Section 2: Parametric differentiation and integration

Section test

1. The gradient function $\frac{dy}{dx}$ for the curve $x = 4 + 2\cos\theta, y = 2 + 2\sin\theta$ is

(a) $-2\cot\theta$ (b) $-\cot\theta$
(c) $-\tan\theta$ (d) $-2\tan\theta$

2. A curve is defined by the parametric equations $x = 4\cos\theta, y = \sin 2\theta$.
The value of $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$ is:

(a) $\frac{1}{2\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}$
(c) $2\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

3. A curve is defined by the parametric equations
$x = \theta - \cos\theta, y = 1 - \sin\theta$.
The value of $\frac{dy}{dx}$ when $\theta = \frac{\pi}{6}$ is:

(a) -3 (b) $-3\sqrt{3}$
(c) $-\sqrt{3}$ (d) $-\frac{\sqrt{3}}{3}$

4. The equation of the tangent to the curve from question 3 at the point where $\theta = \frac{\pi}{6}$ is

(a) $18y + 6\sqrt{3}x - \sqrt{3}\pi = 0$ (b) $18y + 6\sqrt{3}x + \sqrt{3}\pi - 6 = 0$
(c) $18y - 6\sqrt{3}x - \sqrt{3}\pi = 0$ (d) $18y + 6\sqrt{3}x + \sqrt{3}\pi + 6 = 0$

5. The equation of the normal to the curve from question 3 at the point where $\theta = \frac{\pi}{6}$ is:

(a) $6y = 6\sqrt{3}x - \sqrt{3}\pi + 6$ (b) $6y = 6\sqrt{3}x - \sqrt{3}\pi + 12$
(c) $6y = -6\sqrt{3}x + \sqrt{3}\pi + 12$ (d) $6y = -6\sqrt{3}x + \sqrt{3}\pi + 6$

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6. A curve has parametric equations $x = 2t - 3, y = t^3 - t^2$.

   Find the gradient function $\frac{dy}{dx}$ for the curve.

   One of the turning points on the curve has coordinates (-3, 0). Find the coordinates of the other turning point.

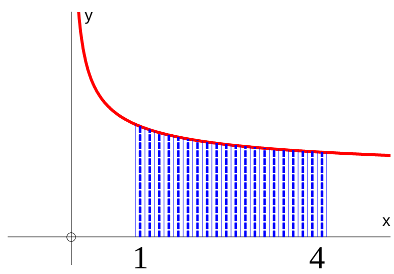
   What is the nature of the turning point at (-3, 0)?

7. A curve has parametric equations $x = 4t^2, y = 8t$.

   Find the equation of the tangent to the curve at the point with parameter t.

   Find the equation of the normal to the curve at the point with parameter t.

8. The diagram shows the curve with parametric equations $x = \frac{1}{t^2}, \quad y = t + 1$, for $t > 0$.



   Find the area of the shaded region.

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Section test solutions

1. $x = 4 + 2\cos\theta \Rightarrow \dfrac{dx}{d\theta} = -2\sin\theta$

$y = 2 + 2\sin\theta \Rightarrow \dfrac{dy}{d\theta} = 2\cos\theta$

Using the chain rule: $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{2\cos\theta}{(-2\sin\theta)} = -\cot\theta$

2. $x = 4\cos\theta \Rightarrow \dfrac{dx}{d\theta} = -4\sin\theta$

$y = \sin 2\theta \Rightarrow \dfrac{dy}{d\theta} = 2\cos 2\theta$

Using the chain rule: $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{2\cos 2\theta}{(-4\sin\theta)} = -\dfrac{\cos 2\theta}{2\sin\theta}$

When $\theta = \frac{\pi}{3}$: $\dfrac{dy}{dx} = -\dfrac{\cos\left(\frac{2\pi}{3}\right)}{2\sin\left(\frac{\pi}{3}\right)} = -\dfrac{\left(-\frac{1}{2}\right)}{\left(2 \times \frac{\sqrt{3}}{2}\right)} = \dfrac{1}{2\sqrt{3}}$

3. $x = \theta - \cos\theta \Rightarrow \dfrac{dx}{d\theta} = 1 + \sin\theta$

$y = 1 - \sin\theta \Rightarrow \dfrac{dy}{d\theta} = -\cos\theta$

Using the chain rule: $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{-\cos\theta}{(1 + \sin\theta)}$

When $\theta = \frac{\pi}{6}$: $\dfrac{dy}{dx} = \dfrac{-\cos\left(\frac{\pi}{6}\right)}{\left(1 + \sin\left(\frac{\pi}{6}\right)\right)} = \dfrac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(1 + \frac{1}{2}\right)} = -\dfrac{\sqrt{3}}{3}$

4. The gradient of the tangent is $-\dfrac{\sqrt{3}}{3}$.

The line goes through the point: $x = \frac{\pi}{6} - \cos\left(\frac{\pi}{6}\right) = \dfrac{\pi - 3\sqrt{3}}{6}$

$y = 1 - \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

Using $y - y_1 = m(x - x_1)$, the equation is: $y - \frac{1}{2} = -\frac{\sqrt{3}}{3}\left(x - \left(\frac{\pi - 3\sqrt{3}}{6}\right)\right)$

$$= -\frac{\sqrt{3}}{3}\left(x - \frac{\pi}{6} + \frac{3\sqrt{3}}{6}\right)$$

$$= -\frac{\sqrt{3}}{3}x + \frac{\sqrt{3}\pi}{18} - \frac{1}{2}$$

$$18y = -6\sqrt{3}x + \sqrt{3}\pi$$

$$18y + 6\sqrt{3}x - \sqrt{3}\pi = 0$$

5. The gradient of the normal is $\frac{3}{\sqrt{3}} = \sqrt{3}$

Using, $y - y_1 = m(x - x_1)$, the equation of the normal is:

$$y - \frac{1}{2} = \sqrt{3}\left(x - \left(\frac{\pi - 3\sqrt{3}}{6}\right)\right)$$

$$= \sqrt{3}\left(x - \frac{\pi}{6} + \frac{3\sqrt{3}}{6}\right)$$

$$= \sqrt{3}x - \frac{\sqrt{3}\pi}{6} + \frac{3}{2}$$

$$y = \sqrt{3}x - \frac{\sqrt{3}\pi}{6} + \frac{3}{2} + \frac{1}{2}$$

$$6y = 6\sqrt{3}x - \sqrt{3}\pi + 12$$

6. $x = 2t - 3 \Rightarrow \frac{dx}{dt} = 2$

$y = t^3 - t^2 \Rightarrow \frac{dy}{dt} = 3t^2 - 2t$

Using the chain rule: $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3t^2 - 2t}{2} = \frac{3}{2}t^2 - t$

At turning points, gradient = 0 $\Rightarrow \frac{3}{2}t^2 - t = 0$

$$\Rightarrow 3t^2 - 2t = 0$$

$$\Rightarrow t(3t - 2) = 0$$

$$t = 0 \text{ or } t = \tfrac{2}{3}$$

When t = 0, x = -3 and $y = 0$

When $t = \frac{2}{3}$, $x = \frac{4}{3} - 3 = -\frac{5}{3}$ and $y = \left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right)^2 = \frac{8}{27} - \frac{4}{9} = -\frac{4}{27}$

The coordinates of the second turning point are $\left(-\frac{5}{3},-\frac{4}{27}\right)$.

For $x < 3$, $t < 0$

At $t = -0.1$, $\frac{dy}{dx}=\frac{3}{2}\times(-0.1)^2-(-0.1)=0.115>0$

For $x > 3$, $t > 0$

At $t = 0.1$, $\frac{dy}{dx}=\frac{3}{2}\times(0.1)^2-(0.1)=-0.015<0$

The gradient goes from positive to negative.
Therefore, this turning point is a maximum.

7. $x=4t^2 \Rightarrow \frac{dx}{dt}=8t$

$y=8t \Rightarrow \frac{dy}{dt}=8$

Using the chain rule: $\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{8}{8t}=\frac{1}{t}$

Using $y-y_1=m(x-x_1)$, the general equation is:

$$y-8t=\frac{1}{t}\left(x-4t^2\right)$$
$$ty-8t^2=x-4t^2$$
$$ty=x+4t^2$$

The gradient of the normal at the point t is $-\frac{1}{1/t}=-t$

Using $y-y_1=m(x-x_1)$, the general equation of the normal is:

$$y-8t=-t\left(x-4t^2\right)=4t^3-tx$$
$$y+tx-4t^3-8t=0$$

8. When $x=1$, $1=\frac{1}{t^2}$ $\Rightarrow t^2=1$ $\Rightarrow t=1$ (since $t > 0$)

When $x=4$, $4=\frac{1}{t^2}$ $\Rightarrow t^2=\frac{1}{4}$ $\Rightarrow t=\frac{1}{2}$ (since $t > 0$)

$x=t^{-2}$ $\Rightarrow \frac{dx}{dt}=-2t^{-3}$

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$$\begin{aligned}
\text{Area} &= \int_1^{\frac{1}{2}} y\frac{dx}{dt}\,dt \\
&= \int_1^{\frac{1}{2}} (t+1)\times -2t^{-3}\,dt \\
&= \int_1^{\frac{1}{2}} \left(-2t^{-2}-2t^{-3}\right)dt \\
&= \left[2t^{-1}+t^{-2}\right]_1^{\frac{1}{2}} \\
&= (4+4)-(2+1) \\
&= 5 \text{ square units}
\end{aligned}$$