

Section 3: Further techniques for integration

Crucial points

1. Remember when to use logarithms in integration

Some students make the mistake of wrongly using logarithms when integrating inverse powers of linear functions of x , especially in the context of integration using partial fractions.

Example Find $\int \frac{1}{(x+1)^2} dx$.

✗ **Wrong** $\int \frac{1}{(x+1)^2} dx = \ln(x+1)^2 + c$.

✓ **Right** $\int \frac{1}{(x+1)^2} dx = \int (x+1)^{-2} dx = \frac{(x+1)^{-1}}{-1} + c = -\frac{1}{x+1} + c$.

2. Make sure you use the correct integration technique when dealing with polynomial fractions

Small changes in the function you are integrating can produce quite different results. For example: $\int \frac{2x+1}{x^2+x-2} dx = \ln|x^2+x-2| + c$, as the numerator of the fraction is the derivative of the denominator.

$$\begin{aligned} \text{However: } \int \frac{3x}{x^2+x-2} dx &= \int \frac{3x}{(x+2)(x-1)} dx \\ &= \int \left(\frac{2}{x+2} + \frac{1}{x-1} \right) dx \\ &= 2 \ln|x+2| + \ln|x-1| + c \end{aligned}$$

using partial fractions