

**The Bronx Science Geometry Teachers Proudly  
Present...**

**THE (ULTIMATE)  
GEOMETRY REVIEW  
SHEET..WITH COMMON  
CORE GOODNESS**

**(2016 Edition)**

## Some General Information

### The Common Core Regents Exam Basics:

**Time: 3 hours**

**Problems: 36**

- Part I: 24 multiple choice problems (2 pts each) = 48 pts
- Part II: 7 short answer problems (2 pts each) = 14 pts
- Part III: 3 short answer problems (4 pts each) = 12 pts
- Part IV: 2 long answer problems (6 pts each) = 12 pts
- Total: 86 pts

Conceptual Category	Domains in Geometry	Percent of Test By Credit
Geometry	Congruence (G-CO)	27% - 34%
	Similarity, Right Triangles, and Trigonometry (G-SRT)	29% - 37%
	Circles (G-C)	2% - 8%
	Expressing Geometric Properties with Equations (G-GPE)	12% - 18%
	Geometric Measurement & Dimensions (G-GMD)	2% - 8%
	Modeling with Geometry (G-GMD)	8% - 15%

**General Breakdown**

Domain	Cluster	Cluster Emphasis
Congruence 27% - 34%	Experiment with transformations in the plane	Supporting
	Understand congruence in terms of rigid motions	Major
	Prove geometric theorems	Major
	Make geometric constructions	Supporting
Similarity, Right Triangles, & Trigonometry 29% - 37%	Understand similarity in terms of similarity transformations	Major
	Prove theorems involving similarity	
	Define trigonometric ratios and solve problems involving right triangles	
Circles 2% - 8%	Understand and apply theorems about circles	Additional
	Find arc lengths and areas of sectors of circles	
Expressing Geometric Properties with Equations 12% - 18%	Translate between the geometric description and the equation for a circle, ellipse	Additional
	Use coordinates to prove simple geometric theorems algebraically	Major
Geometric Measurement & Dimensions 2% - 8%	Explain volume formulas and use them to solve problems	Additional
	Visualize relationships between two-dimensional and three-dimensional objects	
Modeling with Geometry 8% - 15%	Apply geometric concepts in modeling situations	Major

**More Specific Breakdown**

The following playlist is useful, since it has most of the topics from geometry in one compact place:

<http://www.youtube.com/playlist?list=PL26812DF9846578C3&feature=plcp>

Many thanks to the users of Khan Academy for their work here!

Below is the link to the Regents Prep site (feel free to poke around for other subjects as well!) This deals with the majority of Geometry:

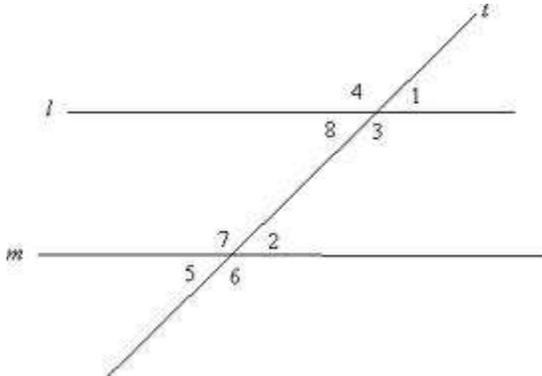
<http://www.regentsprep.org/Regents/math/geometry/math-GEOMETRY.htm>

<http://www.nysedregents.org/Geometry/> This is a link to all existing Geometry Regents exams—replete with answer keys, rubrics, and scaling paraphernalia for your perusal. A wealth of practice here!

<http://www.nysedregents.org/SequentialMathematicsII/> This is a link to Regents exams back to 1998—back when Geometry was folded into something known as “Course II.” (*Note: there will be some topics on these exams that are not in Geometry right now, and one notable topic—circles—is absent completely. Nevertheless, these are excellent resources for most of the other topics.*)

## PARALLEL LINES

Make sure you know how to identify the different types of angles formed when two lines are cut by a transversal:



- The angle pairs  $\{2, 8\}$  and  $\{3, 7\}$  are **alternate interior angles**—you can remember this because they form a sort of “Z” shape or reversed “Z” shape.
- The angle pairs  $\{1, 2\}$ ,  $\{4, 7\}$ ,  $\{5, 8\}$ , and  $\{3, 6\}$  are **corresponding angles**—you can remember these because they form a sort of “F” shape—whether upside-down, reversed, or both!
- The angle pairs  $\{1, 5\}$  and  $\{4, 6\}$  are **alternate exterior angles**.

These lines are only parallel if:

- alternate interior angles are congruent
- alternate exterior angles are congruent
- corresponding angles are congruent
- same side interior angles are supplementary.

If you're uncomfortable with those terms, you can visit:

<http://www.mathsisfun.com/geometry/parallel-lines.html> for more information.

You can get some practice with solving for angles of parallel lines with this video:

<http://www.khanacademy.org/math/geometry/angles/v/angles-of-parallel-lines-2>

Or: <http://www.regentsprep.org/Regents/math/geometry/GP8/PracParallel.htm> (These are more traditional, practice-like problems.)

## CONGRUENT TRIANGLES

- **SSS Postulate** - If two triangles have three pairs of corresponding sides that are congruent, then the triangles are congruent.
- **SAS Postulate** - Triangles are congruent if any pair of corresponding sides and their included angles are congruent in both triangles.
- **ASA Postulate** - Triangles are congruent if any two angles and their included side are congruent in both triangles.
- **Hyp. Leg Theorem** - Two right triangles are congruent if the hypotenuse and one corresponding leg are congruent in both triangles.
- **AAS Theorem** - Triangles are congruent if two pairs of corresponding angles and a pair of non-included sides are equal in both triangles.
- Corresponding sides of congruent triangles are congruent.
- **Isosceles Triangle Theorem** - If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

- **Converse of the Isosceles Triangle Theorem** - If two angles of a triangle are congruent, then sides opposite those angles are congruent.
- If a triangle is equiangular, then it is equilateral.
- If a triangle is equilateral, then it is equiangular.
- Complements (supplements) of congruent angles are congruent.
- **Angle Bisector Theorem** - If  $\overline{BX}$  is an angle bisector of  $\angle ABC$ , then  $m\angle ABX = \frac{1}{2}m\angle ABC$  and  $m\angle XBC = \frac{1}{2}m\angle ABC$ .
- **Converse of the Angle Bisector Theorem** - If  $m\angle ABX = \frac{1}{2}m\angle ABC$  and  $m\angle XBC = \frac{1}{2}m\angle ABC$ , then  $\overline{BX}$  is an angle bisector of  $\angle ABC$ .
- **Perpendicular Bisector Theorem** - If a point lies on the perpendicular bisector of a segment, then the point is equidistant from the endpoints of the segment.
- **Converse of the Perpendicular Bisector Theorem** - If a point is equidistant from the endpoints of a line segment, then the point lies on the perpendicular bisector of the line segment.
- The median, angle bisector, and altitude drawn to the base of an isosceles triangle (equilateral triangle) are the same segment.
- The medians (angle bisectors, perpendicular bisectors, altitudes) of a triangle are concurrent.
- The centroid of a triangle divides the median in the ratio of 2:1.

#### Some information and practice problems:

[http://www.khanacademy.org/search?page\\_search\\_query=congruent+triangles](http://www.khanacademy.org/search?page_search_query=congruent+triangles)

[http://www.khanacademy.org/math/geometry/triangles/e/congruency\\_postulates](http://www.khanacademy.org/math/geometry/triangles/e/congruency_postulates)

#### Videos:

[http://www.youtube.com/watch?v=CJrVOF\\_3dN0](http://www.youtube.com/watch?v=CJrVOF_3dN0) (SSS Postulate)

<http://www.youtube.com/watch?v=8Ld8Csu4sEs> (The other major ones, aside from Hyp-Leg)

<http://www.youtube.com/watch?v=Xc3oHzKXVh8> (An example)

#### Non-Video Practice:

<http://www.regentsprep.org/Regents/math/geometry/GP4/indexGP4.htm>

[http://www.mathwarehouse.com/geometry/congruent\\_triangles/](http://www.mathwarehouse.com/geometry/congruent_triangles/)

## INEQUALITIES

Make sure that you know the following facts about inequalities:

- **“The whole is greater than any of its parts.”**
- **The Trichotomy Postulate:** “Given two numbers,  $a$  and  $b$ , exactly one of the following is true— $a > b$ ,  $a < b$ , or  $a = b$ .”
- **Transitive Property:** “If  $a > b$  and  $b > c$ , then  $a > c$ .”
- **The Addition Postulate of Inequality:** “If  $a > b$  and  $c \geq d$ , then  $a + c > b + d$ . The same is true if the signs are reversed.
- **The Subtraction Postulate of Inequality:** “If  $a > b$  and  $c = d$ , then  $a - c > b - d$ . The same is true if the signs are reversed.
- **The Multiplication Postulate of Inequality:** If  $a > b$  and  $c > 0$ , then  $ac > bc$ . Similarly, if  $a > b$  and  $c < 0$ , then  $ac < bc$ .
- **The Triangle Inequality:** “The sum of the lengths of two sides of a triangle is greater than that of the third.”

- “The measure of an exterior angle of a triangle is greater than the measure of either of the two remote interior angles.”
- “If the lengths of two sides of a triangle are unequal, then the larger angle is opposite the longer side, and vice versa.”
- “If the measures of two angles of a triangle are unequal, then the longer side is opposite the larger angle, and vice versa.”

### Some Information and Practice Problems:

<http://www.regentsprep.org/Regents/math/geometry/GP7/LTriIneq.htm> A brief review (with diagrams) of the material in this section.

<http://www.regentsprep.org/Regents/math/geometry/GPB/theorems.htm> A listing of these theorems.

<http://www.mathopenref.com/triangleinequality.html> More on the Triangle Inequality.

<http://www.regentsprep.org/Regents/math/geometry/GP7/PTriIneq.htm> More traditional review problems.

### Videos:

<http://www.youtube.com/playlist?list=PLFBA79EE6E52D0C49> A playlist of some videos involving the topics here. Best to parse through the list first to see what topic you want to focus on.

## QUADRILATERALS

\*You must be able to apply the properties of all of the special quadrilaterals in algebraic problems as well as proofs.

1. *Properties of Parallelograms*
  - a. 2 pairs of parallel sides
  - b. 2 pairs of opposite sides congruent
  - c. 2 pairs of opposite angles congruent
  - d. consecutive angles are supplementary
  - e. diagonals bisect each other
  - f. each diagonal creates 2 congruent triangles
2. *Properties of Rhombi*
  - a. All properties of parallelograms
  - b. Consecutive sides congruent (equilateral quadrilateral)
  - c. Diagonals are perpendicular
  - d. Diagonals bisect the angles at each vertex
3. *Properties of Rectangles*
  - a. All properties of parallelograms
  - b. Contains a right angle (equiangular quadrilateral)
  - c. Diagonals are congruent
4. *Properties of Squares*
  - a. All properties of rectangles and rhombi
5. *Properties of Trapezoids*
  - a. At least one pair of parallel sides
  - b. The median of a trapezoid is parallel to both bases and its length is the average of the bases.**
6. *Properties of Isosceles Trapezoids*
  - a. Non-parallel sides (legs) are congruent
  - b. Base angles are congruent
  - c. Diagonals are congruent
  - d. Opposite angles are supplementary

Video: <http://www.khanacademy.org/math/geometry/polygons-quads-parallelograms/v/quadrilateral-properties>

Practice:

1. <http://www.regentsprep.org/Regents/math/geometry/GP9/PracQuad.htm>
2. <http://www.regentsprep.org/Regents/math/geometry/GP9/PracQuadPf.htm>
3. <http://www.phschool.com/webcodes10/index.cfm?wcprefix=aua&wcsuffix=0652&fuseaction=home.gotoWebCode&x=7&y=14>

<b>TOPIC: <u>CONSTRUCTIONS</u></b>	<b><u>PRACTICE</u></b>
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1. [Copying segments](#)
2. [Copying angles](#)
3. [Bisecting angles](#)
4. [Bisecting segments](#)
5. **Constructing perpendicular segments given a point on the line**
6. **Constructing perpendicular segments given a point not on the line**  
(For these two, try looking <http://www.regentsprep.org/Regents/math/geometry/GC1/perp.htm> here.)
7. [Copying triangles](#)
8. **Constructing altitudes of a triangle**
9. [Constructing parallel lines through a point](#)
10. [Dividing a segment into n congruent parts](#)
11. [Inscribing a square inside of a given circle](#)
12. [Inscribing a regular hexagon inside of a given circle](#)
13. **Inscribing an equilateral triangle inside of a give circle: There is no nice video here, but just follow the inscribed regular hexagon construction steps and connect every other construction mark along the circle.**
14. [Inscribing a regular pentagon inside of a given circle](#)

You should also be familiar with constructing the centers of a triangle and the properties of each.

- **Centroid:** Concurrency of the three medians of a triangle. **\*\*The centroid is the center of mass of a triangle and it divides each median into a ratio of 2:1 (vertex to centroid : centroid to midpoint = 2:1)\*\*.**
- **Orthocenter:** Concurrency of the three altitudes of a triangle. The orthocenter can be inside (acute triangle), outside (obtuse triangle), or on (right triangle) the triangle.
- **Incenter:** Concurrency of the angle bisectors of a triangle. The incenter is also the center of the incircle, which is the circle that is inscribed within the triangle. **\*\*This means that the incenter is equidistant from all three sides of the triangle\*\*.**

- **Circumcenter: Concurrency of the perpendicular bisectors of all three sides of the triangle. The circumcenter is the center of the circumcircle, the circle that circumscribes the triangle. \*\*This means that the circumcenter is equidistant from all three vertices of the triangle\*\*.**

## SIMILAR TRIANGLES/POLYGONS

- In a proportion, the product of the means is equal to the product of the extremes.
- In a proportion, the means may be interchanged.
- In a proportion, the extremes may be interchanged.
- **AA Postulate** - When 2 angles of one triangle are congruent to 2 corresponding angles of the other triangle, the two triangles must be similar
- **SSS Similarity Theorem** - If the corresponding sides of two triangles are in proportion, then the triangles must be similar.
- **SAS Similarity Theorem** - In two triangles, if two pairs of corresponding sides are in proportion, and their included angles are congruent, then the triangles are similar.
- If two polygons are similar, then the ratio of the perimeters is the same as the ratio of similitude (scale factor).
- If two polygons are similar, then the ratio of the areas is the square of the ratio of similitude (scale factor).
- **Triangle Proportionality Theorem** - If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.
- If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.
- If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.
- **Triangle Angle Bisector Theorem (side proportions)** - If a ray bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides.
- **Right Triangle Altitude Theorem** - If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and each other.
- **Corollary 1 of Right Triangle Altitude Theorem** - When the altitude is drawn to the hypotenuse of a right triangle, the length of the altitude is the geometric mean between the segments of the hypotenuse.
- **Corollary 2 of Right Triangle Altitude Theorem** - When the altitude is drawn to the hypotenuse of a right triangle, each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.
- If two triangles are similar, then the ratio of any two corresponding segments (such as medians, altitudes, angle bisectors) equals the ratio of any two corresponding sides.
- **Pythagorean Theorem** - In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.
- **Converse of Pythagorean Theorem** - If the squares of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

### Some Resources:

- [http://www.khanacademy.org/search?page\\_search\\_query=Similar+triangles](http://www.khanacademy.org/search?page_search_query=Similar+triangles) (A listing of materials.)
- [http://www.khanacademy.org/math/geometry/triangles/e/similar\\_triangles\\_1](http://www.khanacademy.org/math/geometry/triangles/e/similar_triangles_1) (More specific listings.)
- <http://www.regentsprep.org/Regents/math/geometry/GP11/indexGP11.htm> (More general problems.)
- <http://www.mathwarehouse.com/geometry/similar/triangles/>

**Videos:**

<http://www.youtube.com/watch?v=9ThXDY9Y3oU> (The basics.)

[http://www.youtube.com/watch?v=R-6CAr\\_zEEk](http://www.youtube.com/watch?v=R-6CAr_zEEk) (Examples.)

<http://www.youtube.com/watch?v=BI-rtfZVXy0> (More of the general ideas.)

## CIRCLES

You should be very comfortable with finding the area of a circle, area of a circular sector, the length of an intercepted arc, and working with diameter, radius, and circumference. You should be able to describe a circle's center and radius by looking at the standard equation of a circle and you should know how to "complete the square" to get to the standard form of a circle. In addition, you should understand the conversion between radians and degrees.

<http://www.khanacademy.org/math/geometry/circles-topic/v/parts-of-a-circle>

A brief outline of the rules for different (secants, tangents, segments) in a circle:

<http://www.regentsprep.org/Regents/math/geometry/GP14/CircleSegments.htm>

**Get some practice with:**

*Chords and circles:* <http://www.regentsprep.org/Regents/math/geometry/GP14/PracCircleChords.htm>

*Tangents and circles:* <http://www.regentsprep.org/Regents/math/geometry/GP14/PracCircleTangents.htm>

*Mixed practice (secants, tangents, and chords):*

<http://www.regentsprep.org/Regents/math/geometry/GP14/PracCircleSegments.htm>

Know the key theorems for angles both inside and outside of a circle. You should have a graphic organizer with these formulas, but you can also check here for an overview of all the angles of a circle:

<http://www.regentsprep.org/Regents/math/geometry/GP15/CircleAngles.htm>

**Some practice with:**

*Angles inside the circle:* <http://www.regentsprep.org/Regents/math/geometry/GP15/PcirclesN2.htm>

*Angles outside the circle:* <http://www.regentsprep.org/Regents/math/geometry/GP15/PcirclesN3.htm>

You should also be able to solve problems involving arc measure and arc length. Check here for some examples:

<http://www.regentsprep.org/Regents/math/geometry/GP15/PcirclesN4.htm>

*Completing the square:* [https://www.khanacademy.org/math/algebra2/conics\\_precalc/circles-tutorial-prec calc/v/completing-the-square-to-write-equation-in-standard-form-of-a-circle](https://www.khanacademy.org/math/algebra2/conics_precalc/circles-tutorial-prec calc/v/completing-the-square-to-write-equation-in-standard-form-of-a-circle)

*Arc length:* <http://www.mathopenref.com/arclength.html>

*Area of a circular sector:* <http://www.regentsprep.org/regents/math/geometry/gp16/circlesectors.htm>

*Converting degrees to radians:* [https://www.khanacademy.org/math/trigonometry/unit-circle-trig-func/radians\\_tutorial/v/radians-and-degrees](https://www.khanacademy.org/math/trigonometry/unit-circle-trig-func/radians_tutorial/v/radians-and-degrees)

## 3D GEOMETRY

\*You must be able to calculate the volumes and surface areas of prisms, pyramids, cylinders, cones and spheres. Some of the formulas are printed on the Reference Sheet.

Formulas that are **NOT** on the Reference Sheet are:

Volume of a Prism:  $V = Bh$ , where  $B$  is the area of the base.

Lateral Area of a Prism:  $L.A. = Ph$ , where  $P$  is the perimeter of the base.

Lateral Area of a Regular Pyramid:  $L.A. = \frac{1}{2}Pl$ , where  $P$  is the perimeter of the base and  $l$  is the slant height (height of one of the lateral faces)

\*To determine the SURFACE AREA (or total area) of prisms, pyramids, cones or cylinders you must add the area of the bases(s) to the lateral area.

### Density:

Density is defined as the mass of an object per unit<sup>3</sup>. For our purposes, we can think of density as weight of an object per its volume. It is useful to know the relationship:  $\text{Density of Solid} = \frac{\text{Weight of Solid}}{\text{Volume of Solid}}$ .

### Cavalieri's Principle:

The basic idea: Let's say that there are two solids equal in height, with congruent bases such that the bases in each figure are parallel. If the corresponding cross-sections of each figure are congruent, then the two figures must have the same volume. A good way to visualize this is with a stack of coins:



The two stacks in the picture have congruent bases (all four bases are coins), that are parallel to each other. Their heights are the same (due to the same number of coins), and each corresponding cross-section is congruent (since each cross-section is just another coin). Therefore, these two stacks of coins must have the same volume, even though their respective shapes are quite different.

**Video:** <http://www.khanacademy.org/math/geometry/basic-geometry/v/solid-geometry-volume>

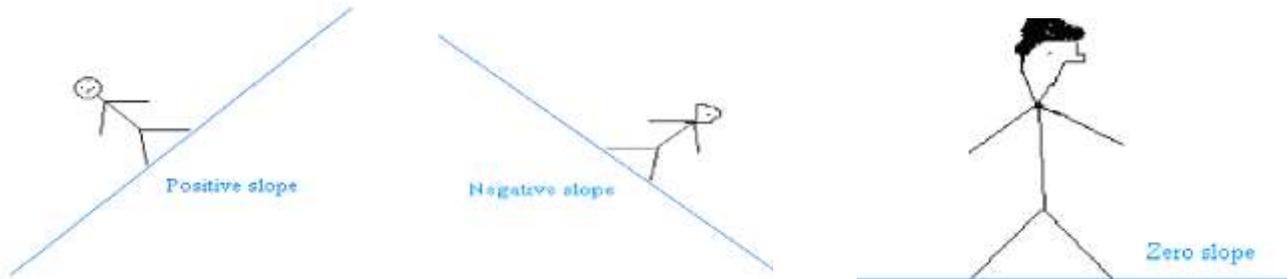
### Practice:

1. <http://www.regentsprep.org/Regents/math/geometry/GG1/PracPLANES.htm>
2. <http://www.regentsprep.org/Regents/math/geometry/GG2/Prac3d.htm>
3. <http://www.phschool.com/webcodes10/index.cfm?wcprefix=aua&wcsuffix=1152&fuseaction=home.gotoWebCode&x=7&y=14>

# COORDINATE GEOMETRY

-slope:  $m = \frac{y_2 - y_1}{x_2 - x_1}$

-horizontal lines ( $y = \dots$ ) have 0 slope, and vertical lines ( $x = \dots$ ) have no slope - slope is undefined



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$y = mx + b$ : **slope-intercept form** where  $m = \text{slope}$  and  $b = \text{y-intercept}$

$(y - y_1) = m(x - x_1)$ : **point-slope form** where  $(x_1, y_1)$  are the coordinates of a point on the line and  $m = \text{slope}$

$Ax + By = C$ : **standard form** where  $A$  and  $B$  are coefficients and *cannot* be 0, and where  $C$  is a constant

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to graph a line in slope-intercept form, graph the y-intercept ( $b$ ) on the y-axis, and use the slope,

$$m = \frac{\text{rise}}{\text{run}}, \text{ to find more points.}$$

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finding intercepts:

-to find the y-intercept, let  $x = 0$  and solve for  $y$ , when given the equation of the line

-to find the x-intercept, let  $y = 0$  and solve for  $x$ , when given the equation of the line

-otherwise, the y-intercept is the point on the line that "meets" or "hits" the y-axis, and the x-intercept "meets" or "hits" the x-axis.

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Thm: Two nonvertical lines are parallel iff their slopes are equal

Thm: Two nonvertical lines are perpendicular iff the product of their slopes is  $-1$ .

ex:  $y = 3x + 4$  and  $y = 3x - 5$  are parallel lines, where  $y = 2x + 1$  and  $y = -\frac{1}{2}x$  are perpendicular lines

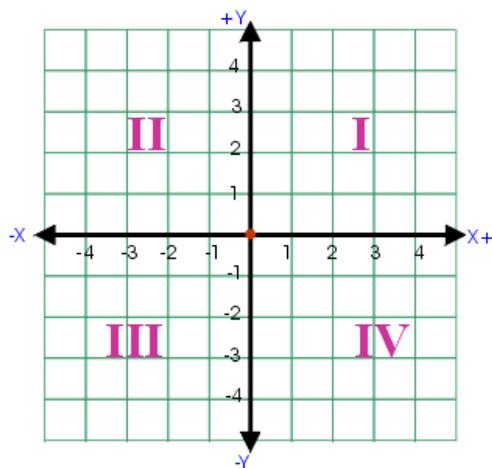
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Distance formula: when given two points,  $(x_1, y_1)$  and  $(x_2, y_2)$ ...  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

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Midpoint formula: when given two points,  $(x_1, y_1)$  and  $(x_2, y_2)$ ...  $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$  quadrants of

the coordinate plane:



Systems of linear equations: solve graphically OR by elimination or substitution

\*refer to the following websites for help:

Graphically...

<http://www.regentsprep.org/Regents/math/ALGEBRA/AE3/GrSys.htm>

Substitution method...

<http://www.regentsprep.org/Regents/math/ALGEBRA/AE3/AlgSys.htm>

Elimination method...

<http://www.regentsprep.org/Regents/math/ALGEBRA/AE3/AlgSysAdd.htm>

*Part I Reminders:*

-label your x- and y-axes

-label your graphs (your lines, parabolas, etc.) using the *original* equation given

### Equation of a circle:

The equation of a circle:  $(x - h)^2 + (y - k)^2 = r^2$ , where  $(h, k)$  is the center and  $r$  represents the radius.

Be able to convert the equation of a circle from standard form to general form by completing the square. For example:

$$2x^2 + 2y^2 - 8x + 20y - 14 = 0$$

Equation in standard form

$$x^2 - 4x + y^2 + 10y = 7$$

Divide by the coefficient of  $x^2$  and  $y^2$ .  
Group together the  $x$ 's and the  $y$ 's. Move the constant to the other side of the equation.

$$x^2 - 4x + 4 + y^2 + 10y + 25 = 7 + 4 + 25$$

Complete the square for  $x$  and  $y$ . Add to both sides to balance the equation. Note: In order to complete the square given  $x^2 + bx$ , add the value  $\left(\frac{b}{2}\right)^2$

$$(x - 2)^2 + (y + 5)^2 = 36$$

Factor and simplify. The center is  $(2, -5)$  and the radius is 6.

Here is a link to more details about completing the square:

<http://www.regentsprep.org/regents/math/algtrig/ate12/completesqlesson.htm>

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Quadratic-Linear Systems:

\*refer to the following website for help...

Graphically...

<http://www.regentsprep.org/Regents/math/geometry/GCG5/LQReview.htm>

**Proof Methods:**

<b>To prove a figure is a(n)...</b>	<b>Methods</b>
Parallelogram	Show <i>one</i> of the following: -diagonals bisect each other -both pairs of opposite sides are parallel -both pairs of opposite sides are congruent -one pair of opposite sides is congruent and parallel
Rectangle	Show that the figure is a parallelogram using any one of the four methods above AND <i>one</i> of the following: -the figure has one right angle -the diagonals are congruent
Rhombus	Show that the figure is a parallelogram using any one of the four methods above AND <i>one</i> of the following: -the diagonals are perpendicular -two adjacent sides are congruent
Square	Show the figure is a rectangle AND two adjacent sides are congruent ...OR... Show that the figure is a rhombus AND one angle is a right angle.
Trapezoid	Show that the quadrilateral has only one pair of opposite sides parallel
Isosceles Trapezoid	Show the figure is a trapezoid AND <i>one</i> of the following: -the diagonal are congruent -the legs are congruent
Right Trapezoid	Show the figure is a trapezoid

	AND one of the legs is perpendicular to a base of the trapezoid
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### General Methods of Proof and Formulas

To Prove	Formula to Use
that line segments are congruent, show that the lengths are equal	Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
that line segments bisect each other, show that the midpoints are the same	Midpoint Formula: $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
that lines are parallel, show that the slopes are equal	Slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m_1 = m_2$
that lines are perpendicular, show that the slopes are negative reciprocals	Products of slopes = -1 $m_1 \cdot m_2 = -1$ OR $m_1 = \frac{-1}{m_2}$

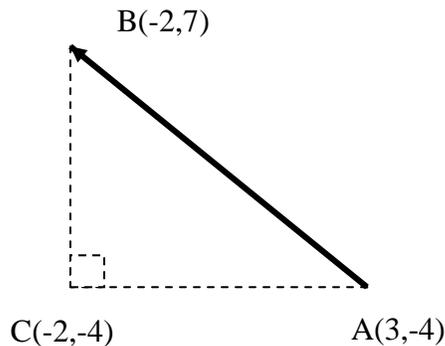
Solving a linear-quadratic system of equations Algebraically...

- solve both equations for y
- equate the linear equation and the quadratic equation
- solve for x
- plug the values for x back into either equation to find the values of y
- your solutions should be coordinates (x, y) representing the point(s) of intersection of the line and the parabola.

### Dividing a given line segment into a desired ratio:

Example: Given the line segment with endpoints A(3, -4) and B(-2, 7), find the point along segment AB starting from A that divides the segment into a ratio of 2:3.

Solution:



To travel the complete distance from point A to point B, we must simultaneously travel -5 units horizontally (5 units left) from A and also +11 units vertically (11 units up) from point A. Now, in order to travel to a point that divides  $\overline{AB}$  into a ratio of 2:3, we want to travel  $\frac{2}{5}$  of the previously mentioned distances, while still starting from A. So, we set of the following equations to find the x and y coordinates of the desired point:

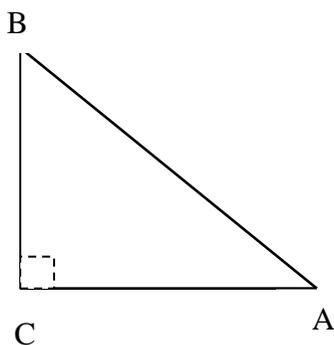
$x = 3 - 5\left(\frac{2}{5}\right) = 1$ ...Here the 3 is the starting x coordinate (from point A) and the “-” appears because we are moving left.

$y = -4 + 11\left(\frac{2}{5}\right) = \frac{2}{5}$ ...Here, the -4 is the starting y coordinate (from point B) and the “+” appears because we are moving up. So, the point that divides  $\overline{AB}$  into a ratio of 2:3 is  $\left(1, \frac{2}{5}\right)$ .

## TRIGONOMETRY

In a given right triangle the angles can be expressed in the following way:

$m\angle A = \theta^\circ$ ,  $m\angle B = (90 - \theta)^\circ$ ,  $m\angle C = 90^\circ$ . In other words, angles A and B are complimentary (they sum to 90 degrees).



### Key trig ratios using the triangle to the left:

1.  $\sin(m\angle A) = \sin(\theta) = \frac{BC}{AB} = \cos(90 - \theta) = \cos(m\angle B)$
2.  $\cos(m\angle A) = \cos(\theta) = \frac{AC}{AB} = \sin(90 - \theta) = \sin(m\angle B)$
3.  $\tan(\theta) = \frac{BC}{AC}$

The first two illustrate the **cofunction relationship** between sine and cosine. It is saying that the sine of a given angle will always equal the cosine of the complement to the given angle. In fact, "cosine" is short for "complementary sine".

Ex: If  $\sin((3x + 2)^\circ) = \cos((4x - 10)^\circ)$ , then what is the value of  $x$  to the nearest tenth?

Solution: To solve this problem, you must use the cofunction relationship as described above. Since sine of some angle is equal to cosine of another angle, this must mean that the two angles are complimentary. So, we can set up the equation:  $(3x + 2) + (4x - 10) = 90 \rightarrow 7x - 8 = 90 \rightarrow x = \frac{98}{7} = 14.0$ .

Key vocabulary for trigonometric application problems:

Angle of elevation: The measure of the angle which is above the horizontal

angle of depression: The measure of the angle which is below the horizontal

**Practice:** [http://www.algebra.com/Word/Word.aspx?file=Trigonometry\\_AnglesElevDepression.xml](http://www.algebra.com/Word/Word.aspx?file=Trigonometry_AnglesElevDepression.xml)

## LOCUS

### Remember:

- A locus is just a fancy term for a shape or set of points that satisfy a given condition or conditions. Think of it as coloring points.
- There are five basic loci: refer to here!  
<http://www.regentsprep.org/Regents/math/geometry/GL1/indexGL1.htm>
- You can also apply these loci to the coordinate plane—just sketch out the locus first, and then see how it interacts with the points and other objects on the coordinate plane.

- A compound locus is just a locus with multiple conditions: Sketch each separately, then see where your sketches overlap—this is what will satisfy the compound locus!
- The locus of points that are equidistant from a given point (focus) and given line (directrix) is a parabola.
  - If the vertex of the parabola is placed at (0,0) and the focus is located at (0, p), then the directrix is the line  $y = -p$ . The equation of such a parabola is  $y = \frac{x^2}{4p}$  OR  $x^2 = (4p)y$ .
  - Derivation of the formula: [Parabola with vertex at origin](#)
  - If you are deriving the equation of a parabola with the vertex at some point that is not the origin, then you must use the distance formula to find an expression for the distance from a point (x,y) on the parabola to the focus, and use the distance formula again to find an expression to find the distance from the point (x,y) on the parabola to the directrix.
    - Once you find both expressions, set them equal and solve for y. You can see an example in the link below.
  - Derivation with focus not at the origin: [Finding the equation of a parabola given a focus and directrix](#)

### Some Practice:

<http://www.regentsprep.org/Regents/math/geometry/GL1/PracLoc1.htm>  
<http://www.regentsprep.org/Regents/math/geometry/GL1/PracLoc2.htm>  
<http://www.regentsprep.org/Regents/math/geometry/GL1/PracLoc3.htm>  
<http://www.regentsprep.org/Regents/math/geometry/GL1/PracLoc4.htm>  
<http://www.regentsprep.org/Regents/math/geometry/GL1/PracLoc5.htm>

These are all links to problems on each of the five basic loci.

<http://www.regentsprep.org/Regents/math/geometry/GL3/PracLocC.htm> (Compound locus.)

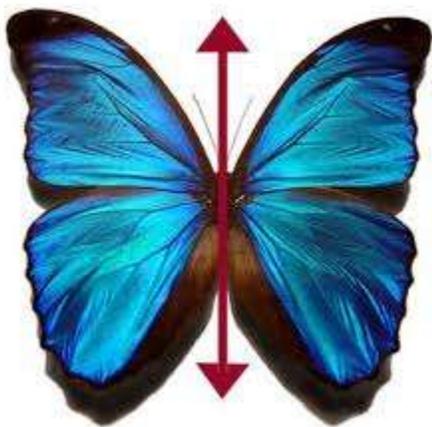
### Videos:

<http://www.youtube.com/watch?v=w-H1kdzLzVw> A good overview of the locus material, and some practice.

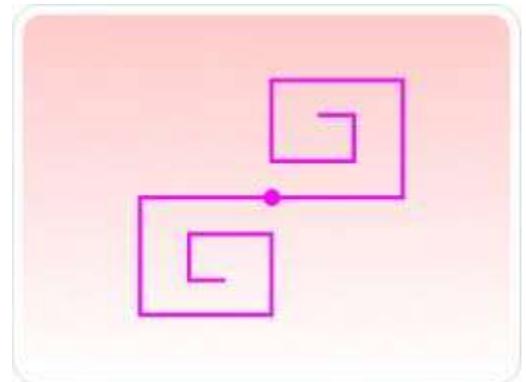
## TRANSFORMATIONS

- A transformation is just some change to the plane—it can even be zero change!
- You should know the different classes of transformations and you should know how to construct each:
  - Line reflections
    - **The line of reflection is the perpendicular bisector of the line segment with endpoints that are the pre-image of a point and the image of that point after the reflection.**
    - **This is an opposite isometry (a rigid motion that does not preserve orientation). It will map a figure onto a congruent figure.**
  - Point reflections
    - **The point of reflection is the midpoint of the line segment with endpoints that are the pre-image of a point and the image of that point after the reflection.**
    - **This is a direct isometry (a rigid motion that does preserve orientation). It will map a figure onto a congruent figure.**
  - Rotations (remember a positive angle is counterclockwise and negative is clockwise!)
    - **The rotocenter is equidistant from the pre-image of a point and the image of a point.**
    - **This is a direct isometry (a rigid motion that does preserve orientation). It will map a figure onto a congruent figure.**

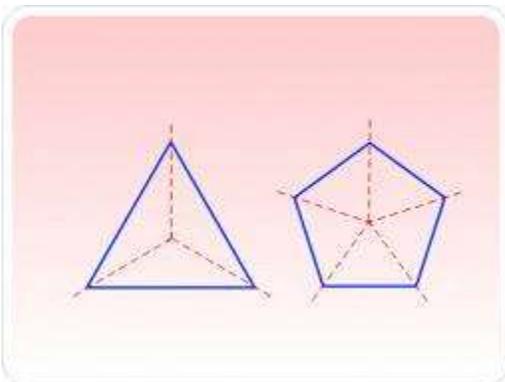
- Translations
  - All points of the pre-image move along a translation vector which defines the horizontal and vertical distances.
  - This is a direct isometry (a rigid motion that does preserve orientation). It will map a figure onto a congruent figure.
- Glide Reflections
  - A composition of a line reflection and a translation. The translation vector must be parallel to the line of reflection.
  - This is an opposite isometry (a rigid motion that does not preserve orientation). It will map a figure onto a congruent figure.
- Dilations
  - Both the x and y coordinates of all points on the pre-image will be multiplied by the scale factor k.
  - In general, this is not an isometry. It will map a figure onto a similar figure.
- Compositions: Remember that these are to be followed **right to left!** No exceptions!
- A transformation or a composition of transformations that maps a pre-image to a congruent image is known as an isometry (also known as rigid motion). **If there exists a series of rigid motion transformations that maps pre-image A onto image B, then A must be congruent to B.**
- **If there exists a series of similarity transformations (transformations including non-isometries) that maps pre-image A onto image B, then A must be similar to B.**
- Symmetry: An object is symmetrical if it has its own image after a transformation. As such:



**Line Symmetry**



**Point Symmetry  
(180° Rotational)**



**Rotational Symmetry**

(The triangle has  $120^\circ$  symmetry, since  $1/3$  of a turn will yield the identical image, and the pentagon has  $72^\circ$  symmetry, since  $1/5$  of a turn yields the same image as the original.)

**Resources:**

- <http://www.regentsprep.org/Regents/math/geometry/GT1a/indexGT1a.htm> Symmetries of all sorts
- <http://www.regentsprep.org/Regents/math/geometry/GT1/Prac1.htm> Practice with Line Reflections

<http://www.regentsprep.org/Regents/math/geometry/GT1/PtPrac.htm> Point Reflections

<http://www.regentsprep.org/Regents/math/geometry/GT2/PracT.htm> Translations

<http://www.regentsprep.org/Regents/math/geometry/GT4/PracRot.htm> Rotations

<http://www.regentsprep.org/Regents/math/geometry/GT3/PracCoD.htm> Dilations

<http://www.regentsprep.org/Regents/math/geometry/GT6/PracGlide.htm> Compositions

<http://www.regentsprep.org/Regents/math/geometry/GT5/RuleSheet.pdf> A sheet that summarizes all the rules of transformations.

**Video:**

<http://www.youtube.com/watch?v=I0d-z7Lauq0> A sampling of the transformations.