Section 1: Introduction to differential equations

Crucial points

1. **Make sure you integrate with respect to the correct variable**

Example Solve $\frac{dy}{dx} = 2y$

✗ ***Wrong*** $y = \int 2y \, dx = y^2 + c$

✓ ***Right*** $\int \frac{1}{y} dy = \int 2dx \Rightarrow \ln y = 2x + c$

2. **Remember to add the arbitrary constant after integrating, and be careful not to make algebraic errors in dealing with it**

Example Solve $\frac{dy}{dx} = \frac{x^2}{2y}$

✗ ***Wrong*** $\int 2y \, dy = \int x^2 dx$

$\Rightarrow \quad y^2 = \frac{1}{3}x^3 + c$

$\Rightarrow \quad y = \sqrt{\frac{1}{3}x^3} + c$

✓ ***Right*** $\int 2y \, dy = \int x^2 dx$

$\Rightarrow \quad y^2 = \frac{1}{3}x^3 + c$

$\Rightarrow \quad y = \sqrt{\frac{1}{3}x^3 + c}$

The '+c' must be inside the square root sign here – the 'green' family of solutions is quite different to the 'red' one above!