

Section 2: Numerical integration

Section test

1. A function f has the following values:

x	0	0.5	1	1.5	2
$f(x)$	2.4	3.6	5.1	7.4	9.8

Using the trapezium rule, find an approximate value for the integral of the function f between $x = 0$ and $x = 2$. Give your answer to 1 decimal place.

2. A function f has the following values:

x	10	11	12	13	14	15	16
$f(x)$	10.1	4.5	5.4	10.7	14.7	16.6	17.5

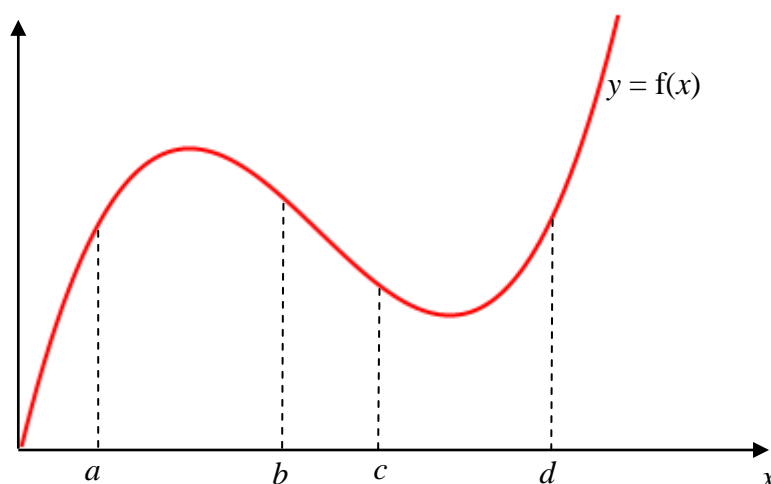
Using the trapezium rule, find an approximate value for the integral of the function f between $x = 10$ and $x = 16$. Give your answer to 1 decimal place.

3. The value of the integral $\int_0^2 \sqrt{1+x^3} \, dx$, correct to four decimal places, is 3.2413.

Using the trapezium rule with two strips, find the approximate value of the integral to four decimal places, and the percentage error in this use of the trapezium rule.

Using the trapezium rule with four strips, find the approximate value of the integral to four decimal places, and the percentage error in this use of the trapezium rule.

4. The diagram below shows the curve $y = f(x)$.



Edexcel A level Num methods 2 section test solns

The trapezium rule is used to find approximations for $A_1 = \int_a^b f(x) dx$ and

$A_2 = \int_c^d f(x) dx$. Which of the following statements is correct?

- (a) The trapezium rule gives an underestimate for both A_1 and A_2
- (b) The trapezium rule gives an overestimate for A_1 and an underestimate for A_2
- (c) The trapezium rule gives an underestimate for A_1 and an overestimate for A_2
- (d) The trapezium rule gives an overestimate for both A_1 and A_2

5. This question is about the integral $\int_1^{1.6} \frac{e^{2x}}{x^2} dx$.

Using the trapezium rule with $h = 0.3$, find an approximate value for the integral, to 4 decimal places.

Using the trapezium rule with $h = 0.2$, find an approximate value for the integral, to 4 decimal places.

Using the trapezium rule with $h = 0.1$, find an approximate value for the integral, to 4 decimal places.

6. This question is about the integral $\int_0^2 \ln(1+x^2) dx$.

Find an underestimate of the value of this integral using rectangles of width 0.5.

Find an overestimate of the value of this integral using rectangles of width 0.5.

Edexcel A level Num methods 2 section test solns

Solutions to section test

1. $h = 0.5$

$$\begin{aligned} A &\approx \frac{1}{2}h[f_0 + f_4 + 2(f_1 + f_2 + f_3)] \\ &= \frac{1}{2} \times 0.5 [2.4 + 9.8 + 2(3.6 + 5.1 + 7.4)] \\ &= 11.1 \end{aligned}$$

2. $h = 1$

$$\begin{aligned} A &\approx \frac{1}{2}h[f_0 + f_7 + 2(f_1 + \dots + f_6)] \\ &= \frac{1}{2} \times 1 [10.1 + 17.5 + 2(4.5 + 5.4 + 10.7 + 14.7 + 16.6)] \\ &= 65.7 \end{aligned}$$

3.

x	0	1	2
$f(x)$	1	$\sqrt{2}$	3

$h = 1$

$$\begin{aligned} A &\approx \frac{1}{2}h[f_0 + f_2 + 2f_1] \\ &= \frac{1}{2} \times 1 [1 + 3 + 2\sqrt{2}] \\ &= 3.4142 \text{ (4 d.p.)} \end{aligned}$$

$$\begin{aligned} \text{Percentage error} &= \frac{3.4142 - 3.2413}{3.2413} \times 100 \\ &= 5.33\% \end{aligned}$$

x	0	0.5	1	1.5	2
$f(x)$	1	$\sqrt{1.125}$	$\sqrt{2}$	$\sqrt{4.375}$	3

$h = 0.5$

$$\begin{aligned} A &\approx \frac{1}{2}h[f_0 + f_4 + 2(f_1 + f_2 + f_3)] \\ &= \frac{1}{2} \times 0.5 [1 + 3 + 2(\sqrt{1.125} + \sqrt{2} + \sqrt{4.375})] \\ &= 3.2833 \text{ (4 d.p.)} \end{aligned}$$

$$\begin{aligned} \text{Percentage error} &= \frac{3.2833 - 3.2413}{3.2413} \times 100 \\ &= 1.30\% \end{aligned}$$

Edexcel A level Num methods 2 section test solns

4. For A_1 , the top lines of the trapezia all lie below the curve, so the trapezium rule gives an underestimate.
For A_2 , the top lines of the trapezia all lie above the curve, so the trapezium rule gives an overestimate.

5.

x	1	1.3	1.6
$f(x)$	7.389056	7.966709	9.583020

$$h = 0.3$$

$$\begin{aligned} A &\approx \frac{1}{2}h[f_0 + f_2 + 2f_1] \\ &= \frac{1}{2} \times 0.3 [7.389056 + 9.583020 + 2 \times 7.966709] \\ &= 4.9358 \text{ (4 d.p.)} \end{aligned}$$

x	1	1.2	1.4	1.6
$f(x)$	7.389056	7.654984	8.390126	9.583020

$$h = 0.2$$

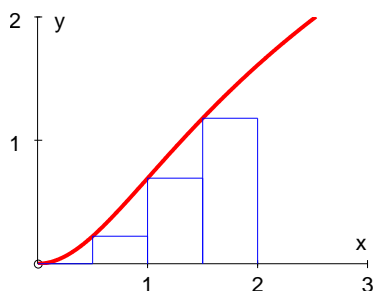
$$\begin{aligned} A &\approx \frac{1}{2}h[f_0 + f_3 + 2(f_1 + f_2)] \\ &= \frac{1}{2} \times 0.2 [7.389056 + 9.583020 + 2(7.654984 + 8.390126)] \\ &= 4.9062 \text{ (4 d.p.)} \end{aligned}$$

x	1	1.1	1.2	1.3	1.4	1.5	1.6
$f(x)$	7.389056	7.458689	7.654984	7.966709	8.390126	8.926905	9.583020

$$h = 0.1$$

$$\begin{aligned} A &\approx \frac{1}{2}h[f_0 + f_6 + 2(f_1 + \dots + f_5)] \\ &= \frac{1}{2} \times 0.1 \left[7.389056 + 9.583020 \right. \\ &\quad \left. + 2(7.458689 + 7.654984 + 7.966709 + 8.390126 + 8.926905) \right] \\ &= 4.8883 \text{ (4 d.p.)} \end{aligned}$$

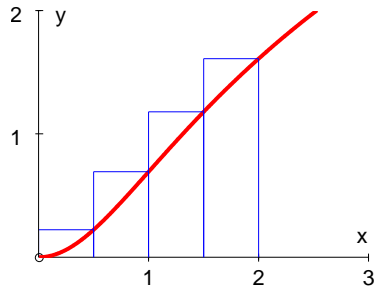
6.



Edexcel A level Num methods 2 section test solns

$$\begin{aligned}\text{Area of rectangles} &= 0.5(\ln 1 + \ln(1 + 0.5^2) + \ln 2 + \ln(1 + 1.5^2)) \\ &= 1.05\end{aligned}$$

An underestimate for the value of the integral is 1.05.



$$\begin{aligned}\text{Area of rectangles} &= 0.5(\ln(1 + 0.5^2) + \ln 2 + \ln(1 + 1.5^2) + \ln 5) \\ &= 1.85\end{aligned}$$

An overestimate for the value of the integral is 1.85.