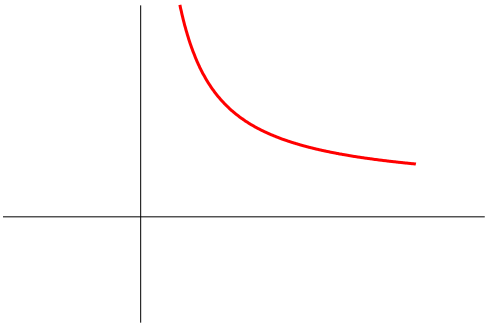
Section 1: The shape of curves

Section test

1. The diagram below shows part of the graph of $y = \mathrm{f}(x)$



For the interval shown on the graph, which of the following statements is true?

(a) $\mathrm{f}'(x) < 0$ and $\mathrm{f}''(x) > 0$ (b) $\mathrm{f}'(x) < 0$ and $\mathrm{f}''(x) < 0$

(c) $\mathrm{f}'(x) > 0$ and $\mathrm{f}''(x) > 0$ (d) $\mathrm{f}'(x) > 0$ and $\mathrm{f}''(x) < 0$

2. For the curve $y = x^3 - 2x^2 - 3x + 1$, at which of the following points is the curve convex?

(a) $x = 0$ (b) $x = 1$

(c) $x = -1$ (d) $x = 2$

3. Find the values of x for which the curve $y = 2x^3 - 3x^2 + 4x - 1$ is concave.

4. Which of the following statements are true?

(a) Where $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$, the gradient of the curve is positive and getting more positive

(b) Where $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$, the curve is concave

(c) Where $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$, the tangent to the curve lies below the curve

5. The point A on the graph of $y = \mathrm{f}(x)$ has x-coordinate a.

If, when $x = a$, $\frac{\mathrm{d}y}{\mathrm{d}x} < 0$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$, which of these statements could be true?

(a) The point A is a local maximum

(b) The point A is a local minimum

(c) The point A is a stationary point of inflection

(d) The point A is a non-stationary point of inflection

(e) The point A is a stationary point but could be a local maximum, a local minimum or a stationary point of inflection

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6. Which of the following are points of inflection for the curve $y = x^5 - 5x^4 + 15x + 4$?
   (a) $x = 0$
   (b) $x = 1$
   (c) $x = 3$
   (d) $x = -3$

7. Find the coordinates of the point of inflection for the curve $y = -x^3 + 3x^2 - 1$.

8. The curve $y = x^3 + px^2 + qx - 4$ has a stationary point of inflection at $x = 2$. Find the values of p and q.

9. The point A on the graph of $y = \mathrm{f}(x)$ has x-coordinate a.
   If, when $x = a$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$, which of these statements is true?
   (a) The point A is a local maximum
   (b) The point A is a local minimum
   (c) The point A is a stationary point of inflection
   (d) The point A is a non-stationary point of inflection
   (e) The point A is a stationary point but could be a local maximum, a local minimum or a stationary point of inflection

10. Find the stationary point(s) of the curve $y = x^4 + 4x^3 + 7$.

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Solutions to section test

1. The graph is decreasing so $\frac{dy}{dx} < 0$.

   The gradient of the graph is becoming less negative, so $\frac{d^2y}{dx^2} > 0$.

2. $y = x^3 - 2x^2 - 3x + 1$

   $\frac{dy}{dx} = 3x^2 - 4x - 3$

   $\frac{d^2y}{dx^2} = 6x - 4$

   The curve is convex when $\frac{d^2y}{dx^2} > 0$. This is the case when $x = 1$ and when $x = 2$.

3. $y = 2x^3 - 3x^2 + 4x - 1$

   $\frac{dy}{dx} = 6x^2 - 6x + 4$

   $\frac{d^2y}{dx^2} = 12x - 6$

   The curve is concave when $\frac{d^2y}{dx^2} < 0$

   $12x - 6 < 0$

   $2x < 1$

   $x < \frac{1}{2}$

4. (a) is false – although it is true that if the gradient of the curve is positive and getting more positive, then $\frac{d^2y}{dx^2} > 0$, it could also be the case that the gradient is negative and getting less negative
   (b) is false – it is convex
   (c) is true

5. The correct answer is that the point A is a non-stationary point of inflection.

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6. $y = x^5 - 5x^4 + 15x + 4$

$\frac{dy}{dx} = 5x^4 - 20x^3 + 15$

$\frac{d^2y}{dx^2} = 20x^3 - 60x^2 = 20x^2(x-3)$

The points at which $\frac{d^2y}{dx^2} = 0$ are $x = 0$ and $x = 3$.

However at $x = 0$ the second derivative does not change sign, so the only point of inflection is $x = 3$.

7. $y = -x^3 + 3x^2 - 1$

$\frac{dy}{dx} = -3x^2 + 6x$

$\frac{d^2y}{dx^2} = -6x + 6$

At point of inflection $\frac{d^2y}{dx^2} = 0 \Rightarrow -6x + 6 = 0 \Rightarrow x = 1$

When $x = 1, y = -1 + 3 - 1 = 1$

so the point of inflection is $(1, 1)$

8. $y = x^3 + px^2 + qx - 4$

$\frac{dy}{dx} = 3x^2 + 2px + q$

$\frac{d^2y}{dx^2} = 6x + 2p$

Since there is a stationary point of inflection at $x = 2$, both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ must be zero at $x = 2$.

When $x = 2$, $\frac{d^2y}{dx^2} = 12 + 2p$ so $p = -6$

When $x = 2$, $\frac{dy}{dx} = 12 + 4p + q = 12 - 24 + q$ so $q = 12$

9. The correct answer is that the point A is a stationary point but could be a local maximum, a local minimum or a stationary point of inflection

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10. $y = x^4 + 4x^3 + 7$

$\frac{dy}{dx} = 4x^3 + 12x^2$

At stationary points, $4x^3 + 12x^2 = 0$

$4x^2(x+3) = 0$

$x = 0$ or -3

When $x = 0$, $y = 7$ so there is a stationary point at (0, 7)

When $x = -3$, $y = 81 - 108 + 7 = -20$ so there is a stationary point at (-3, -20)

When x = -1, $\frac{dy}{dx} = 4 + 12 > 0$

When x = 1, $\frac{dy}{dx} = -4 + 12 > 0$

so (0, 7) is a point of inflection.

When x = -4, $\frac{dy}{dx} = -256 + 192 < 0$

so at (-3, -20) gradient goes from negative to positive, so it is a minimum point.