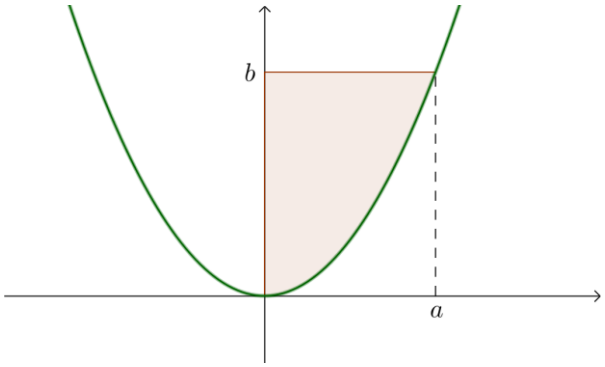
Section 1: Finding areas

Section test

1. Which of the expressions below give the area of the shaded region?



(a) $\int_0^a y\,\mathrm{d}x$

(b) $\int_0^a x\,\mathrm{d}y$

(c) $\int_0^b y\,\mathrm{d}x$

(d) $\int_0^b x\,\mathrm{d}y$

(e) $ab-\int_0^a y\,\mathrm{d}x$

(f) $ab-\int_0^a x\,\mathrm{d}y$

(g) $ab-\int_0^b y\,\mathrm{d}x$

(h) $ab-\int_0^b x\,\mathrm{d}y$

2. Find the area in square units enclosed by the curve $y=x^2$, the line $y=4$ and the y-axis.

3. Find the area in square units enclosed by the curve $y=\sqrt{4-x}$ and the coordinate axes.

4. Find the area in square units enclosed by the curve $y=x^2-2x+4$ and the line $y=4$.

5. Find the area in square units enclosed by the curve $y=x^2+2$ and the line $y=3x+2$.

6. Find the area in square units enclosed between the curves $y=x^2+3x+1$ and $y=2x^2+x+1$.

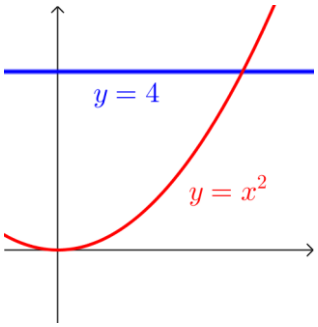
7. Find the area in square units enclosed between the curves $y=x^3$ and $y=x^3-x^2+4$.

Edexcel A level Maths Integration 2 section test

Section test solutions

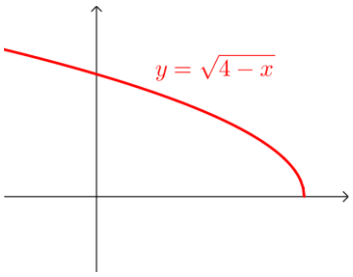
1. If the area is calculated by considering the area between the curve and the y-axis, the area is given by $\int_o^b x\,dy$.
   Alternatively, the area can be calculated by subtracting the area between the curve and the y-axis from the area of a rectangle, so in this case the area is given by $ab - \int_o^a y\,dx$.

2.



$$\begin{aligned}\text{Area} &= \int_o^4 x\,dy \\ &= \int_o^4 y^{\frac{1}{2}}\,dy \\ &= \left[\tfrac{2}{3}y^{\frac{3}{2}}\right]_o^4 \\ &= \tfrac{2}{3}\times 8 \\ &= \tfrac{16}{3}\end{aligned}$$

3.



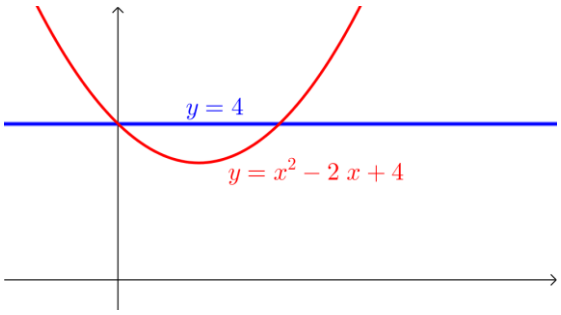
$y = \sqrt{4-x} \Rightarrow y^2 = 4 - x \Rightarrow x = 4 - y^2$

When $x = 0, y = 2$

Edexcel A level Maths Integration 2 section test

$$\text{Area} = \int_0^2 x\,dy$$
$$= \int_0^2 (4 - y^2)\,dy$$
$$= \left[4y - \tfrac{1}{3}y^3\right]_0^2$$
$$= 8 - \tfrac{8}{3}$$
$$= \tfrac{16}{3}$$

4.



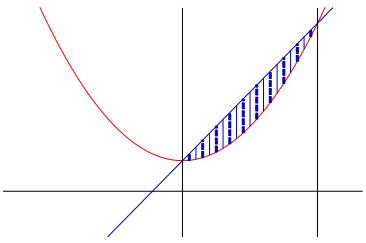
At intersection points, $x^2 - 2x + 4 = 4$

$$x^2 - 2x = 0$$
$$x(x-2) = 0$$
$$x = 0 \text{ or } 2$$

$$\text{Area} = \int_0^2 \left(4 - (x^2 - 2x + 4)\right)dx$$
$$= \int_0^2 \left(-x^2 + 2x\right)dx$$
$$= \left[-\tfrac{1}{3}x^3 + x^2\right]_0^2$$
$$= -\tfrac{8}{3} + 4$$
$$= \tfrac{4}{3}$$

5. At intersection points $x^2 + 2 = 3x + 2$

$$x^2 - 3x = 0$$
$$x(x-3) = 0$$
$$x = 0 \text{ or } x = 3$$

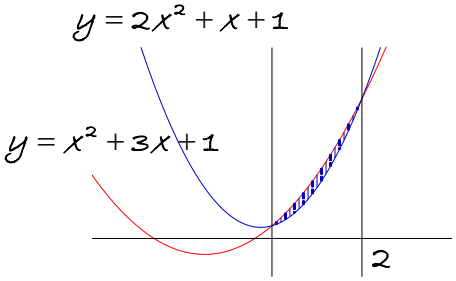


$$\text{Area} = \int_0^3 \left(3x + 2 - (x^2 + 2)\right)dx$$
$$= \int_0^3 \left(3x - x^2\right)dx$$
$$= \left[\tfrac{3}{2}x^2 - \tfrac{1}{3}x^3\right]_0^3$$
$$= \left(\tfrac{27}{2} - 9\right) - 0$$
$$= 4.5$$

Edexcel A level Maths Integration 2 section test

6. At intersection points $x^2+3x+1=2x^2+x+1$

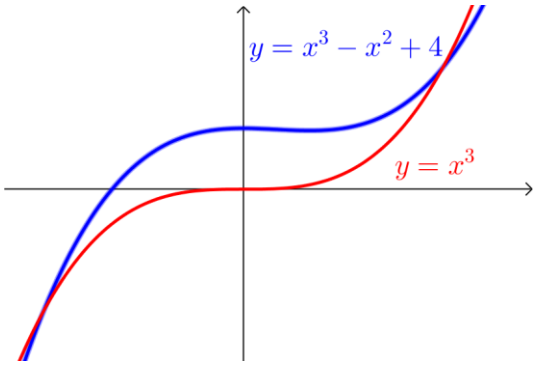
$$0=x^2-2x$$
$$0=x(x-2)$$
$$x=0 \text{ or } x=2$$



Area under $y=x^2+3x+1$ $=\int_0^2 (x^2+3x+1)\,dx$

$$=\left[\tfrac{1}{3}x^3+\tfrac{3}{2}x^2+x\right]_0^2$$
$$=\left(\tfrac{8}{3}+6+2\right)-0$$
$$=10\tfrac{2}{3}$$

7.



At intersection points, $x^3=x^3-x^2+4$

$$x^2-4=0$$
$$(x-2)(x+2)=0$$
$$x=2 \text{ or } -2$$

Area $=\int_{-2}^{2}\left(x^3-x^2+4-x^3\right)dx$

$$=\int_{-2}^{2}\left(-x^2+4\right)dx$$
$$=\left[-\tfrac{1}{3}x^3+4x\right]_{-2}^{2}$$
$$=-\tfrac{8}{3}+8-\left(\tfrac{8}{3}-8\right)$$
$$=\tfrac{32}{3}$$