Reflection:

Follow up: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

students do not have an integrated sense of what the value of the limit is other than memorization

INTRODUCE

NAG ⟷ Graphical method

↓ ↳ Algebraic

numeric method

↓

most are familiar w/ calculator.

- Importance of squeeze or pinching theorem is NICE wrt this limit.

- For Topic of Linearization (Ch 4) this is a Nice concept to cycle back to! $f(x) \approx x$ when x is close to 0.
a small neighborhood of 0
$f(x) = \sin x \approx x$.

- Also connect to Taylor Polynomial Ch 9

Ch2

Squeeze or Pinching Thm $\quad \lim_{x\to a} f(x) = ?$

If $\quad g(x) \le f(x) \le h(x)$ for a neighbourhood near $x=a$, and if

$$\lim_{x\to a} g(x) = L, \text{ and } \lim_{x\to a} h(x) = L$$

then $\lim_{x\to a} f(x) = L$.

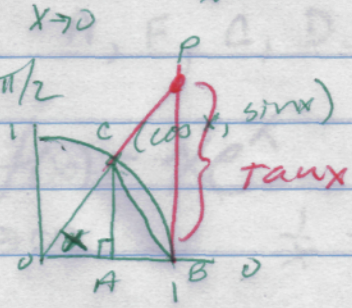
Geometric (Analytic) proof of

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

see p68 #75

group problem in notes

(G)



$0 \le x \le \pi/2$

$r=1$

area ΔCOB < area sector COB ≤ area ΔPOB

$$\tfrac{1}{2}(1)\sin x \le \tfrac{1}{2} r^2\theta \le \tfrac{1}{2}(1)\cdot\tan x$$

(A)

$$\sin x \le x \le \frac{\sin x}{\cos x}$$

$$1 \le \frac{x}{\sin x} \le \frac{1}{\cos x}$$

$$1 \ge \frac{\sin x}{x} \ge \cos x$$

$\lim_{x\to 0} 1 = 1 \qquad \lim_{x\to 0} \cos x = 1$

$$\therefore \lim_{x\to 0} \frac{\sin x}{x} = 1$$