solution.

Also called the <u>integral</u> curve

Use alamos.math.arizona.edu/ODEApplet/OdeApplet.html
See my website ; Slope field <u>1</u>

What do the solution curves tell us?

- General behavior perhaps - as $x \to \infty$ or as $x \to 0$.
- Helpful when we cannot find a reasonable formula for y.

See p 304 _ SLOPE FIELD for

$$\frac{dy}{dt} = .056y$$

Skip

* p 306 SLOPE FIELD for

$$\frac{dy}{dx} = -\frac{2xy}{1+x^2}$$

$y' = 0$ @ $x = 0$

as $x \to \pm\infty$ the solutions $\to 0$

6.8.12

$$\frac{dy}{dx} = 2x$$

Explore slope fields

~~http://mathviewer.com~~

$$\cancel{(x+2y) = \frac{dy}{dx}}$$

Enter and draw slope field for:

$$dy/dx = -2xy/(1+x^2)$$

~~Close alpha key~~

ignore

initial x = -4	-3	-2	-4
initial y = 1	||	1	+.1
step = .05		.05	.05

6.8.13

Separate to solve:

$$\frac{dy}{dx} = \frac{-2xy}{(1+x^2)}$$

$$\int \frac{dy}{y} = \int \frac{-2x}{1+x^2}\,dx$$

$u = 1+x^2$

$du = 2x\,dx$

$-du = -2x\,dx$

$$\ln y = -\int \frac{du}{u}$$

$$\ln y = -\ln u + C$$

$$\ln y = -\ln(1+x^2) + C$$

$$e^{\ln y} = e^{-\ln(1+x^2)+C}$$

$$\boxed{\frac{4}{1+x^2} = y}$$

$$y = e^{C} e^{-\ln(1+x^2)}$$

$$= \frac{e^C}{e^{\ln(1+x^2)}} = \frac{e^C}{(1+x^2)} = y$$

Numerical Methods of Solutions to First order Initial Value PROBLEMS

Euler's Method

Spse we are given a differential equation $\frac{dy}{dx} = f(x,y)$ and some initial condition (x_0, y_0) ie $y(x_0) = y_0$.

We can approximate the solution $y = y(x)$ by

$$L(x) = y(x_0) + \underbrace{y'(x_0)}_{m}(x - x_0)$$

OR

$$L(x) = y_0 + f(x_0, y_0)(x - x_0)$$

$L(x)$ gives a good approximation to the solution $y(x)$ in a small neighborhood around x_0.

Basic idea : String together several linearizations to approximate the cure –

Numerical Methods of solutions to first-order equations do not produce a general Solution of $\frac{dy}{dx} = f(x,y)$ – They produce tables of values of y for preselected values of x.

Helpful when no explicit algebraic solution is available.

Here it is →

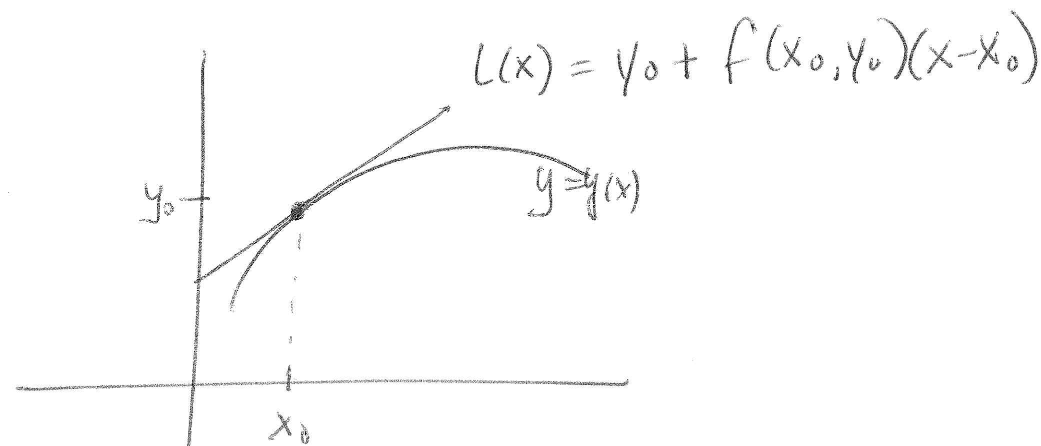
The initial condition (x_0, y_0) represents a point that lies on the solution curve –

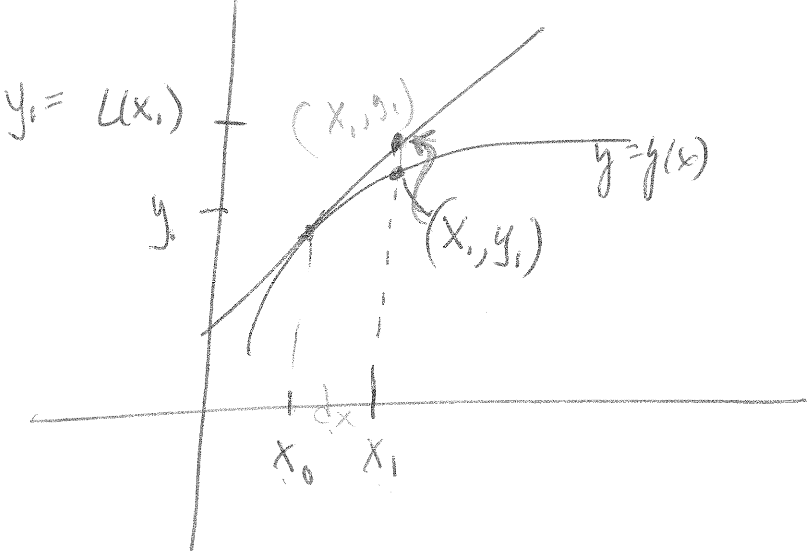
Now choose a point, $x_1 = x_0 + dx$ to be close to x_0. So, dx is small.

Then

$$y_1 = L(x_1) = y_0 + f(x_0, y_0)(x_1 - x_0)$$

$$y_1 = L(x_1) = y_0 + f(x_0, y_0)\,dx$$



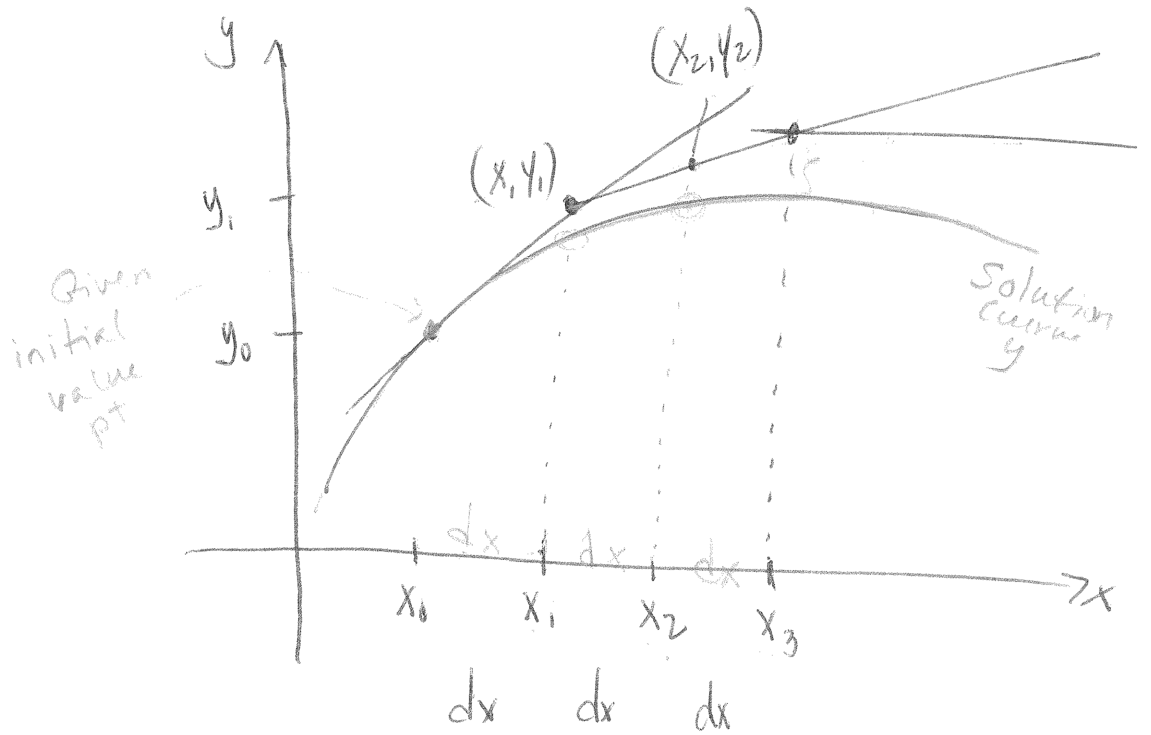


Now use (x_1, y_1) which is very close to the actual $(x_1, y(x_1))$.

Use (x_1, y_1) and slope $f(x_1, y_1)$ of the solution curve that point (x_1, y_1) and write

$$y_2 = y_1 + f(x_1, y_1)dx \quad (x_2, y_2)$$

$$dx = x_2 - x_1 \quad \text{or} \quad x_2 = x_1 + dx$$



$$y_3 = y_2 + f(x_2, y_2)dx$$

$$\vdots$$

<u>Ex</u> Find by using Eulers Method an approximation for $y(.3)$ given

$\frac{dy}{dx} = y' = -y + x + 1 \qquad y(0) = 1$

$dx = 0.1 \qquad x \in [0,1]$

(step)

use the software to see the slope field and hit the table feature to see the results for Euler —

$$y_1 = y_0 + f(x_0, y_0)\,dx$$

$$y_1 = 1 + (-1 + 0 + 1)(.1)$$

$$y_1 = 1 \qquad x_1 = .1 \quad y_1 = 1$$

$(.1, 1)$

$$y_2 = y_1 + f(x_1, y_1)dx$$

$$y_2 = 1 + (-1 + .1 + 1)(.1) \qquad x_2 = x_1 + dx$$

$.1 + .1$

$$y_2 = 1.01 \qquad x_2 = .2$$

$$y_3 = 1.01 + f(.2, 1.01)dx$$

$$= 1.01 + (-1.01 + .2 + 1)(.1)$$

$$y_3 = 1.029 \qquad y(.3) \approx 1.029$$