Section 3: Implicit differentiation

Section test

1. The derivative of u^3 with respect to v is:

(a) $3u^2\dfrac{\mathrm{d}u}{\mathrm{d}v}$ (b) $3u^2\dfrac{\mathrm{d}v}{\mathrm{d}u}$

(c) $3u^2$ (d) $3v^2$

2. The derivative of xy^3 with respect to x is:

(a) $3xy^2\dfrac{\mathrm{d}y}{\mathrm{d}x}$ (b) $3xy^2\dfrac{\mathrm{d}y}{\mathrm{d}x}+y^3$

(c) y^3 (d) $3xy^2$

3. The derivative of e^{-2y} with respect to x is:

(a) $\mathrm{e}^{-2y}\dfrac{\mathrm{d}y}{\mathrm{d}x}$ (b) $-2\mathrm{e}^{-2y}$

(c) $-2\mathrm{e}^{-3y}\dfrac{\mathrm{d}y}{\mathrm{d}x}$ (d) $-2\mathrm{e}^{-2y}\dfrac{\mathrm{d}y}{\mathrm{d}x}$

4. Find the gradient of the tangent to the circle $x^2+y^2+2x-4y=0$ at the point (0, 0).

5. Given that $\cos y = x$, then $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ =

(a) $-\dfrac{1}{\sin y}$ (b) $\dfrac{1}{\sin y}$

(c) $\sin y$ (d) $-\sin y$

6. Given that $x^2-2xy+y^3=10$, then $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ =

(a) $\dfrac{x^2}{2x-3y^2}$ (b) $\dfrac{x^2-2y+3y^2}{2x}$

(c) $\dfrac{2(y-x)}{3y^2-2x}$ (d) $\dfrac{2(x-y)}{3y^2-2x}$

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7. Given that $y = \dfrac{x^3}{(1-2x)}$, then $\dfrac{1}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x}$ =

   (Hint: take logarithms of both sides and then differentiate implicitly).

(a) $\dfrac{3}{x} - \dfrac{1}{1-2x}$ (b) $\dfrac{3}{x} + \dfrac{2}{1-2x}$

(c) $\dfrac{3}{x} - \dfrac{2}{1-2x}$ (d) $\dfrac{3}{x} + \dfrac{1}{1-2x}$

8. Given that $\tan y = \ln 2x$, then $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ =

(a) $\dfrac{1}{2x\sec^2 y}$ (b) $\dfrac{1}{2x}\cos^2 y$

(c) $\dfrac{1}{x}\cos^2 y$ (d) $\dfrac{1}{2x}\cos^2 x$

9. Find the gradient of the curve $y^2 = \dfrac{1}{1+\cos x}$ at the point $\left(\frac{\pi}{2}, 1\right)$.

10. A curve has equation $x^2 + xy + y^2 - 2y = 3$.

    Find the equation of the tangent to the curve at the point (1, 2).

    Find the equation of the normal to the curve at the point (0, -1).

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Solutions to section test

1. $\frac{d}{dv}\left(u^3\right) = 3u^2 \frac{du}{dv}$

2. $u = x \Rightarrow \frac{du}{dx} = 1$

$v = y^3 \Rightarrow \frac{dv}{dx} = 3\frac{dy}{dx}y^2$

$\frac{d}{dx}\left(xy^3\right) = \left(y^3\right)(1) + (x)\left(3\frac{dy}{dx}y^2\right)$

$= y^3 + 3xy^2\frac{dy}{dx}$

3. $\frac{d}{dx}\left(e^{-2y}\right) = -2\frac{dy}{dx}e^{-2y}$

$= -2e^{-2y}\frac{dy}{dx}$

4. $x^2 + y^2 + 2x - 4y = 0$

Differentiating: $2x + 2y\frac{dy}{dx} + 2 - 4\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left(2y - 4\right) = -2x - 2$

$\frac{dy}{dx} = \frac{-2x - 2}{2y - 4}$

At the point $(0,0)$: $\frac{dy}{dx} = \frac{-2}{-4} = \frac{1}{2}$

5. $\cos y = x$

Differentiating: $-\sin y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = -\frac{1}{\sin y}$

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6. $x^2 - 2xy + y^3 = 10$

$$2x - \left(2y + 2x\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0$$

$$2x\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 2x - 2y$$

$$\frac{dy}{dx}\left(2x - 3y^2\right) = 2x - 2y$$

$$\frac{dy}{dx} = \frac{2x - 2y}{2x - 3y^2}$$

$$= \frac{2\left(x - y\right)}{2x - 3y^2}$$

$$= \frac{2\left(y - x\right)}{3y^2 - 2x}$$

7. $y = \dfrac{x^3}{1 - 2x}$

$$\ln y = \ln\left(\frac{x^3}{1 - 2x}\right) = 3\ln x - \ln(1 - 2x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{x} + \frac{2}{1 - 2x}$$

8. $\tan y = \ln 2x = \ln 2 + \ln x$

Differentiating: $\sec^2 y\dfrac{dy}{dx} = \dfrac{1}{x}$

$$\frac{dy}{dx} = \frac{1}{x\sec^2 y} = \frac{1}{x}\cos^2 y$$

9. $\dfrac{dy}{dx}2y = \dfrac{\sin x}{(1 + \cos x)^2}$

$$\frac{dy}{dx} = \frac{\sin x}{2y(1 + \cos x)^2}$$

At the point $\left(\dfrac{\pi}{2}, 1\right)$: $\dfrac{dy}{dx} = \dfrac{\sin\left(\frac{\pi}{2}\right)}{2(1)\left(1 + \cos\left(\frac{\pi}{2}\right)\right)^2} = \dfrac{1}{2(1)} = \dfrac{1}{2}$

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10. $x^2 + xy + y^2 - 2y = 3$

Differentiating: $2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} - 2\frac{dy}{dx} = 0$

$$\frac{dy}{dx}(2 - x - 2y) = 2x + y$$

$$\frac{dy}{dx} = \frac{2x + y}{2 - x - 2y}$$

When $x = 1$ and $y = 2$: $\frac{dy}{dx} = \frac{2+2}{2-1-4} = \frac{4}{-3} = -\frac{4}{3}$

Equation of tangent is $y - 2 = -\frac{4}{3}(x - 1)$

$$y - 2 = -\tfrac{4}{3}x + \tfrac{4}{3}$$

$$y = -\tfrac{4}{3}x + \tfrac{10}{3}$$

When $x = 0$ and $y = -1$: $\frac{dy}{dx} = \frac{-1}{2+2} = -\frac{1}{4}$

Gradient of normal $= 4$

Equation of normal is $y - (-1) = 4(x - 0)$

$$y + 1 = 4x$$

$$y = 4x - 1$$