Edexcel A level Mathematics Vectors

Section 1: Vectors in three dimensions

Section test

1. The vectors **a** and **b** are given by

$$\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix}$$

Find the vector $3\mathbf{b} - 2\mathbf{a}$.

2. The magnitude of the vector $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ is

(a) 3
(b) $\sqrt{29}$
(c) $\sqrt{11}$
(d) $\sqrt{20}$

3. The vectors $\begin{pmatrix} 2 \\ p \\ -3 \end{pmatrix}$ and $\begin{pmatrix} q \\ -2 \\ 6 \end{pmatrix}$ are parallel. Find the values of p and q.

4. The points P and Q have position vectors $\overrightarrow{OP} = \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$.

The point T lies on PQ such that the ratio of PT to TQ is 2:1.
Find the coordinates of T.

5. Points A, B and C have coordinates (2, -1, 0), (3, 2, -4) and (-1, 5, 2) respectively.
Find the value of $\left|\overrightarrow{AB}\right|$.

Find the vector $\overrightarrow{CB}$.
Find the unit vector in the direction CA.

6. The points P (1, 2, -3), Q (2, 0, 3), R (-1, 1, 0) and S form a parallelogram PQRS.
Find the coordinates of S and the lengths PQ and QR.

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Section test solutions

1. $3\underline{b} - 2\underline{a} = 3\begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix} - 2\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 9 \\ -6 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 13 \\ -8 \end{pmatrix}$

2. $\sqrt{2^2 + (-3)^2 + 4^2} = \sqrt{29}$

3. Since the vectors are parallel, $k\begin{pmatrix} 2 \\ p \\ -3 \end{pmatrix} = \begin{pmatrix} q \\ -2 \\ 6 \end{pmatrix}$ for some value k.

$$\begin{pmatrix} 2k \\ pk \\ -3k \end{pmatrix} = \begin{pmatrix} q \\ -2 \\ 6 \end{pmatrix}$$

$-3k = 6 \quad \Rightarrow k = -2$

$pk = -2 \quad \Rightarrow -2p = -2 \quad \Rightarrow p = 1$

$2k = q \quad \Rightarrow q = -4$

4. $\overrightarrow{PT} = \frac{2}{3}\overrightarrow{PQ}$

$\overrightarrow{OT} - \overrightarrow{OP} = \frac{2}{3}(\overrightarrow{OQ} - \overrightarrow{OP})$

$\overrightarrow{OT} = \overrightarrow{OP} + \frac{2}{3}\overrightarrow{OQ} - \frac{2}{3}\overrightarrow{OP}$

$= \frac{2}{3}\overrightarrow{OQ} + \frac{1}{3}\overrightarrow{OP}$

$= \frac{2}{3}\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} \frac{7}{3} \\ 1 \\ -\frac{2}{3} \end{pmatrix}$

So the coordinates of T are $\left(\frac{7}{3}, 1, -\frac{2}{3}\right)$

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5. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$

$\left|\overrightarrow{AB}\right| = \sqrt{1^2 + 3^2 + (-4)^2} = \sqrt{26}$

$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ -6 \end{pmatrix}$

$\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \\ -2 \end{pmatrix}$

$\left|\overrightarrow{CA}\right| = \sqrt{3^2 + (-6)^2 + (-2)^2} = \sqrt{49} = 7$

So a unit vector in the direction CA is $\begin{pmatrix} \frac{3}{7} \\ -\frac{6}{7} \\ -\frac{2}{7} \end{pmatrix}$.

6. Since PQRS is a parallelogram, $\overrightarrow{QP} = \overrightarrow{RS}$

$\overrightarrow{OP} - \overrightarrow{OQ} = \overrightarrow{OS} - \overrightarrow{OR}$

$\overrightarrow{OS} = \overrightarrow{OP} - \overrightarrow{OQ} + \overrightarrow{OR}$

$= \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 3 \\ -6 \end{pmatrix}$

So the coordinates of S are (-2, 3, -6).

$PQ = \sqrt{(2-1)^2 + (0-2)^2 + (3-(-3))^2} = \sqrt{1+4+36} = \sqrt{41}$

$QR = \sqrt{(-1-2)^2 + (1-0)^2 + (0-3)^2} = \sqrt{9+1+9} = \sqrt{19}$