

Design of Time Truncated Single Acceptance Sampling Plan Based on Rayleigh Distribution as a Product Life using R Software

*O.J. Braimah

Department of Statistics
Abdul-Raheem College of Advanced Studies
Igbaja, Kwara State
(An Affiliate of Al-Hikmah University, PMB 1601) Ilorin, Nigeria
E-mail: ojbraimah2012@gmail.com
Phone: +2347036708840

P.A. Osanaiye

Department of Statistics
University of Ilorin
Ilorin, Kwara State, Nigeria

*Corresponding Author

ABSTRACT

This paper is aimed at carrying a reliability sampling plan assuming that lifetimes of items to be tested follow a compound Rayleigh distribution and the life test is terminated at a pre-fixed time (t). This type of sampling plan is used to minimize and save the test time in real life situations. Using R-Software, the minimum sample size required for ensuring the specified mean life at specified consumer's confidence level is been determined and tabulated. The operating characteristic curve values of the sampling plan was obtained by varying ratio of the true mean life (μ) to the specified life (μ_0). The values of minimum mean ratios were also obtained to minimize the producer's risk at the specified level. Numerical example were also discussed for illustrative purpose.

Key Words: R- Software, Mean life, Producer's Risk, Consumer's Risk, Operating Characteristic, Truncated Life Test.

Abbreviations

n - Sample size, c - Acceptance number, t - Termination time/Maximum test duration. p - Failure probability, p^* - Maximum Allowable Percent Defective (MAPD), μ - True Mean life, μ_0 - Specified Mean life,

$Pa = L(p) = \sum_{i=0}^c \binom{n}{i} p^i (1-p)^{n-i}$ - Probability of Acceptance, $\frac{t}{\mu_0}$ - Test termination ratio, α - Producer's risk, β -

Consumer's risk, σ - Scale Parameter, d - Number of defectives, t - Termination time, δ - Shape parameter.

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1. INTRODUCTION

Quality control has become one of the most important tools that distinguish different commodities in a global business market. Two important techniques for ensuring quality are the statistical product control in the form of acceptance sampling and statistical process control. Acceptance sampling is an important field of statistical quality control used to accept or reject products slated for inspection. This field was popularized by Dodge and Roming [2].

Dodge summarised the procedure of Acceptance Sampling as "a sample is randomly taken from a lot and the fate of the products depends on the information obtained from this sample. Acceptance sampling is concerned with inspection and decision making regarding lots of product and constitutes one of the oldest techniques in quality assurance.

1.1 Product Life Time

Life data refers to measurements of product life. Product life-time can either be measured in hours, miles, cycles or any other metric that applies to the period of successful operation of a particular product. Since time is a common measure of life, life data points are often called *times-to-failure*. There are different types of life products. With complete data, the exact time-to-failure for the unit is known (for example, a unit may fail at 100th hour of operation).

Statistical distributions have been formulated by various authors (statisticians, mathematicians and engineers) to mathematically model or represent certain behavior of products. The probability density function (*pdf*) and cumulative distribution function (*cdf*) is a mathematical function that describes the distribution.

Flowchart

The following is the operating procedure of single sampling for a life test.

This paper is aimed to determine an optimal sample size to provide desired levels of protection for both consumer as well as the producer when unit to be tested follow a compound Rayleigh distribution using R software.

2. MATERIALS AND METHOD

2.1 Operating Procedure of Single Acceptance Sampling Plan for Truncated Life Tests

1. A random sample is selected and put on the tests.
2. An experimenter runs this test for a pre-decided experiment time t_0 .
3. An acceptance number c is fixed for the experiment and the test is then truncated if more than c defectives (d) are recorded before the end of the experimental time t_0 , otherwise accept the lot (i.e if $c \leq d$).

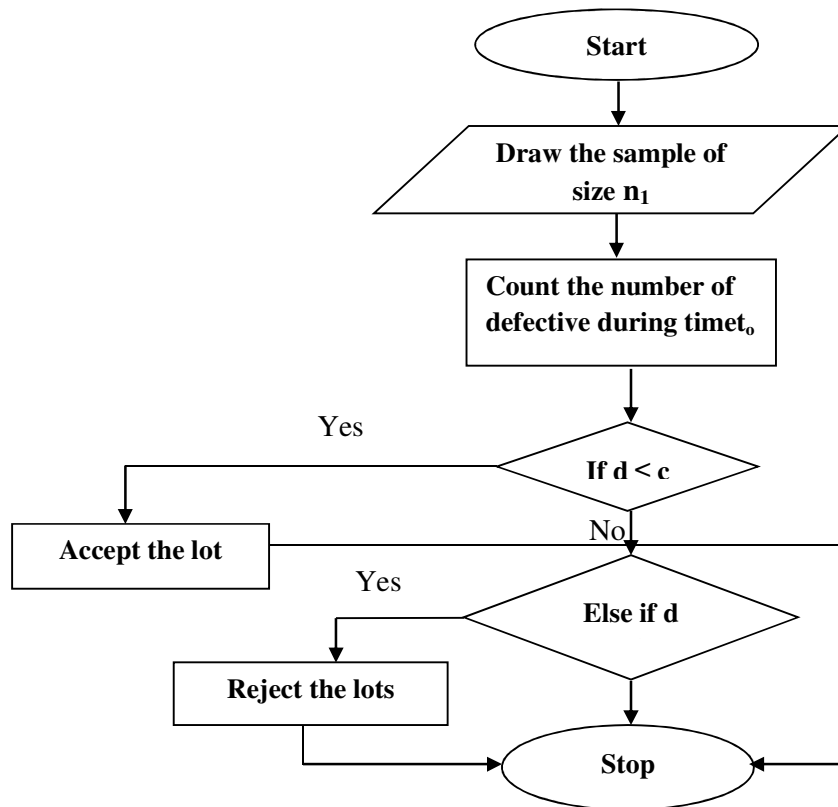


Fig. 1: Flow chart for truncated single acceptance sampling plan.

2.2 Rayleigh Distribution

The Rayleigh distribution as proposed by Rayleigh (1880) is widely applied in studies on reliability and survival analysis, engineering and communication technology. For example, Balakrishnan, Leiva and Lopez [2] identified the importance of the Rayleigh distribution in electro vacuum devices.

If we let X denotes a random variable arising from a Rayleigh distribution with probability distribution Function (pdf):

$$f(t, \mu) = 2\mu t e^{-\mu t^2}, \text{ where } t > 0 \text{ is the life time and } \mu > 0.$$

The cumulative distribution function (cdf) of the Rayleigh model is given by

$$F(t, \mu) = 1 - e^{-\frac{1}{2}(\frac{t}{\mu})^2} \tag{1}$$

The mean life time and variance of the Rayleigh model is given by

$$E(x) = \frac{1}{2} \sqrt{\frac{\pi}{\mu}} \text{ and } V(x) = \frac{4-\pi}{2} \sigma^2 \text{ respectively.}$$

It is of note that in life testing experiments, it is expected that the environmental conditions cannot remained same during the testing time. Therefore, it seems logical to treat the parameters involved in the life time model as random variables. In view of this, if the parameter μ is itself a random variable, then the distribution of lifetime of each item is a compound Rayleigh distribution.

The particular form of μ considered here is the gamma p.d.f.

$$G(\mu, \beta, \delta) = \frac{\beta^\delta \mu^{\delta-1} e^{-\beta\mu}}{\Gamma\delta}, \mu, \beta, \delta > 0$$

The parameters β and δ are scale and shape parameters respectively. The resulting compound Rayleigh distribution has p.d.f.

$$f(t, \beta, \gamma) = \int_0^\infty 2\mu t e^{-\mu t^2} \frac{\beta^\delta \mu^{\delta-1} e^{-\beta\mu}}{\Gamma\delta} d\mu$$

$$= 2\delta\beta^\delta t(\beta + t^2)^{-(\delta+1)}$$

Therefore, the cumulative distribution function of the compound Rayleigh model is given by

$$F(t, \beta, \delta) = 1 - \beta^\delta (\beta + t^2)^{-\delta}, t > 0 \tag{2}$$

If the mean life time and cumulative distribution function of the compound Rayleigh model are given by

$$\mu = (E) = \frac{\sqrt{\beta\pi}}{2\Gamma\delta} \Gamma(\delta - \frac{1}{2}) t > 0 \tag{3}$$

$$F(t; \beta, \delta) = 1 - \beta^\delta (\beta + t^2)^{-\delta}, t > 0 \tag{4}$$

Where $P = F(t; \beta, \delta)$ is the probability that an item fails before time t .

$$P = 1 - \beta^\delta (\beta + t^2)^{-\delta}$$

$$= 1 - \frac{1}{(1 + \frac{t^2}{\beta})^\delta} \tag{5}$$

From equation (3), we have

$$\mu = \frac{\sqrt{\beta\pi}\Gamma(\delta - \frac{1}{2})}{2\Gamma\delta}$$

$$\Rightarrow \sqrt{\beta} = \frac{2\mu\Gamma\delta}{\sqrt{\pi}\Gamma(\delta - \frac{1}{2})} \tag{6}$$

If we let the termination time be a multiple of the specified life μ_0 , i.e. $t_0 = \alpha\mu_0$ for a specified multiplier ' α '. The sampling plan then consists of the following parameters: the number of units ' n ', put on test, an acceptance number ' c ', and experiment time ratio $\frac{t_0}{\mu_0}$.

After putting the value of $\sqrt{\beta}$ from equation (6) and $t = \alpha\mu_0$ in equation (5), one gets

$$P = 1 - \frac{1}{1 + \left(\frac{\alpha\sqrt{\pi}\Gamma(\delta - \frac{1}{2})}{2\mu_0\Gamma\delta}\right)^2} \tag{7} \text{ Where } \alpha = \frac{t_0}{\mu_0}$$

2.2 Determination of Minimum Sample Size (n)

Suppose we fix the probability of accepting a bad lot (consumer's risk), i.e., the one for which the true mean life μ is below the specified mean life say μ_0 , not to exceed $1 - p^*$. Balakrishnan, Victor, and Lopez, [2] cite

Stephens (2001) that, assuming that the lot size (N) is large enough to be considered infinite (for example, $\frac{n}{N} \leq 0.10$), so that the binomial distribution can be used. Thus, the acceptance and non acceptance criteria for the lot are equivalent to the decisions of accepting or rejecting the hypothesis $\mu \geq \mu_0$. We want to find the minimum sample size (n) such that:

$$\sum_{i=0}^c \binom{n}{i} p^i (1-p)^{n-i} \leq 1-p^* \tag{8}$$

where $p = F(t, \mu)$ is monotonically increasing on $\frac{t}{\mu_0}$ and decreasing on μ , for fixed t , which is easy to establish for any distribution. Thus, $p = F(t, \mu)$ depends only on the ratio $\frac{t}{\mu_0}$. Hence, it is always sufficient to specify this ratio. Therefore, if the number of observed failures is at most c , from (8) we can establish with probability p^* that $F_T(\frac{t}{\mu}) \leq F_T(\frac{t}{\mu_0})$, which implies that $\mu \geq \mu_0$.

The choices of p^* and $\frac{t}{\mu_0}$ will give us the chances to compare results with those obtained by Authors like Gupta and Groll [3] Kantam *et al.* [4] and Baklizi and El Masri [1]

The minimum sample size (n) for various distributions were obtained using trial and error method, and using monotonicity property on n with respect to p .

If p is very small, and n is large, then binomial distribution is approximated to Poisson distribution with $\mu = \lambda = np$, [5] [6]. Therefore, (8) can be rewritten as:

$$\sum_{i=0}^c \frac{e^{-\lambda} \lambda^i}{i!} \leq 1-p^* \tag{9}$$

where $\lambda = np = nF(t; \mu)$.

Suppose $\sum_{i=0}^c \frac{e^{-\lambda} \lambda^i}{i!} = 1 - G_{c+1}(\lambda, 1)$

If $G_k(x, \mu)$ denotes the cumulative distribution function of a gamma distribution with the scale and shape parameters as k and μ respectively, i.e.

$$G_k(x, \mu) = \frac{\mu^k}{\Gamma(k)} \int_0^x t^{k-1} e^{-\mu t} dt \tag{10}$$

Therefore, if γ_{c+1, p^*} denotes the p^* percentage point of a standardized gamma variable with the shape parameter $c+1$, then

$$n \approx \left[\frac{\gamma_{c+1, p^*}}{1-(1-q)^{\frac{1}{2}}} \right] + 1 \tag{11}$$

where q = specified probability of failure, γ_{c+1, p^*} is the P^* percentage point at a standardized gamma variable with shape parameter $r = c+1$ or one-half time the P^* percentage of a χ^2 with $2c+2$ degree of freedom.

If $p = 1 - q$ is close to 0.5, the approximation for n was given to be

$$n \approx \left[\frac{(2c+1+PZ_{\frac{p^*}{2}}) \sqrt{\frac{1-(2c+1)^2}{(2c+1+PZ_{\frac{p^*}{2}})^2}}}{2(1-p)} \right] + 1 \tag{12}$$

This approximation was discussed by several authors and was modified by several authors including Muhammad *et al* (2012).

$$n \approx \left[\frac{\chi^2_{2c+2, p^*}}{2(1-p)} \right] + 1 \tag{13}$$

where p = Acceptance Probability (assuming either Binomial or Poisson distribution) using trial and error method.

We in this paper modified formula for minimum sample size is approximately given as:

$$n \approx \left[\frac{\gamma_{c+1, p^*}}{2F(c; \mu)} \right] + 1 \tag{14}$$

Now using the relationship between the gamma and χ^2 random variables, we in this paper then modified the formula as:

$$n \approx \left\lceil \frac{\chi^2_{2c+2, p^*}}{2F(t; \mu)} \right\rceil + 1 \tag{15}$$

Where $F(t; \mu)$ is the cumulative distribution function (cdf) of the assumed distribution.

Here χ^2_{2c+1, p^*} denotes the p^* percentage point of a χ^2 variable with $2c + 2$ degree of freedom. In order to ensure accuracy in our computation, a current software (GRETLL) is used to obtain the chi-square values.

2.3 Operating Characteristics (OC)

An acceptance sampling plan is best described in graphical terms on an operating characteristic curve (OC curve). An OC curve is a plot of the actual number of nonconforming units in a lot (expressed as a percentage) against the probability that the lot will be accepted when sampled according to the plan. The shape of an OC curve is determined primarily by sample size, n, and acceptance number, c, although there is a small effect of lot size, N. The OC function of the sampling plan $(n, c, \frac{t}{\mu_0})$ is the probability of accepting a lot and is given by

$$L(P) = \sum_{i=0}^c \binom{n}{i} p^i (1-p)^{n-i} \tag{16}$$

where $p = F(t, \mu)$ is considered as a function of μ , i.e., the lot quality parameter. It can be seen that the operating characteristic is an increasing function of μ . For given $P^*, \frac{t}{\mu_0}$, the choice of c and n is made on the basis of operating characteristics.

2.4 Minimum Ratio Value

In order to calculate the minimum required ratio values, the producer's risk is been considered. The producer's risk is the probability of rejection of the lot when $\mu \geq \mu_0$, it can be computed as follows;

$$PR(p) = P(\text{Rejecting a lot}) = 1 - P(\text{Accepting the Lot} / \mu \geq \mu_0)$$

$$= \sum_{i=c+1}^n \binom{n}{i} p^i (1-p)^{n-i} \tag{17}$$

For the given sampling plan and for a given value of the producer's risk, say γ , one may be interested in knowing the minimum value of $\frac{\mu}{\mu_0}$, that will ensure the producer's risk to beat most γ . The $\frac{\mu}{\mu_0}$, is the smallest quantity for which p satisfies the inequality:

$$PR(p) = \sum_{i=c+1}^n \binom{n}{i} p^i (1-p)^{n-i} \leq \gamma \tag{18}$$

The following algorithm is utilized to compute the Minimum Ratio values of acceptance sampling plan:

1. Set a given probability of accepting a bad lot $(1 - P^*)$.
2. Find the smallest sample size n for each predetermined value of acceptance number c satisfying the inequality (8).
3. For a given producer's risk α , find the smallest value of $\frac{\mu}{\mu_0}$ which satisfies the inequality

$$\sum_{i=0}^c \binom{n}{i} p^i (1-p)^{n-i} \geq 1 - \alpha \tag{19}$$

In order to highlight the practical aspects of the theoretical work, we need to discuss a numerical example. Assuming that the lifetimes of the a product/equipments follow compound Rayleigh distribution with known shape parameter δ , the numerical results are presented in tables 1 to 3. In table-1, we provide the minimum sample size/number required to affirm that the true mean life μ exceeds a specified mean life μ_0 with consumer's confidence level P^* and corresponding acceptance number c when $\delta = 1$. We presents the OC values in table 2 for different combinations of the values of probability P^* , mean ratio $\frac{\mu}{\mu_0}$ and experiment time ratio $\frac{t}{\mu_0}$ for $\delta = 1$ and $c = 0$. Table 3 presents the minimum ratios of the true mean life μ to the specified mean life μ_0 for the acceptance of the lot with producer's risk of 0.05.

For all numerical computations, R-software was used to develop the programs which has not been used by any author in this field of study.

3. RESULT AND ANALYSIS

3.1 Illustrative Example

For all numerical computations, a program has been developed in R-statistical software by simulating existing values. We discuss a numerical example for highlighting the real life aspects of the hypothetical developments. Suppose we assume that the lifetimes of the testing equipments follow compound Rayleigh distribution with known shape parameter, the numerical results are presented in Tables 1 to 3.

In Table 1, we have provided the minimum sample size required to state that the mean life exceeds at a mean value (μ_0) with consumer’s confidence level P^* and corresponding acceptance number c when $\delta = 1$.

Table 2 presents the Operating characteristics values for different combined probability values P^* , mean ratio $\frac{\mu}{\mu_0}$ and experiment time ratio $\frac{t}{\mu_0}$ for $\delta = 1$ and $c=0$.

$\frac{\mu}{\mu_0}$	1	2	4	6	8	10
OC Values	0.058	0.368	0.753	0.879	0.929	0.954

From the tabulated (OC function) values above, it is observed that if the true mean lifetime is double the specified lifetime ($\frac{\mu}{\mu_0} = 2$), then the producer’s risk will be $(1 - 0.3685=0.6315)$, while it is about 0.046 when the true mean lifetime is ten times of the specified mean life. Thus, the producer’s risk tends to decrease for the higher values of the mean ratios. Consequently, we can also get the smallest values of $\frac{\mu}{\mu_0}$ for various choices of c and $\frac{t}{\mu_0}$ from table 3 in order to claim that the producer’s risk is less than or equal to 0.05. The smallest value of $\frac{\mu}{\mu_0}$ is 9.57 for $c = 0$, $\frac{t}{\mu_0} = 0.8$ and $P^* = 0.90$. This means that the item should have a mean lifetime of at least 9.57 times of the specified mean life of 30 days in order that the lot will be accepted with the probability 0.95. Thus, this sampling plan can be utilized to maintain the quality of the product in terms of its average life according to the consumer’s standard at fixed producer’s risk.

The minimum ratio values for this sampling plan (compound Rayleigh) life time distributions is also computed for varying values of P^* , $\frac{t}{\mu_0}$, $\frac{\mu}{\mu_0}$ and δ are listed in tables 3.

3.2 Statistical Analysis

Suppose the control statistician is interested in establishing a sampling plan to ensure that the mean lifetime is at least say 30 days with confidence level of 90%, if he wishes to stop the experiment at time $t = 24$ days. Therefore, for an acceptance number $c = 0$, the required sample size (n) corresponding to the values of $P^* = 0.90$, $\frac{t}{\mu_0} = 0.8$ is 5 (Table 1). Thus, we can say that if 5 units have to be put on test and no more than 0 failures out of 5 is observed during 24 days, then the control statistician can assert that the mean lifetime of the product is at least 30 days with a confidence level of 0.90. For the sampling plan ($n = 5$, $c = 0$, $\frac{t}{\mu_0} = 0.8$) and confidence level $P^* = 0.90$ under this distribution, the OC values can be found from Table-2 and are as follows:



Table 1: Minimum sample size n to be tested for a time t in order to assert with probability P^* acceptance number c (when shape parameter $\delta = 1$).

P^*	c	$\frac{t}{\mu_0}$							
		0.628	0.942	1.257	1.571	2.356	3.141	3.972	4.713
0.75	0	3	3	3	3	3	3	4	4
	1	5	5	4	4	4	4	5	5
	2	7	7	6	6	5	5	4	4
	3	9	9	7	7	6	6	6	6
	4	11	11	8	8	7	7	7	7
	5	12	12	9	9	8	8	8	8
	6	14	14	10	10	8	8	9	9
	7	16	16	11	11	9	9	10	10
	8	18	18	12	12	10	10	11	11
	9	19	19	13	13	11	11	11	11
0.90	0	3	3	3	3	3	3	4	4
	1	5	5	3	4	4	4	5	5
	2	7	7	4	4	4	4	3	3
	3	9	9	6	7	6	6	6	6
	4	11	11	7	8	7	7	7	7
	5	12	12	8	9	8	8	8	8
	6	14	14	9	10	8	8	9	9
	7	16	16	10	11	9	9	9	10
	8	18	18	11	12	10	10	11	11
	9	19	19	12	13	11	11	11	11
0.95	0	3	3	14	3	3	3	4	4
	1	5	5	3	4	4	4	5	5
	2	7	7	6	6	5	5	4	4
	3	9	9	6	7	6	6	6	6
	4	11	11	8	8	7	7	7	7
	5	12	12	9	9	8	8	8	8
	6	14	14	10	10	8	8	9	9
	7	16	16	11	11	9	9	10	10
	8	18	18	12	12	10	10	11	11
	9	19	19	13	13	11	11	11	11
0.99	0	3	3	3	3	3	3	4	4
	1	5	5	4	4	4	4	5	5
	2	7	7	6	6	5	5	4	4
	3	9	9	7	7	6	6	6	6
	4	11	11	8	8	7	7	7	7
	5	12	12	9	9	8	8	8	8
	6	14	14	10	10	8	8	9	9
	7	16	16	11	11	9	9	10	10
	8	18	18	12	12	10	10	11	11
	9	19	19	13	13	11	11	11	11
10	21	21	14	14	12	12	12	12	



Table 2: Value of operating characteristic function of the sampling plans for compound Rayleigh Distribution (when $\delta=1$ and $c=0$)

P^*	$\frac{t}{\mu_0}$						
		1	2	4	6	8	10
75	0.4	0.189	0.625	0.885	0.946	0.969	0.980
	0.6	0.148	0.548	0.850	0.929	0.959	0.973
	0.8	0.150	0.514	0.828	0.917	0.952	0.969
	1	0.083	0.382	0.750	0.875	0.927	0.952
	1.5	0.152	0.419	0.742	0.866	0.920	0.947
	2	0.092	0.288	0.618	0.784	0.866	0.910
	2.5	0.060	0.206	0.509	0.700	0.805	0.866
	3	0.043	0.153	0.418	0.618	0.742	0.818
90	0.4	0.097	0.517	0.843	0.926	0.957	0.972
	0.6	0.078	0.448	0.805	0.907	0.946	0.965
	0.8	0.058	0.368	0.753	0.879	0.929	0.954
	1	0.083	0.383	0.750	0.875	0.927	0.952
	1.5	0.023	0.175	0.551	0.750	0.846	0.897
	2	0.092	0.288	0.618	0.784	0.866	0.910
	2.5	0.061	0.206	0.509	0.700	0.805	0.866
	3	0.043	0.153	0.418	0.618	0.742	0.818
95	0.4	0.035	0.390	0.783	0.896	0.940	0.961
	0.6	0.042	0.367	0.763	0.885	0.933	0.956
	0.8	0.023	0.264	0.686	0.842	0.907	0.939
	1	0.024	0.237	0.650	0.819	0.892	0.929
	1.5	0.023	0.175	0.551	0.750	0.846	0.897
	2	0.008	0.083	0.382	0.615	0.750	0.828
	2.5	0.003	0.042	0.259	0.490	0.649	0.750
	3	0.043	0.153	0.418	0.618	0.742	0.818
99	0.4	0.009	0.267	0.710	0.858	0.917	0.946
	0.6	0.006	0.201	0.649	0.822	0.895	0.931
	0.8	0.008	0.189	0.624	0.806	0.885	0.924
	1	0.006	0.146	0.563	0.767	0.859	0.907
	1.5	0.004	0.073	0.409	0.650	0.779	0.850
	2	0.008	0.083	0.382	0.615	0.750	0.828
	2.5	0.004	0.042	0.259	0.490	0.649	0.750
	3	0.002	0.023	0.175	0.382	0.551	0.669

Table 3: Minimum ratio of true value μ to specified μ_0 for the acceptability of a lot with producer's risk 0.05

P*	C	$\alpha = \left(\frac{t}{\mu}\right)$								
		0.628	0.942	1.257	1.571	2.356	3.141	3.972	4.713	
0.75	0	6.188	7.177	7.797	9.746	10.271	13.694	17.118	20.541	
	1	3.038	3.276	3.821	3.971	5.956	5.854	7.318	8.781	
	2	2.363	2.666	2.955	3.252	4.097	5.462	5.142	6.171	
	3	2.067	2.244	2.571	2.575	3.264	4.352	4.147	4.976	
	4	1.898	2.116	2.169	2.463	2.78	3.706	4.632	4.269	
	5	1.787	1.941	2.06	2.165	2.884	3.275	4.094	3.793	
	6	1.709	1.818	1.981	2.142	2.603	2.963	3.704	4.444	
0.90	0	7.327	8.297	9.57	9.746	14.619	13.694	17.118	20.541	
	1	3.699	3.97	4.368	4.777	5.956	7.941	9.926	8.781	
	2	2.829	3.049	3.269	3.694	4.877	5.462	6.828	8.193	
	3	2.391	2.645	2.79	3.214	3.862	4.352	5.44	6.528	
	4	2.192	2.32	2.517	2.711	3.274	3.706	4.632	5.559	
	5	2.026	2.19	2.34	2.575	3.247	3.846	4.094	4.912	
	6	1.909	2.031	2.214	2.476	2.926	3.471	3.704	4.444	
0.95	0	8.762	9.282	11.062	11.962	14.619	19.492	24.365	20.541	
	1	3.988	4.557	4.853	5.46	7.165	7.941	9.926	11.911	
	2	2.995	3.222	3.555	4.087	4.877	5.462	6.828	8.193	
	3	2.567	2.766	2.992	3.214	3.862	5.149	5.44	6.528	
	4	2.326	2.508	2.673	2.937	3.694	4.365	4.632	5.559	
	5	2.17	2.341	2.467	2.756	3.247	3.846	4.807	4.912	
	6	2.031	2.16	2.321	2.626	3.213	3.471	4.339	4.444	
0.99	0	10.371	11.752	12.376	13.827	17.943	19.492	24.365	29.238	
	1	4.751	5.077	5.698	6.066	8.19	9.553	11.941	11.911	
	2	3.515	3.839	4.065	4.443	5.541	6.503	8.128	9.754	
	3	2.935	3.203	3.527	3.739	4.82	5.826	6.436	7.723	
	4	2.61	2.846	3.094	3.342	4.067	4.925	5.456	6.548	
	5	2.433	2.616	2.814	3.084	3.863	4.33	5.412	5.768	
	6	2.255	2.455	2.616	2.902	3.473	3.902	4.877	5.206	

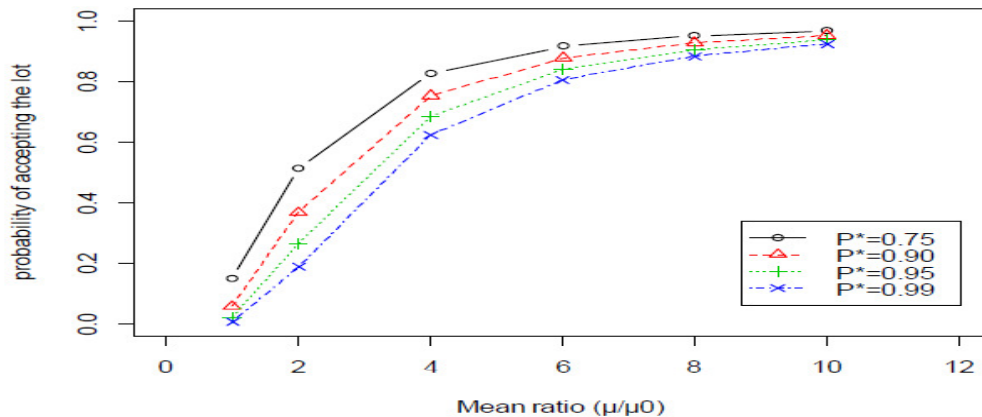


Fig. 1: OC values vs. mean ratio $\frac{\mu}{\mu_0}$ with Experiment time ratio (α) = 0.8

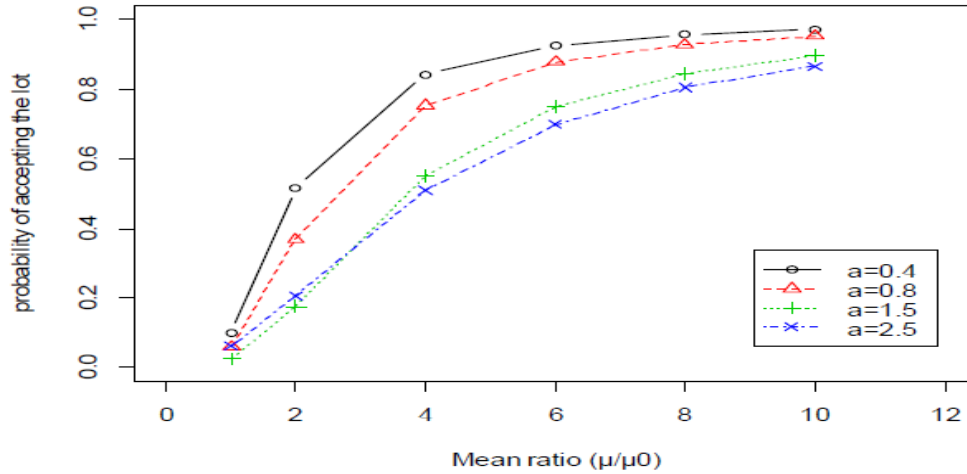


Fig. 2: OC values vs. mean ratio $\frac{\mu}{\mu_0}$ with confidence level (P^*) = 0.90

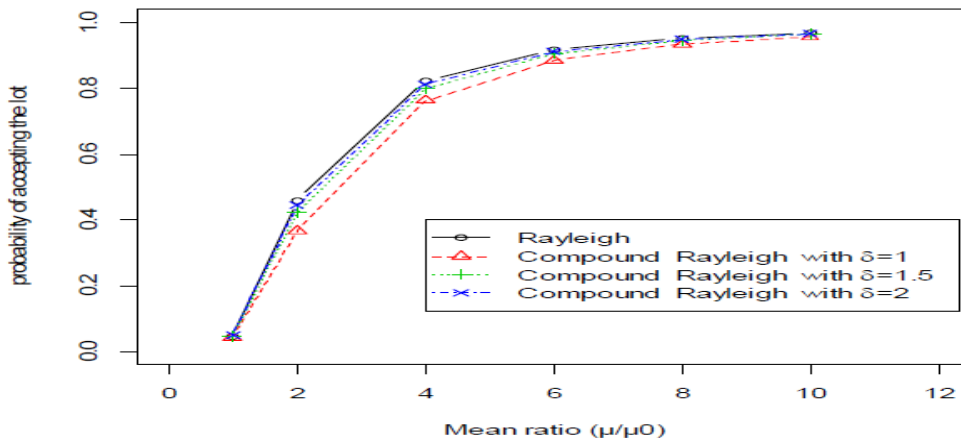


Fig. 3: OC values vs. mean ratio $\frac{\mu}{\mu_0}$ with confidence level (P^*) = 0.95 and Experiment time ratio (a) = 0.6

4. CONCLUSION

Acceptance sampling plans have been particularly used in industries to decide whether a certain lot of manufactured or purchased items satisfy a pre specified quality. The following are the general interpretations of the numerical findings.

4.1 Interpretation of required sample size:

1. The minimum sample size for zero acceptance sampling plan need to be very low as compare to one and more acceptance number for any combination of confidence level and experiment time ratio.
2. For fixed confidence level and acceptance number, when we increase experiment time ratio the minimum sample size required to reach the decision tend to low.

3. For fixed acceptance number and varying experiment time ratio, the minimum sample size required to reach a decision tend to uniformly high as we increase the confidence level.

4.2 Interpretation of OC curve behaviour

1. Varying mean ratio and fix experiment time ratio, the probability of acceptance is uniformly decreasing with an increase in the confidence level.
2. For experiment time ratio and fixed confidence level, the probability of acceptance tends to increase as we increase the mean ratio.

3. On comparing the operating characteristics (OC) curve corresponding to Rayleigh and Compound Rayleigh lifetime distributions, it is observed that for any fixed value of consumer's confidence level and experiment time ratio, the OC curve has uniformly low values in case of compound Rayleigh distribution as compare to Rayleigh distribution. Therefore, for Rayleigh distribution, the probabilities of acceptance of lot are higher as compared to the compound Rayleigh distribution.

Conclusively, the results obtained in this study are more efficient compared to the result obtained in earlier studies who used FORTRAN and other software because the minimum sample sizes and experimental termination time obtained in table 1 and 3 respectively are smaller while the probability of acceptance values in table 2 are higher. These table values give a maximum protection to both the producer and consumer.

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