Edexcel A level Mathematics Integration

Section 2: Integration by substitution

Section test

1. $\int \dfrac{1}{\sqrt{2x-1}}\,dx =$

   (a) $\frac{1}{2}\sqrt{2x-1}+c$ (b) $\sqrt{2x-1}+c$

   (c) $2\sqrt{2x-1}+c$ (d) $-(2x-1)^{-\frac{3}{2}}+c$

2. Find the value of $\displaystyle\int_{1}^{2}(3x-1)^{3}\,dx$.

3. Find the integral of e^{5x} with respect to x.

4. Find the exact value of $\displaystyle\int_{1}^{2}\dfrac{1}{3x-1}\,dx$.

5. The integral of $\sin\frac{1}{2}x$ with respect to x is

   (a) $-2\cos\frac{1}{2}x+c$ (b) $2\cos\frac{1}{2}x+c$

   (c) $\cos\frac{1}{2}x+c$ (d) $-\frac{1}{2}\cos\frac{1}{2}x+c$

6. $\int \dfrac{x^{2}}{(1-x^{3})^{2}}\,dx =$

   (a) $\dfrac{1}{3(1-x^{3})}+c$ (b) $\dfrac{1}{(1-x^{3})}+c$

   (c) $-\dfrac{1}{3(1-x^{3})}+c$ (d) $-\dfrac{1}{(1-x^{3})}+c$

7. $\int \dfrac{e^{-3x}}{1-e^{-3x}}\,dx =$

   (a) $\frac{1}{3}\ln\left|1-e^{-3x}\right|$ (b) $\ln\left|1-e^{-3x}\right|$

   (c) $-\frac{1}{3}\ln\left|1-e^{-3x}\right|$ (d) $3\ln\left|1-e^{-3x}\right|$

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8. $\int \cos^3 x \sin x \, dx =$

(a) $\frac{1}{4}\cos^4 x + c$

(b) $-\frac{1}{2}\cos^2 x + c$

(c) $-\frac{1}{4}\cos^4 x + c$

(d) $-\frac{1}{8}\cos^4 x \sin^2 x + c$

9. $\int (e^x - e^{-x})^2 \, dx =$

(a) $\frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} + c$

(b) $2e^{2x} - 2e^{-2x} + c$

(c) $\frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x} + c$

(d) $2e^{2x} - 2x - 2e^{-2x} + c$

10. Evaluate $\int_0^1 \frac{x}{x^2+1} \, dx$, giving your answer in exact form.

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Section test solutions

1. Let $u = 2x - 1 \Rightarrow \dfrac{du}{dx} = 2 \Rightarrow du = 2\ dx \Rightarrow dx = \tfrac{1}{2}du$

$$\int \frac{1}{\sqrt{2x-1}}dx = \int \frac{1}{\sqrt{u}} \times \tfrac{1}{2}du$$
$$= \int \tfrac{1}{2}u^{-\frac{1}{2}}du$$
$$= \tfrac{1}{2} \times 2u^{\frac{1}{2}} + c$$
$$= u^{\frac{1}{2}} + c$$
$$= \sqrt{2x-1} + c$$

2. Let $u = 3x - 1 \Rightarrow \dfrac{du}{dx} = 3 \Rightarrow du = 3\ dx \Rightarrow dx = \tfrac{1}{3}du$

When $x = 1,\ u = 3 \times 1 - 1 = 2$
$\qquad x = 2,\ u = 3 \times 2 - 1 = 5$

$$\int_1^2 (3x-1)^3 dx = \int_2^5 u^3 \times \tfrac{1}{3}du$$
$$= \left[\tfrac{1}{12}u^4\right]_2^5$$
$$= \tfrac{1}{12}\left(5^4 - 2^4\right)$$
$$= 50.75$$

3. $\displaystyle \int e^{kx}dx = \frac{1}{k}e^{kx} + c$

$\Rightarrow \displaystyle \int e^{5x}dx = \tfrac{1}{5}e^{5x} + c$

4. $\displaystyle \int_1^2 \frac{1}{3x-1}\ dx = \tfrac{1}{3}\int_1^2 \frac{3}{3x-1}\ dx$
$$= \left[\tfrac{1}{3}\ln(3x-1)\right]_1^2$$
$$= \tfrac{1}{3}[\ln 5 - \ln 2]$$
$$= \tfrac{1}{3}\ln 2.5$$

5. $\displaystyle \int \sin kx\ dx = -\frac{1}{k}\cos kx + c$

$\Rightarrow \displaystyle \int \sin \tfrac{1}{2}x\ dx = -2\cos \tfrac{1}{2}x + c$

6. Let $u = 1 - x^3 \implies \dfrac{du}{dx} = -3x^2 \implies dx = -\dfrac{1}{3x^2}du$

$$\int \frac{x^2}{\left(1 - x^3\right)^2} = \int \frac{x^2}{u^2} \times -\frac{1}{3x^2} du$$

$$= \int -\tfrac{1}{3} u^{-2} du$$

$$= -\tfrac{1}{3} \times -u^{-1} + c$$

$$= \frac{1}{3u} + c$$

$$= \frac{1}{3(1 - x^3)} + c$$

7. Let $u = 1 - e^{-3x} \implies \dfrac{du}{dx} = 3e^{-3x} \implies dx = \dfrac{1}{3e^{-3x}}du$

$$\int \frac{e^{-3x}}{1 - e^{-3x}} dx = \int \frac{e^{-3x}}{u} \times \frac{1}{3e^{-3x}} du$$

$$= \int \tfrac{1}{3} \times \frac{1}{u} du$$

$$= \tfrac{1}{3} \ln|u| + c$$

$$= \ln\left|1 - e^{-3x}\right| + c$$

8. Let $u = \cos x \implies \dfrac{du}{dx} = -\sin x \implies dx = -\dfrac{1}{\sin x}du$

$$\int u^3 \sin x \left(-\frac{1}{\sin x} \right) du = -1 \int u^3 du$$

$$= -\tfrac{1}{4} u^4 + c$$

$$= -\tfrac{1}{4} \cos^4 x + c$$

9. $\displaystyle \int \left(e^x - e^{-x} \right)^2 dx = \int \left(e^{2x} + e^{-2x} - 1 - 1 \right) dx$

$$= \tfrac{1}{2} e^{2x} - \tfrac{1}{2} e^{-2x} - 2x + c$$

10. Let $u = x^2 + 1 \implies \dfrac{du}{dx} = 2x \implies dx = \dfrac{1}{2x} du$

When $x = 0, u = 1$

When $x = 1, u = 2$

$$\int_0^1 \left(\frac{x}{x^2+1} \right) dx = \int_1^2 \frac{x}{u} \times \frac{1}{2x} \, du$$

$$= \frac{1}{2} \int_1^2 \frac{1}{u} \, du$$

$$= \frac{1}{2} \left[\ln u \right]_1^2$$

$$= \frac{1}{2} \ln 2$$