Section 2: Integration by substitution

Crucial points

1. **Remember to substitute for dx in the integral when integrating by substitution**

   Example Find $\int x(2x-1)^4 dx$

   ✗ ***Wrong*** Let $u = 2x-1 \Rightarrow x = \frac{1}{2}(u+1)$

   $$\Rightarrow \int x(2x-1)^4 = \int \frac{u+1}{2} \times u^4$$
   $$= \frac{1}{2}\int u^5 + u^4$$
   $$= \frac{1}{12}u^6 + \frac{1}{10}u^5 + c$$

   ✓ ***Right*** Let $u = 2x-1 \Rightarrow x = \frac{1}{2}(u+1)$

   also $\frac{du}{dx} = 2 \Rightarrow dx = \frac{1}{2}du$

   $$\Rightarrow \int x(2x-1)^4 dx = \int \frac{u+1}{2} u^4 \times \tfrac{1}{2} du$$
   $$= \frac{1}{4}\int (u^5 + u^4) du$$
   $$= \frac{1}{24}u^6 + \frac{1}{20}u^5 + c$$

2. **Remember to change the limits of a definite integral when making a substitution**

   When you change the variable in an integration (from x to u say) by making a substitution, you must change the limits of the integration from values of x to the equivalent values of u.

3. **Don't mix up the derivatives and integrals of $\sin x$ and $\cos x$.**

   The derivative of $\sin x$ is $\cos x$, the integral is $-\cos x$

   Example Find $\int \sin x \, dx$

   ✗ ***Wrong*** $\int \sin x \, dx = \cos x + c$

   ✓ ***Right*** $\int \sin x \, dx = -\cos x + c$

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4. Be careful with signs when substituting values into definite integrals

Example Evaluate $\int_0^{\pi/3} \sin x \, dx$

✗ ***Wrong***

$$\int_0^{\pi/3} \sin x \, dx = \left[-\cos x\right]_0^{\pi/3} = -\cos\tfrac{\pi}{3} + \cos 0 = -\tfrac{1}{2}$$

✓ ***Right***

$$\int_0^{\pi/3} \sin x \, dx = \left[-\cos x\right]_0^{\pi/3} = -\cos\tfrac{\pi}{3} + \cos 0 = -\tfrac{1}{2} + 1 = \tfrac{1}{2}$$

5. Make sure that you never integrate across an asymptote when evaluating an integral.

Example Find $\int_1^3 \frac{1}{(x-2)^2} \, dx$

✗ ***Wrong***

$$\int_1^3 \frac{1}{(x-2)^2} \, dx = \left[-(x-2)^{-1}\right]_1^3$$
$$= -1 - 1$$
$$= -2$$

✓ ***Right***

The integral is not defined, as it represents an area between $x = 1$ and $x = 3$; but the integrand is not defined when $x = 2$.