Section 1: Solving equations numerically

Section test

1. How many of roots of the equation $3x^3 - 12x^2 + 8x - 1 = 0$ are there in the range $0 \le x \le 5$?

2. There is a root of $x^3 + 3x - 2 = 0$ between 0 and 1. Give an interval of width 0.1 in which the root lies.

3. The equation $y = x^3 - 4x - 1$ has three roots: α, which is in the interval (-2, -1); β, which is in the interval (-1, 0); and γ, which is in the interval (2, 3).
   The equation is to be solved using the method of iteration.

   The first rearrangement used is $x = \mathrm{g}(x) = \dfrac{x^3 - 1}{4}$.

   Using first approximation x = -1, the iterations
   (a) diverge (b) converge to root α
   (c) converge to root γ (d) converge to root β

   Using first approximation x = 3, the iterations
   (a) diverge (b) converge to root α
   (c) converge to root γ (d) converge to root β

   Another rearrangement used is $x = \sqrt[3]{4x+1}$.
   Using first approximation x = -1, the iterations
   (a) diverge (b) converge to root α
   (c) converge to root γ (d) converge to root β

   Using first approximation x = 3, the iterations
   (a) diverge (b) converge to root α
   (c) converge to root γ (d) converge to root β

4. The equation $\mathrm{e}^x = 3x + 2$ has two roots. The equation can be rearranged into the forms $x = \dfrac{\mathrm{e}^x - 2}{3}$ or $x = \ln(3x+2)$. Use the iteration method to find the two roots, correct to 4 decimal places.

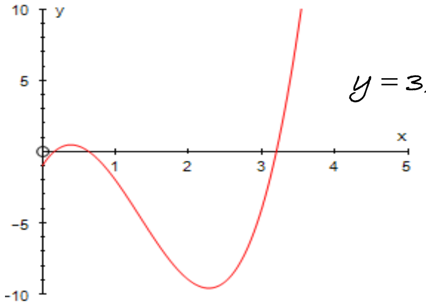
5. The equation $x^3 + x^2 + 1 = 0$ has a root in the interval (-2, -1).
   Using a first approximation of $x = -1.5$, use the Newton-Raphson method to find a second and third approximation to 6 d.p, and give the value of the root correct to 4 d.p.

6. The equation $\mathrm{e}^x - x - 2 = 0$ has a root in the interval (1, 2).
   Using a first approximation of $x = 1$, use the Newton-Raphson method to find a second and third approximation to 6 d.p, and give the value of the root correct to 4 d.p.

Edexcel A level Num methods 1 section test solns

Solutions to section test

1.



$y = 3x^3 - 12x^2 + 7x - 1$

From the graph, the equation $3x^3 - 12x^2 + 7x - 1 = 0$ has three roots.

2. Let $f(x) = x^3 + 3x - 2$

$f(0.5) = 0.5^3 + 3 \times 0.5 - 2 = -0.375$

$f(0.6) = 0.6^3 + 3 \times 0.6 - 2 = 0.016$

There is a change of sign, and therefore a root, between 0.5 and 0.6.

3. $g(x) = \dfrac{x^3 - 1}{4}$

$x_0 = -1$	$g(x_0) = -0.5$
$x_1 = -0.5$	$g(x_1) = -0.28125$
$x_2 = -0.28125$	$g(x_2) = -0.255562$
$x_3 = -0.255562$	$g(x_3) = -0.254173$
$x_4 = -0.254173$	$g(x_4) = -0.254105$
$x_5 = -0.254105$	$g(x_5) = -0.254102$
$x_6 = -0.254102$	$g(x_6) = -0.254102$

The iterations are converging towards the root in the interval (-1, 0), which is the root β.

$g(x) = \dfrac{x^3 - 1}{4}$

$x_0 = 3$	$g(x_0) = 6.5$
$x_1 = 6.5$	$g(x_1) = 68.40625$
$x_2 = 68.40625$	$g(x_2) = 80025.06$

The iterations are diverging.

$g(x) = \sqrt[3]{4x + 1}$

$x_0 = -1$	$g(x_0) = -1.442250$
$x_1 = -1.442250$	$g(x_1) = -1.683226$

$x_2 = -1.683226$	$g(x_2) = -1.789747$
$x_3 = -1.789747$	$g(x_3) = -1.833031$
$x_4 = -1.833031$	$g(x_4) = -1.850048$
$x_5 = -1.850048$	$g(x_5) = -1.856654$
$x_6 = -1.856654$	$g(x_6) = -1.859206$
$x_7 = -1.859206$	$g(x_7) = -1.860190$
$x_8 = -1.860190$	$g(x_8) = -1.860569$

The iterations are converging towards the root in the interval (-2, -1), which is the root α.

$g(x) = \sqrt[3]{4x+1}$

$x_0 = 3$	$g(x_0) = 2.351335$
$x_1 = 2.351335$	$g(x_1) = 2.183159$
$x_2 = 2.183159$	$g(x_2) = 2.135060$
$x_3 = 2.135060$	$g(x_3) = 2.120898$
$x_4 = 2.120898$	$g(x_4) = 2.116692$
$x_5 = 2.116692$	$g(x_5) = 2.115439$
$x_6 = 2.115439$	$g(x_6) = 2.115066$
$x_7 = 2.115066$	$g(x_7) = 2.114955$

The iterations are converging towards the root in the interval (2, 3), which is the root γ.

4. Let $f(x) = e^x - 3x - 2$

$f(-1) = 1.36$

$f(0) = -1$

$f(1) = -2.28$

$f(2) = -0.61$

$f(3) = 9.09$

The smaller root is between -1 and 0, and the larger root is between 2 and 3.

Using the rearrangement $x = \dfrac{e^x - 2}{3}$

$g(x) = \dfrac{e^x - 2}{3}$

$x_0 = 0$	$g(x_0) = -0.666667$
$x_1 = -0.666667$	$g(x_1) = -0.495528$
$x_2 = -0.495528$	$g(x_2) = -0.463584$
$x_3 = -0.463584$	$g(x_3) = -0.456992$
$x_4 = -0.456992$	$g(x_4) = -0.455605$
$x_5 = -0.455605$	$g(x_5) = -0.455311$

$x_6 = -0.455311 \qquad g(x_6) = -0.455250$

$x_7 = -0.455250 \qquad g(x_7) = -0.455237$

The root appears to be -0.4552 correct to 4 decimal places.

Check: $f(-0.45515) = -0.000197$

$f(-0.45525) = 0.000039$

so the root is -0.4552 correct to 4 decimal places.

Using the rearrangement $x = \ln(3x+2)$

$g(x) = \ln(3x+2)$

$x_0 = 2 \qquad g(x_0) = 2.079442$

$x_1 = 2.079442 \qquad g(x_1) = 2.108797$

$x_2 = 2.108797 \qquad g(x_2) = 2.119430$

$x_3 = 2.119430 \qquad g(x_3) = 2.123254$

$x_4 = 2.123254 \qquad g(x_4) = 2.124625$

$x_5 = 2.124625 \qquad g(x_5) = 2.125117$

$x_6 = 2.125117 \qquad g(x_6) = 2.125293$

$x_7 = 2.125293 \qquad g(x_7) = 2.125356$

$x_8 = 2.125356 \qquad g(x_8) = 2.125379$

The root appears to be 2.1254 correct to 4 decimal places.

Check: $f(2.12535) = -0.00022$

$f(2.12545) = 0.00032$

so the root is 2.1254 correct to 4 decimal places.

5. $f(x) = x^3 + x^2 + 1$

$f'(x) = 3x^2 + 2x$

$$x_{n+1} = x_n - \frac{x^3 + x^2 + 1}{3x^2 + 2x}$$

$x_1 = -1.5$

$x_2 = -1.466667$

$x_3 = -1.465572$

Repeating the iterations suggests that the root is -1.4656 to 4 d.p.

Check: $f(-1.46555) > 0$

$f(-1.46565) < 0$

so the root is -1.4656 to 4 d.p.

6. $f(x) = e^x - x - 2$

$f'(x) = e^x - 1$

$$x_{n+1} = x_n - \frac{e^x - x - 2}{e^x - 1}$$

$x_1 = 1$

$x_2 = 1.163953$

$x_3 = 1.146421$

Repeating the iterations suggests that the root is 1.1462 to 4 d.p.

Check: $f(1.14615) < 0$

$f(1.14625) > 0$

so the root is 1.1462 to 4 d.p.