Ch 7 8 9 review

1. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Consider the family of functions $f(x) = \frac{1}{x^2-2x+k}$, where k is a constant.

a) Find the value of k, for $k > 0$, such that the slope of the line tangent to the graph of f at $x = 0$ equals 6.

Please respond on separate paper, following directions from your teacher.

b) For $k = -8$, find the value of $\int_0^1 f(x)\,dx$.

Please respond on separate paper, following directions from your teacher.

c) For $k = 1$, find the value of $\int_0^2 f(x)\,dx$ or show that it diverges.

Please respond on separate paper, following directions from your teacher.

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Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

The first point is earned for a denominator of $\left(x^2-2x+k\right)^2$ or a factor of $\left(x^2-2x+k\right)^{-2}$.

The second point is earned for $f'(x)=\frac{-(2x-2)}{\left(x^2-2x+k\right)^2}$. A response that calculates the derivative correctly but then simplifies incorrectly will earn the second point, but is not eligible for the third point.

0	1	2	3

The student response accurately includes **all three** of the criteria below.

- ☐ denominator of $f'(x)$
- ☐ $f'(x)$
- ☐ answer

Solution:

$$f'(x)=\frac{-(2x-2)}{\left(x^2-2x+k\right)^2}$$

$$f'(0)=\frac{2}{k^2}=6 \;\Rightarrow\; k^2=\frac{1}{3} \;\Rightarrow\; k=\frac{1}{\sqrt{3}}$$

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

The first point is earned for obtaining $A=\frac{1}{6}$ and $B=-\frac{1}{6}$ with denominators $x-4$ and $x+2$, respectively. Other nonzero numerators with these denominators will not earn the first point, but the response is eligible for the second and third points.

A factorization of $(x+4)(x-2)$ resulting in nonzero numerators is eligible to earn the second point, but not the third point.

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Both correct antiderivatives, including use of absolute value, earn the second point.

0	1	2	3

The student response accurately includes **all three** of the criteria below.

- ☐ partial fraction decomposition
- ☐ antiderivatives
- ☐ answer

Solution:

$$\frac{1}{x^2-2x-8} = \frac{1}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$$
$$\Rightarrow \; 1 = A(x+2) + B(x-4)$$
$$\Rightarrow \; A = \frac{1}{6}, \; B = -\frac{1}{6}$$

$$\int_0^1 f(x)\, dx = \int_0^1 \left(\frac{\frac{1}{6}}{x-4} - \frac{\frac{1}{6}}{x+2} \right) dx$$
$$= \left[\frac{1}{6}\ln|x-4| - \frac{1}{6}\ln|x+2| \right]_{x=0}^{x=1}$$
$$= \left(\frac{1}{6}\ln 3 - \frac{1}{6}\ln 3 \right) - \left(\frac{1}{6}\ln 4 - \frac{1}{6}\ln 2 \right) = -\frac{1}{6}\ln 2$$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

The first point is earned for the definite integral $\int_0^2 \frac{1}{x^2-2x+1}\, dx$ split into two integrals at $x=1$, or work that shows that $f(x)$ has a vertical asymptote or infinite discontinuity at $x=1$.

If the first point is not earned, the response is only eligible for the second point.

The second point requires a correct antiderivative with or without reference to a substitution. Correct limits of integration are not required for the second point.

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The third point requires evidence of an improper integral, including use of appropriate limit notation. The third point is earned with a correctly evaluated one-sided limit along with the conclusion of divergence. Only one of the two integrals need to be referenced. Note that any limit that " $= \frac{1}{0}$ " makes the response not eligible to earn the third point.

0	1	2	3

The student response accurately includes **all three** of the criteria below.

- ☐ improper integral
- ☐ antiderivative
- ☐ answer with reason

Solution:

$$\int_0^2 \frac{1}{x^2-2x+1}\,dx = \int_0^2 \frac{1}{(x-1)^2}\,dx = \int_0^1 \frac{1}{(x-1)^2}\,dx + \int_1^2 \frac{1}{(x-1)^2}\,dx$$
$$= \lim_{b\to 1^-} \int_0^b \frac{1}{(x-1)^2}\,dx + \lim_{b\to 1^+} \int_b^2 \frac{1}{(x-1)^2}\,dx$$
$$= \lim_{b\to 1^-} \left(-\tfrac{1}{x-1}\Big|_{x=0}^{x=b}\right) + \lim_{b\to 1^+} \left(-\tfrac{1}{x-1}\Big|_{x=b}^{x=2}\right)$$
$$= \lim_{b\to 1^-} \left(-\tfrac{1}{b-1} - 1\right) + \lim_{b\to 1^+} \left(-1 + \tfrac{1}{b-1}\right)$$

Because $\lim_{b\to 1^-} \left(-\frac{1}{b-1}\right)$ does not exist, the integral diverges.

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2. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(a) For each of the following improper integrals, evaluate the integral or show that the integral diverges.

$$\int_{-3}^{0} \frac{4}{\sqrt{(x+3)^3}}\,dx$$

Please respond on separate paper, following directions from your teacher.

(b) $\int_{1}^{\infty} xe^{-4x}\,dx$

Please respond on separate paper, following directions from your teacher.

(c) $\int_{2}^{\infty} \frac{4}{x^2+4x+3}\,dx$

Please respond on separate paper, following directions from your teacher.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2	3

The student response accurately includes all three of the criteria below.

- ☐ limit
- ☐ antiderivative
- ☐ answer

Solution:

$$\lim_{a\to -3^+}\int_a^0 \frac{4}{\sqrt{(x+3)^3}}\,dx = \lim_{a\to -3^+} \frac{-8}{\sqrt{x+3}}\Bigg|_a^0$$

$$= \lim_{a\to -3^+}\left(\frac{-8}{\sqrt{3}} + \frac{8}{\sqrt{a+3}}\right) = \infty$$

Therefore, the integral diverges.

Part B

At most 2 out of 3 points is earned if no limit notation is used. Numerical answers do not need to be simplified.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3

The student response accurately includes all three of the criteria below.

- ☐ integration by parts
- ☐ antiderivative
- ☐ answer

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Solution:

$u = x \;\Rightarrow\; du = dx$
$dv = e^{-4x}\ dx \;\Rightarrow\; v = \frac{-1}{4}e^{-4x}$

$$\int xe^{-4x}\ dx = x \cdot \tfrac{-1}{4}e^{-4x} - \int \frac{-1}{4}e^{-4x}\ dx$$
$$= \tfrac{-x}{4}e^{-4x} + \int \frac{1}{4}e^{-4x}\ dx$$
$$= \tfrac{-x}{4}e^{-4x} - \tfrac{1}{16}e^{-4x}$$

$$\int_1^\infty xe^{-4x}\ dx = \lim_{b\to\infty}\int_1^b xe^{-4x}\ dx$$
$$= \lim_{b\to\infty}\left(\tfrac{-x}{4}e^{-4x} - \tfrac{1}{16}e^{-4x}\Big|_1^b\right)$$
$$= \lim_{b\to\infty}\left(\left(\tfrac{-b}{4}e^{-4b} - \tfrac{1}{16}e^{-4b}\right) - \left(\tfrac{-1}{4}e^{-4} - \tfrac{1}{16}e^{-4}\right)\right)$$
$$= 0 - \left(\tfrac{-1}{4}e^{-4} - \tfrac{1}{16}e^{-4}\right)$$
$$= \tfrac{1}{4}e^{-4} + \tfrac{1}{16}e^{-4}$$

Part C

At most 2 out of 3 points is earned if no limit notation is used. Numerical answers do not need to be simplified.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2	3

The student response accurately includes all three of the criteria below.

- ☐ partial fraction decomposition
- ☐ antiderivative
- ☐ answer

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Solution:

$$\int \frac{4}{x^2+4x+3}\,dx = \int \frac{4}{(x+1)(x+3)}\,dx = \int \left(\frac{A}{x+1}+\frac{B}{x+3}\right)\,dx$$
$$= \int \frac{(A+B)x+3A+B}{(x+1)(x+3)}\,dx$$

$$\begin{array}{r} A+B=0 \\ 3A+B=4 \end{array} \Rightarrow A=2,\ B=-2$$

$$\int_2^\infty \frac{4}{x^2+4x+3}\,dx = \lim_{b\to\infty}\int_2^b \left(\frac{2}{x+1}-\frac{2}{x+3}\right)\,dx$$
$$= \lim_{b\to\infty}\left(2\ln|x+1|-2\ln|x+3|\,|_2^b\right)$$
$$= \lim_{b\to\infty}\left((2\ln(b+1)-2\ln(b+3))-(2\ln 3-2\ \ln 5)\right)$$
$$= \lim_{b\to\infty}\left(2\ln\tfrac{b+1}{b+3}-2\ln\tfrac{3}{5}\right)$$
$$= 2\ln 1-2\ln\tfrac{3}{5} = -2\ln\tfrac{3}{5}$$

---

3. What is the area of the region enclosed by the graphs of $y=e^x-2$, $y=\sin x$, and $x=0$?

(A) 0.239

(B) 0.506

(C) 0.745 ✔

(D) 2.340

(E) 3.472

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4. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e - 2$ ✔

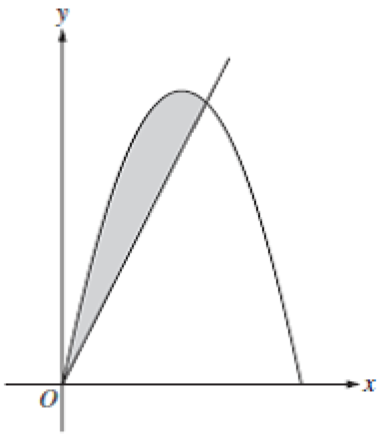
(B) $2e$

(C) $\frac{e}{2} - 1$

(D) $\frac{e-1}{2}$

(E) $e - 1$

5.



The figure above shows the graph of $y = 5x - x^2$ and the graph of the line $y = 2x$. What is the area of the shaded region?

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(A) $\frac{25}{6}$

(B) $\frac{9}{2}$ ✓

(C) 9

(D) $\frac{27}{2}$

(E) $\frac{45}{2}$

---

6. The region bounded by the *x*-axis and the part of the graph of $y = \cos x$ between $x = -\pi/2$ and $x = \pi/2$ is separated into two regions by the line $x = k$. If the area of the region for $-\pi/2 \le x \le k$ is three times the area of the region for $k \le x$, then $k =$

(A) arcsin(1/4)

(B) arcsin(1/3)

(C) $\pi/6$ ✓

(D) $\pi/4$

(E) $\pi/3$

---

7. A region in the plane is bounded by the graph of $y = 1/x$, the *x*-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

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(A) is independent of *m.* ✔

(B) increases as *m* increases.

(C) decreases as *m* increases.

(D) decreases as *m* increases when m < 1/2 ; increases as *m* increases when m > 1/2.

(E) increases as *m* increases when m < 1/2 ; decreases as *m* increases when m >1/2.

---

8. If $0 \leq k < \frac{\pi}{2}$ and the area under the curve *y*=cos *x* from *x*=*k* to $x = \frac{\pi}{2}$ is 0.1, then *k*=

(A) 1.471

(B) 1.414

(C) 1.277

(D) 1.120 ✔

(E) 0.436

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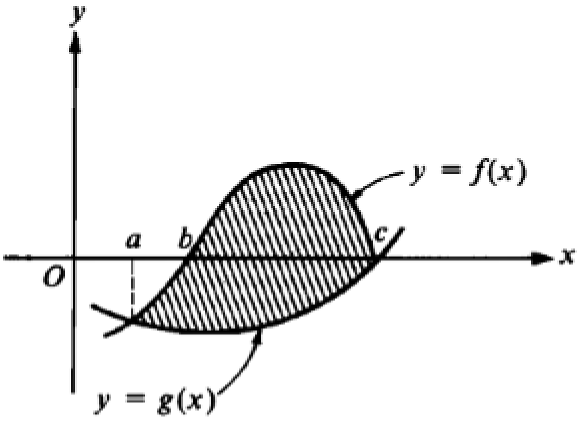
9.



The area of the shaded region in the figure above is represented by which of the following integrals?

(A) $\int_a^c (|f(x)|-|g(x)|)dx$

(B) $\int_b^c f(x)dx - \int_a^c g(x)dx$

(C) $\int_a^c (g(x) - f(x))dx$

(D) $\int_a^c (f(x) - g(x))dx$ ✔

(E) $\int_a^b (g(x) - f(x))dx + \int_b^c (f(x) - g(x))\,dx$

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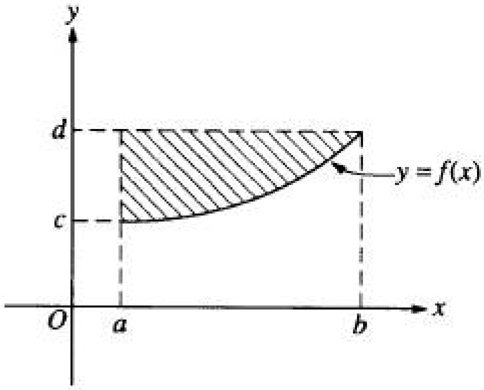
10.



Which of the following represents the area of the shaded region in the figure above?

(A) $\int_c^d f(y)dy$

(B) $\int_a^b (d - f(x))dx$ ✔

(C) $f'(b) - f'(a)$

(D) $(b - a)[f(b) - f(a)]$

(E) $(d - c)[f(b) - f(a)]$

---

11. Let *R* be the region between the graph of $y = e^{-2x}$ and the *x*-axis for$x \geq 3$. The area of *R* is

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(A) $\frac{1}{2e^6}$ ✔

(B) $\frac{1}{e^6}$

(C) $\frac{2}{e^6}$

(D) $\frac{\pi}{2e^6}$

(E) infinite

---

12. The base of a solid is the region in the first quadrant bounded by the y-axis, the graph of $y = \tan^{-1} x$, the horizontal line $y = 3$ and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 2.561

(B) 6.612 ✔

(C) 8.046

(D) 8.755

(E) 20.773

---

13. The region bounded by the graph of $y = 2x - x^2$ and the x-axis is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an equilateral triangle. What is the volume of the solid?

(A) 1.333

(B) 1.067

(C) 0.577

(D) 0.462 ✔

(E) 0.267

---

14. A vase has the shape obtained by revolving the curve $y = 2 + \sin x$ from $x = 0$ to $x = 5$ about the *x*-axis, where x and y are measured in inches. What is the volume, in cubic inches, of the vase?

(A) 10.716

(B) 25.501

(C) 33.666

(D) 71.113

(E) 80.115 ✔

---

15. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y-2}$ and the lines *x* =0 and *y* = 5 is revolved about the *y*-axis?

(A) 3.464

(B) 4.500

(C) 7.854

(D) 10.883

(E) 14.137 ✔

16. Let S be the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$ for $0 \leq x \leq 1$. What is the volume of the solid generated when S is revolved about the line $y = 3$?

(A) $\pi \int_0^1 ((3-2x^2)^2 - (3-2x)^2)dx$ ✔

(B) $\pi \int_0^1 ((3-2x)^2 - (3-2x)^2)dx$

(C) $\pi \int_0^1 (4x^2 - 4x^4)dx$

(D) $\pi \int_0^2 ((3-\frac{y}{2})^2 - (3-\sqrt{\frac{y}{2}})^2)dy$

(E) $\pi \int_0^2 ((3-\sqrt{\frac{y}{2}})^2 - (3-\frac{y}{2})^2)dy$

17. The length of a curve from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1+9x^4}\,dx.$ If the curve contains the point $(1,\ 6)$, which of the following could be an equation for this curve?

(A) $y = 3 + 3x^2$

(B) $y = 5 + x^3$ ✓

(C) $y = 6 + x^3$

(D) $y = 6 - x^3$

(E) $y = \frac{16}{5} + x + \frac{9}{5}x^5$

---

18. Which of the following integrals gives the length of the curve $y = \ln x$ from $x = 1$ to $x = 2$?

(A) $\int_1^2 \sqrt{1 + \frac{1}{x^2}}\, dx$ ✓

(B) $\int_1^2 \left(1 + \frac{1}{x^2}\right)\, dx$

(C) $\int_1^2 \sqrt{1 + e^{2x}}\, dx$

(D) $\int_1^2 \sqrt{1 + (\ln x)^2}\, dx$

(E) $\int_1^2 \left(1 + (\ln x)^2\right)\, dx$

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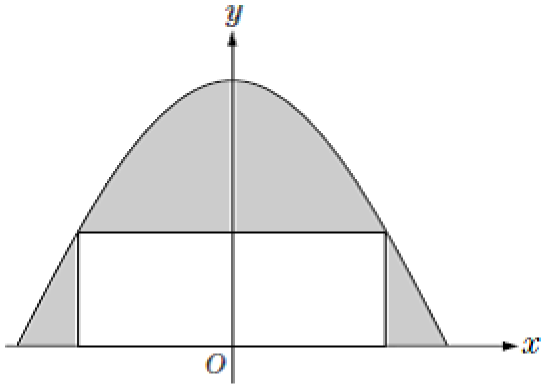
19.



A rectangle with one side on the *x*-axis has its upper vertices on the graph of *y* = cos *x*, as shown in the figure above. What is the minimum area of the shaded region?

(A) 0.799

(B) 0.878 ✔

(C) 1.140

(D) 1.439

(E) 2.000

---

20. A population *y* changes at a rate modeled by the differential equation $\frac{dy}{dt} = 0.2y\,(1000 - y)$, where *t* is measured in years. What are all values of *y* for which the population is increasing at a decreasing rate?

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A) 500 only

B) $0 < y < 500$ only

C) $500 < y < 1000$ only ✓

D) $0 < y < 1000$

E) $y > 1000$

---

21. The speed of a runner, in miles per hour, on a straight trail is modeled by $f(m) = \frac{1}{10}\left(-2m^3 + 9m^2 - 12m\right) + 7$ where *m* is the runner's distance, in miles, from the start of the trail. What is the maximum speed of the runner for $0 \leq m \leq 3$?

A) 6.5

B) 6.6

C) 7.0 ✓

D) 7.5

---

22. $\int x \sin(6x)\, dx =$

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(A) $-x\cos(6x)+\sin(6x)+C$

(B) $-\frac{x}{6}\cos(6x)+\frac{1}{36}\sin(6x)+C$ ✔

(C) $-\frac{x}{6}\cos(6x)+\frac{1}{6}\sin(6x)+C$

(D) $\frac{x}{6}\cos(6x)+\frac{1}{36}\sin(6x)+C$

(E) $6x\cos(6x)-\sin(6x)+C$

---

23. Let *f* be a differentiable function such that $\int f(x)\sin x\,dx=-f(x)\cos x+\int 4x^3\cos x\,dx.$ Which of the following could be $f(x)$?

(A) $\cos x$

(B) $\sin x$

(C) $4x^3$

(D) $-x^4$

(E) x^4 ✔

---

24. If *f* is a function such that $f'(x)=-f(x)$, then $\int x\,f(x)\,dx=$

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(A) $f(x)(x+1)+C$

(B) $-f(x)(x+1)+C$ ✔

(C) $\frac{x^2}{2}f(x)+C$

(D) $-\frac{x^2}{2}f(x)+C$

(E) $-\frac{x^2}{2}f(x)\left(1+\frac{x}{3}\right)+C$

---

25. $\int xf(x)dx =$

(A) $xf(x)-\int xf'(x)dx$

(B) $\frac{x^2}{2}f(x)-\int \frac{x^2}{2}f'(x)dx$ ✔

(C) $xf(x)-\frac{x^2}{2}f(x)+C$

(D) $xf(x)-\int f'(x)dx$

(E) $\frac{x^2}{2}\int f(x)dx$

---

26. $\int xe^{2x}dx=$

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(A) $xe^{2x}/2 - e^{2x}/4 + C$ ✓

(B) $xe^{2x}/2 - e^{2x}/2 + C$

(C) $xe^{2x}/2 + e^{2x}/4 + C$

(D) $xe^{2x}/2 + e^{2x}/2 + C$

(E) $x^2e^{2x}/4 + C$

27.

$\int x^2 sin\ x\ dx$

(A) $-2x\ cos\ x + \int x^2 cos\ x\ dx$

(B) $-x^2 cos + 2\int x\ cos\ x\ dx$ ✓

(C) $x^2 sin\ x + 2\int x\ cos\ x\ dx$

(D) $-1/3\ x^3 cos\ x + C$

(E) $1/3\ x^3 sin\ x - x^2 cos\ x + C$

---

28. $\displaystyle\int \frac{1}{x^2 - 7x + 10}\, dx =$

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(A) $\ln\left|(x-2)(x-5)\right|+C$

(B) $\frac{1}{3}\ln\left|(x-2)(x-5)\right|+C$

(C) $\frac{1}{3}\ln\left|\frac{2x-7}{(x-2)(x-5)}\right|+C$

(D) $\frac{1}{3}\ln\left|\frac{x-2}{x-5}\right|+C$

(E) $\frac{1}{3}\ln\left|\frac{x-5}{x-2}\right|+C$ ✓

---

29. $\displaystyle\int \frac{2x}{(x+2)(x+1)}\,dx =$

(A) $\ln\ |x+2|+\ln\ |x+1|+C$

(B) $\ln\ |x+2|+\ln\ |x+1|-3x+C$

(C) $-4\ln\ |x+2|+2\ln\ |x+1|+C$

(D) $4\ln\ |x+2|-2\ln\ |x+1|+C$ ✓

(E) $2\ln\ |x|+\frac{2}{3}\,x+\frac{1}{2}\,x^2+C$

---

30. $\displaystyle\int_0^1 \frac{5x+8}{x^2+3x+2}\,dx$ is

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(A) $\ln(8)$

(B) $\ln\left(\frac{27}{2}\right)$

(C) $\ln(18)$ ✔

(D) $\ln(288)$

(E) divergent

31.

x	0.0	0.5	1.0	1.5	2.0
$f'(x)$	1.0	0.7	0.5	0.4	0.3

Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = f'(x)$ with initial condition $f(1) = 5$. Selected values of $f'(x)$ are given in the table above. What is the approximation for $f(2)$ if Euler's method is used with a step size of 0.5, starting at $x = 1$?

(A) 5.35

(B) 5.45 ✔

(C) 5.50

(D) 5.90

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32.

x	2	2.2	2.4
$f'(x)$	−0.5	−0.3	−0.1

Let y = f(x) be the solution to the differential equation $\frac{dy}{dx} = f'(x)$ with initial condition f(2) = 3. Selected values of f' are given in the table above. What is the approximation for f(2.4) if Euler's method is used, starting at x = 2 with two steps of equal size?

(A) 2.80

(B) 2.82

(C) 2.84

(D) 2.92

(E) 3.16

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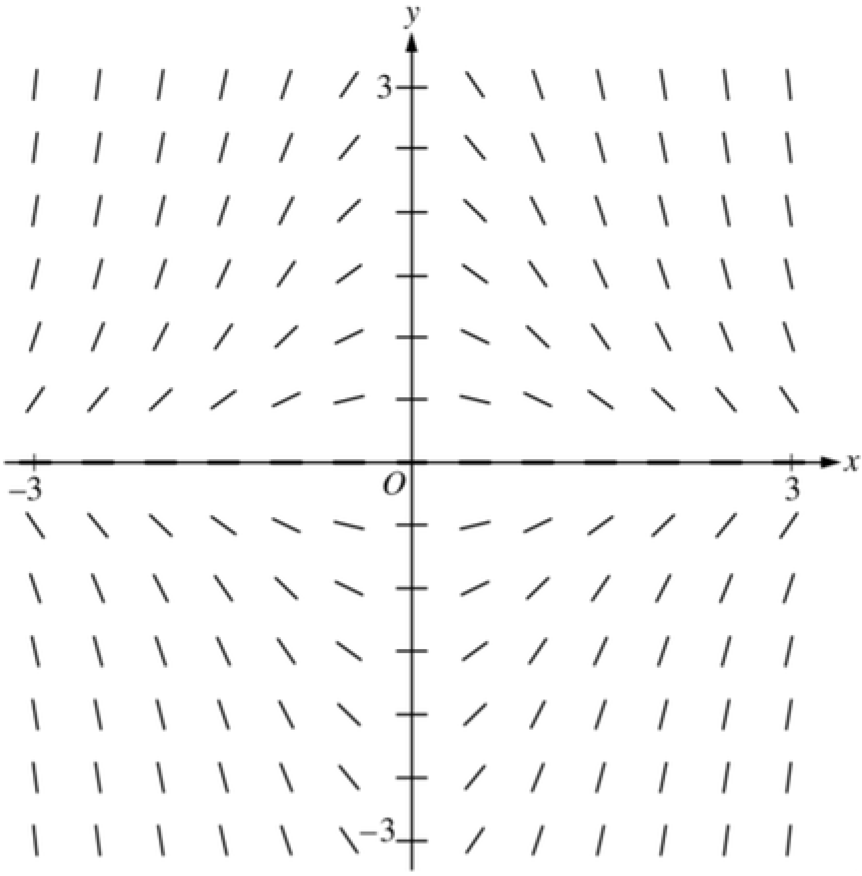
33.



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = x/y$

(B) $\frac{dy}{dx} = -x/y$

(C) $\frac{dy}{dx} = xy$

(D) $\frac{dy}{dx} = -xy$ ✔

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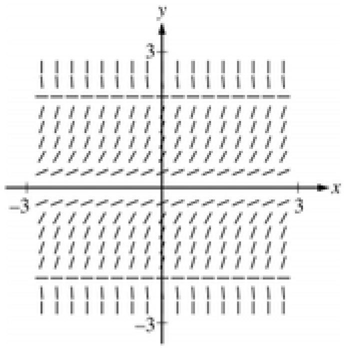
34.



Shown above is a slope field for the differential equation $\frac{dy}{dx} = y^2\left(4 - y^2\right)$. If *y* = *g(x)* is the solution to the differential equation with the initial condition $g(-2) = -1, then, \lim_{x \to \infty} g(x)\, is$

(A) $-\infty$

(B) -2

(C) 0 ✔

(D) 2

(E) 3

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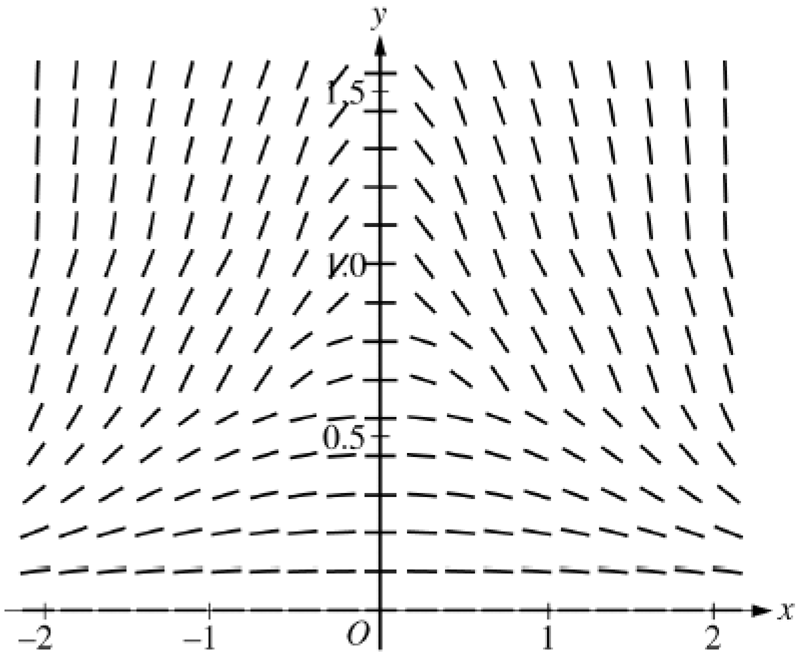
35.



The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 1$?

(A) $y = \cos x$

(B) $y = 1 - x^2$

(C) $y = e^x$

(D) $y = \sqrt{1 - x^2}$

(E) $y = \frac{1}{1+x^2}$

---

36. The temperature of a solid at time $t \geq 0$ is modeled by the nonconstant function H and increases according to the differential equation $\frac{dH}{dt} = 2H + 1$, where $H(t)$ is measured in degrees Fahrenheit and t is measured in hours. Which of the following must be true?

(A) $H = H^2 + t + C$

(B) $ln\ |2H+1| = t/2 + C$

(C) $ln\ |2H+1| = t + C$

(D) $ln\ |2H+1| = 2t + C$ ✔

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37. The number of antibodies *y* in a patient's bloodstream at time *t* is increasing according to a logistic differential equation. Which of the following could be the differential equation?

(A) $\frac{dy}{dt} = 0.025t$

(B) $\frac{dy}{dt} = 0.025t\,(5000 - t)$

(C) $\frac{dy}{dt} = 0.025y$

(D) $\frac{dy}{dt} = 0.025\,(5000 - y)$

(E) $\frac{dy}{dt} = 0.025y\,(5000 - y)$ ✔

---

38. The number of moose in a national park is modeled by the function *M* that satisfies the logistic differential equation $\frac{dM}{dt} = 0.6M\left(1 - \frac{M}{200}\right)$, where *t* is the time in years and $M(0) = 50$. What is $\lim\limits_{t \to \infty} M(t)$?

(A) 50

(B) 200 ✔

(C) 500

(D) 1000

(E) 2000

---

39. The number of students in a cafeteria is modeled by the function P that satisfies the logistic differential equation $\frac{dP}{dt} = \frac{1}{2000}P(200 - P)$, where *t* is the time in seconds and *P(0)* = 25. What is the greatest rate of change, in students per second, of the number of students in the cafeteria?

(A) 5 ✔

(B) 25

(C) 100

(D) 200

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40. A population of wolves is modeled by the function *P* and grows according to the logistic differential equation $\frac{dP}{dt} = 5P\left(1 - \frac{P}{5000}\right)$, where *t* is the time in years and *P(0)* = 1000. Which of the following statements are true?

1.
$\lim_{t \to \infty} P(t) = 5000$

2.
$\frac{dP}{dt} is, positive, for, t > 0.$

3.
$\frac{d^2P}{dt^2} is, positive, for, t > 0.$

(A) I only

(B) II only

(C) I and II only ✔

(D) I and III only

(E) I, II, and III

---

41. If x and y are positive real numbers, which of the following conditions guarantees that the infinite series $\sum_{n=0}^{\infty} \left(\frac{x}{y}\right)\left(\frac{x}{y^2}\right)^n$ converges?

(A) $x > y$

(B) $x > y^2$

(C) $x < y$

(D) $x < y^2$ ✔

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42. Let x be a real number. Which of the following statements about the infinite series $\sum_{k=0}^{\infty} e^{kx}$ is true?

(A) The sum of the series is $\frac{1}{1-e^x}$ if $x < 0$. ✓

(B) The sum of the series is $\frac{1}{1-e^x}$ if $x > 0$.

(C) The sum of the series is $\frac{e^x}{1-e^x}$ if $x < 0$.

(D) The sum of the series is $\frac{e^x}{1-e^x}$ if $x > 0$.

43. $\lim_{x \to \pi} \frac{x + \pi \sec x}{x^2 - \pi^2}$ is

(A) $-\frac{\pi}{2}$

(B) 0

(C) $\frac{1}{2\pi}$ ✓

(D) nonexistent

44. $\lim_{x \to 0} \frac{6e^{4x} - 2e^{3x} - 4}{\sin(2x)} =$

(A) 2

(B) 4

(C) 9 ✔

(D) 18

---

45. Let f be the function defined by $f(x) = 2x + 3e^{-5x}$, and let g be a differentiable function with derivative given by $g'(x) = \frac{1}{x} + 4\cos\left(\frac{5}{x}\right)$. It is known that $\lim\limits_{x\to\infty} g(x) = \infty$. The value of $\lim\limits_{x\to\infty} \frac{f(x)}{g(x)}$ is

(A) 0

(B) $\frac{1}{2}$ ✔

(C) 1

(D) nonexistent

---

46. $\lim\limits_{t\to 0} \frac{\sin t}{\ln(2e^t - 1)} =$

(A) -1

(B) 0

(C) $\frac{1}{2}$ ✔

(D) 1

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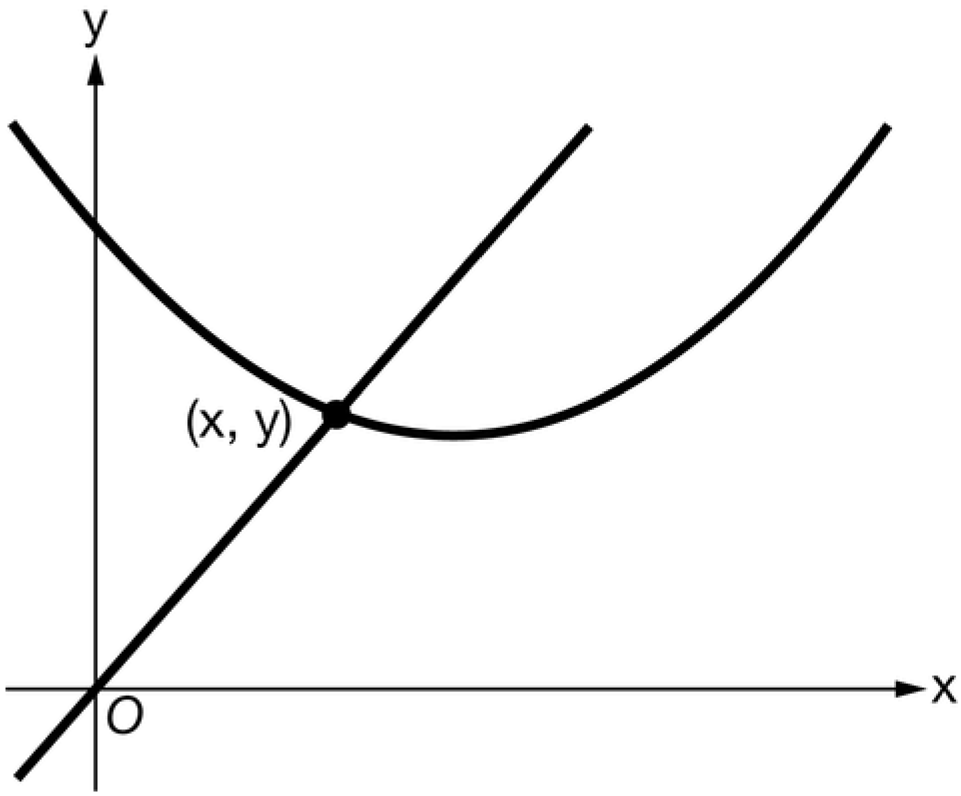
47.



Consider all lines in the xy-plane that pass through both the origin and a point (x, y) on the graph of $y = x^2 - 4x + 9$ for $1 \leq x \leq 4$. The figure above shows one such line and the graph of $y = x^2 - 4x + 9$. Which of the following statements is true?

(A) The line with minimum slope passes through the graph of $y = x^2 - 4x + 9$ at $x = 1$.

(B) The line with minimum slope passes through the graph of $y = x^2 - 4x + 9$ at $x = 2$.

(C) The line with minimum slope passes through the graph of $y = x^2 - 4x + 9$ at $x = 3$. ✔

(D) The line with minimum slope passes through the graph of $y = x^2 - 4x + 9$ at $x = 4$.

48. Let g be a continuously differentiable function with $g(1) = 6$ and $g'(1) = 3$. What is $\lim_{x \to 1} \frac{\int_1^x g(t)\,dt}{g(x) - 6}$?

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2 ✔

(E) The limit does not exist.

49.

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
2	0	0	5	7

The third derivative of the function *f* is continuous on the interval(0,4) . Values for *f* and its first three derivatives atx=2 are given in the table above. What is $\lim_{x\to 2}\frac{f(x)}{(x-2)^2}$?

(A) 0

(B) $\frac{5}{2}$ ✔

(C) 5

(D) 7

(E) The limit does not exist.

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50. Let *f* and *g* be functions that are differentiable for all real numbers, with $g(x) \neq 0$ for $x \neq 0$. If $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = 0$ and $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x\to 0} \frac{f(x)}{g(x)}$ is

(A) 0

(B) $\frac{f'(x)}{g(x)}$

(C) $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$ ✔

(D) $\frac{f'(x)g(x)-f(x)g'(x)}{(f(x))^2}$

(E) nonexistent

---

51. $\int_1^\infty \frac{x^2}{(x^3+2)^2}\, dx$ is

(A) $-\frac{1}{9}$

(B) $\frac{1}{9}$ ✔

(C) $\frac{1}{3}$

(D) 1

(E) divergent

---

52. $\int_1^\infty xe^{-x^2}\, dx$ is

(A) $-\frac{1}{e}$

(B) $\frac{1}{2e}$ ✔

(C) $\frac{1}{e}$

(D) $\frac{2}{e}$

(E) divergent

---

53. $\int_0^3 \frac{dx}{(1-x)^2}$

(A) $-\frac{3}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

(E) divergent ✔

---

54. $\int\limits_1^{\infty} \frac{x}{(1+x^2)^2} dx$ is

(A) $-\frac{1}{2}$

(B) $-\frac{1}{4}$

(C) $\frac{1}{4}$ ✔

(D) $\frac{1}{2}$

(E) divergent

---

55. If R is the unbounded region between the graph of $y = \frac{1}{x(\ln x)^2}$ and the x-axis for $x \geq 3$, then the area of R is

(A) -1 / ln 3

(B) ln(ln 3)

(C) 1 / ln 3 ✔

(D) $\frac{3}{(ln\ 3)^3}$

(E) infinite

---

56. $\int_1^{\infty} \frac{1}{x^p} dx \; and \int_0^1 \frac{1}{x^p} dx$ both diverge when *p* =

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(A) 2

(B) 1 ✓

(C) $\frac{1}{2}$

(D) 0

(E) -1

---

57. Which of the following statements about the integral $\int_0^{\pi} \sec^2 x dx$ is true

(A) The integral is equal to 0.

(B) The integral is equal to 2/3.

(C) The integral diverges because $\lim\limits_{x\to\frac{\pi}{2}} \sec^2 x$ does not exist.

(D) The integral diverges because $\lim\limits_{x\to\frac{\pi}{2}} \tan x$ does not exist. ✓

---

58. What are all values of *p* for which $\int_1^{\infty} \frac{1}{x^{3p+1}} dx$ converges?

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(A) $p < 0$

(B) $p > -1/3$

(C) $p > 0$ ✔

(D) $p > 1$

---

59. What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

(A) 1

(B) 2

(C) 4 ✔

(D) 6

(E) The series diverges.

---

60. What is the sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$?

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(A) $\frac{-2}{e^2-2e}$

(B) $\frac{-2}{e^2+2e}$ ✔

(C) $\frac{-2}{e+2}$

(D) $\frac{e}{e+2}$

(E) The series diverges.

---

61. What is the value of $\sum_{n=0}^{\infty}\left(-\frac{2}{3}\right)^n$?

(A) −2

(B) $-\frac{2}{5}$

(C) $\frac{3}{5}$ ✔

(D) 3

(E) The series diverges.

---

62. Which of the following series converge to 2?

I. $\sum_{n=1}^{\infty}\frac{2n}{n+3}$

II. $\sum_{n=1}^{\infty}\frac{-8}{(-3)^n}$

III. $\sum_{n=0}^{\infty}\frac{1}{2^n}$

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(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only ✔

63. If $f(x) = \sum_{k=1}^{\infty} (\sin^2 x)^k$, then $f(1)$ is

(A) 0.369

(B) 0.585

(C) 2.400

(D) 2.426 ✔

(E) 3.426